Energy Price Shocks, Unemployment, and Monetary Policy*

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Abstract

This paper studies the optimal conduct of monetary policy in the presence of heterogeneous exposure to energy price shocks between the employed and the unemployed, as it is documented by data from the euro-area Consumer Expectations Survey: higher energy prices weigh more on the unemployed, who consume less and devote a higher proportion of their consumption to energy. I account for this evidence into a tractable Heterogeneous-Agent New Keynesian (HANK) model with Search and Matching (S&M) frictions in the labour market, and energy as a complementary input in production and as a non-homothetic consumption good: energy price shocks weigh more on the jobless, who consume less due to imperfect unemployment insurance and, since preferences are non-homothetic, devote a higher share of this lower consumption to energy. Households' heterogeneous exposure to rising energy prices induces an endogenous trade-off for monetary policy, whose optimal response involves partly accommodating core inflation so as to indirectly sustain employment and, therefore, prevent workers from becoming more exposed to the shock through unemployment.

JEL Classification: E21, E24, E31, E32, E52

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1 Introduction

Since the end of 2021 and through mid-2023, euro-area economies have had to cope with exceptionally high inflation, both energy and overall, and depressed real wages (Figure 1). The recent surge in energy prices has not only depressed workers' real income but has also had distributional consequences. Indeed, this type of shock affects consumers heterogeneously: as energy is a necessity good, poorer households devote a larger share of their income to energy consumption and are hit harder when the price increases.[1] Given that workers suffer substantial losses in income and consumption during unemployment spells,[2] the latter constitute a channel through which households can become more exposed to rising energy prices. Exploiting data from the euro-area Consumer Expectations Survey, I document how the unemployed allocate a significantly higher share of their overall expenditure in goods and services to energy-intensive consumption (utilities and transport services) compared to the employed. Moreover, this disparity widened further after the shock hit, despite the fact that most countries implemented several measures aimed at relieving the pressure on poorer and, hence, more exposed households (Arregui et al., 2022).

In light of these facts, it appears crucial to understand how monetary authorities can influence household exposure to energy price shocks via the unemployment rate, given that the latter is one of the critical indicators guiding monetary policy decisions. This paper studies how monetary policy should optimally react to these shocks through the lens of a tractable Heterogeneous-Agent New Keynesian (HANK) model with Search and Matching (S&M) frictions in the labour market and energy as a complementary input in production and a non-homothetic consumption good, wherein employed and unemployed workers are heterogeneously affected by energy price shocks due to the joint presence of imperfect unemployment insurance and non-homothetic preferences. More specifically, and differently from the homothetic case previously considered in the HANK-S&M literature,[3] consumption is limited to what remains after covering subsistence energy needs. Hence, lower consumption will be associated with a higher energy consumption share. In other words, due to the presence of imperfect unemployment insurance, the unemployed are forced to consume less, and, in addition, non-homothetic energy consumption implies that they devote a higher share of their consumption to energy compared to the employed. Therefore, higher energy prices end up weighing more on the jobless and, due to the presence of a frictional labour market, employed

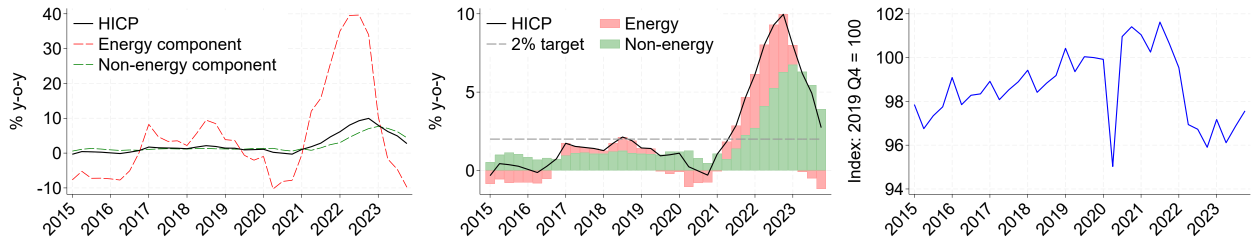
[1] Recent studies that have noted the distributional impact of energy price surges due to their effect on heterogeneous consumption baskets include Ari et al. (2022); Bachmann et al. (2022); Battistini et al. (2022); Curci et al. (2022).

[2] Various studies have documented these consumption losses to range from 14% to 26% (Den Haan et al., 2018).

[3] See, for instance, Ravn and Sterk (2021) and Challe (2020).

Figure 1: Inflation and Real Wages in the Euro Area

(a) Inflation Developments (b) Inflation Contributions (c) Real Wages



Notes: The left panel of the figure reports quarterly averages of monthly year-on-year percentage changes in the euro-area Harmonised Index of Consumer Prices (HICP) and in its energy and non-energy components, while the middle panel reports the weighted contributions of the two components to the changes of the overall index; the HICP "energy" classification includes electricity, gas, liquid fuels, solid fuels, heat energy, and fuels and lubricants for personal transport equipment. The right panel reports the average compensation per employee in the euro area, deflated using the HICP and normalised at 100 in 2019 Q4. Latest observations: 2023 Q4. *Sources:* ECB and Eurostat.

workers also can transition endogenously to unemployment and hence become more exposed to the shock. In turn, the monetary authority can indirectly mitigate the share of more exposed households by trading off higher inflation with lower unemployment.

I show analytically, by relying on a simple positive illustrative case, how rising energy prices induce firms to reduce activity and employment when energy is a complementary input in production. Moreover, in the presence of imperfect insurance, this response is amplified through both an unemployment risk channel and the heterogeneous exposure to the shock between the employed and the unemployed. First, as unemployment risk increases and households cannot perfectly insure against it, precautionary saving desires also increase, and aggregate demand is depressed. This, in turn, sets in motion the feedback loop already highlighted, among others, by Ravn and Sterk (2021), whereby production falls even further, feeding back to even greater unemployment risk, and so on. Second, and on top of this, due to the presence of non-homothetic preferences, the increase in real energy prices raises the consumption losses upon unemployment, strengthening unemployment fears and precautionary saving motives, further depressing aggregate demand and employment.

These two sources of amplification of the unemployment response to rising energy prices — unemployment risk and the heterogeneous exposure to the shock— pose, in turn, the normative question of how monetary policy should optimally react. First, by relying on the same simplified case for fully analytical characterisations, I show that fluctuations in real energy prices endogenously induce a monetary policy trade-off between stabilising core inflation and maintaining unemployment at its desired, constrained-efficient level. This trade-off originates as an endogenous wedge between natural unemployment (the level consistent with stable core inflation) and constrained-efficient unemployment (the welfare-maximising level). Intuitively, the presence of subsistence energy consumption and imperfect unemployment insurance jointly imply that employed and unemployed workers are heterogeneously exposed to energy price shocks. Consequently, rising energy prices induce an increase in the consumption losses upon unemployment. As a result, it is optimal for the monetary authority to partly accommodate core inflation in order to indirectly sustain employment. This helps to limit the increase in unemployment induced by the shock hitting producers, thereby preventing households from becoming more exposed to the shock through unemployment.

Second, I numerically explore the mechanism for the general case, confirming the analytical results: compared to a policy rule aimed at fully stabilising core inflation, the optimal policy is able to achieve a smaller increase in unemployment at the cost of partly accommodating core inflation; this rise in inflation is, however, smaller than if unemployment were instead to be fully stabilised. It is also confirmed that the trade-off arises due to the presence of heterogeneous exposure to the energy price shock between the employed and the unemployed: considering the counterfactual case of homothetic preferences and hence homogeneous direct exposure to the shock, the optimal policy coincides with one aimed at fully stabilising core inflation. In both cases, while the monetary authority reacts more or less aggressively to core inflation, the energy component of inflation is instead fully looked through under the optimal policy. Intuitively, only core inflation endogenously matters for the hiring incentives of producers, being only the core goods produced within the economy. In contrast, energy is treated as a non-produced good, with its (real) price evolving exogenously. This simplifying assumption, in line with the approach of Blanchard and Galí (2007), concisely captures the situation faced by an energy-importing economy such as the euro area.

Related Literature. This paper contributes, first, to the macroeconomic literature on the impact of energy shocks on producers and consumers: earlier studies (Blanchard and Galí, 2007; Bodenstein et al., 2008; Montoro, 2012; Blanchard and Riggi, 2013) have considered representative-agent settings, providing the normative insight that central banks should react aggressively to core inflation but fully accommodate energy price changes (Natal, 2012). My paper moves forward in this context by making a case for not acting too hard on core inflation, as excess unemployment would make heterogeneously exposed households worse off. Gagliardone and Gertler (2023), still using a representative-agent setting with oil as a complementary good in consumption and complementary input in production, show that the recent inflation surge was mainly accounted for by a combination of oil price shocks and accommodative monetary policy. In their framework, which also allows for unemployment, a rationale for part of such accommodation is the fact that monetary policy trade-offs arise due to real wage rigidity, while in my heterogeneous-household setting they arise not only due to this but, first and foremost, because energy shocks weigh more on the unemployed. Recent studies have also taken household heterogeneity into account, for instance in Two-Agent New Keynesian (TANK) frameworks à la Debortoli and Galí (2017): Chan et al. (2023) investigate how energy shocks impacting producers can feed back to depressed demand, but without considering how energy shocks *directly* affect consumers, an aspect which I explicitly model with liquidity-constrained households being more exposed to these shocks due to non-homothetic energy consumption. Corsello and Riggi (2023) develop a TANK framework with energy in production and heterogeneously exposed households, as the hand-to-mouth devote a higher (constant) share of their consumption expenditure to energy,[4] hence ending up experiencing a higher inflation rate. Their analysis is positive rather than normative, offering a historical decomposition of inflation inequality into its main drivers, including the monetary policy stance. Kharroubi and Smets (2023) introduce a minimum subsistence requirement for energy consumption to explore optimal *fiscal* responses to an energy shock in a two-agent framework, and advocate subsidising poorer, constrained households while taxing those that are rich and unconstrained. My analysis delves into optimal monetary policy, keeping fiscal policy constant. Moreover, while these Two-Agent frameworks assume a constant fraction of hand-to-mouth households, in my setting, the fraction of agents ending up being liquidity-constrained is endogenous and microfounded, as it is captured by the unemployment rate. In such context, Kharroubi and Smets' fiscal approach cannot be easily translated since it would involve taxing the employed and subsidising the unemployed, potentially dampening job-search incentives. Other recent studies have accounted for household heterogeneity within quantitative HANK models à la Kaplan et al. (2018): Pieroni (2023) introduces non-homothetic preferences in a similar fashion as I do, but in a rich quantitative model with energy consumption by both households and firms, thus relying only on numerical solutions and focusing on positive analysis. In my work, I consider instead a specific dimension of household heterogeneity —stemming from imperfect unemployment insurance— which allows me to characterise and investigate analytically the mechanisms at the source of monetary policy trade-offs with their normative implications.

Second, I build on the recent literature studying the aggregate effects of incomplete markets in the form of imperfectly insurable unemployment risk within tractable HANK-S&M models (Challe

[4] Their setting aligns with Känzig's (2023), who rationalises his findings that a tighter carbon pricing regime results in higher energy prices, disproportionately affecting the poor due to their higher energy share. Given that Känzig's shocks are, essentially, energy price shocks, their effects can also be rationalised within my framework.

et al., 2017; Ravn and Sterk, 2017, 2021; Den Haan et al., 2018; McKay and Reis, 2021). My work relates in particular to Challe (2020), who has investigated the optimal conduct of monetary policy within this class of models in response to productivity shocks and generic cost-push shocks.

I contribute to these strands of literature, in the first instance, by modelling energy as an input in both production and consumption within a tractable HANK-S&M framework and innovating on the existing studies by uncovering a novel precautionary saving motive and source of monetary policy trade-off, both arising endogenously due to non-homothetic preferences and imperfect insurance, focusing on normative implications analytically as well as quantitatively. In this vein, my work also relates to Acharya et al. (2023), who study how (optimal) monetary policy can mitigate consumption risk arising from households' unequal exposure to aggregate shocks in a tractable HANK model. In a similar spirit, Smirnov (2022) shows numerically, using a rich quantitative HANK model, how optimal policy aims to mitigate this unequal exposure.

Roadmap. The remainder of this paper is structured as follows. Section 2 provides motivating evidence. Section 3 describes the model, and Section 4 characterises its constrained-efficient allocation. Section 5 focuses on the linearised version of the model and deals with positive analysis, while normative analysis is performed in Section 6. Section 7 concludes.

2 Motivating Evidence

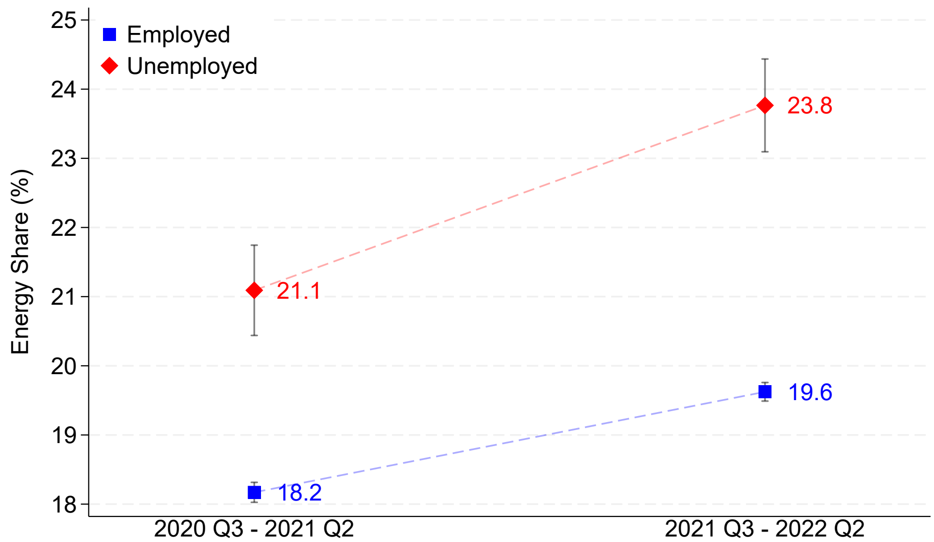
In this section, I exploit the microdata from the European Central Bank's Consumer Expectations Survey (CES) to document some facts about the heterogeneity between the employed and the unemployed in their exposure to energy price shocks. The CES covers the six largest economies in the euro area (Germany, France, Italy, Spain, the Netherlands, and Belgium) over the period since April 2020. On a quarterly basis, it collects information on individual monthly spending in twelve major categories, including energy-intensive ones (utilities and transport services), as well as their employment situation.[5] This allows to compute an individual-level measure of exposure to energy price shocks as the share of energy-intensive categories in overall expenditure on goods and services.

Figure 2 reports the average energy shares of the employed and the unemployed over time, controlling for individual-specific observable characteristics.[6] Between late 2020 and early 2021

[5]The "utilities" category includes water, sewerage, electricity, gas, heating oil, phone, cable, internet; the "transport" category includes fuel, car maintenance, public transportation fares. Information about the employment situation is available from the October 2020 wave of the survey. Appendix A provides a more detailed description of the data.

[6]These include age, gender, birth abroad, education, presence of a partner. See Appendix A for more details.

Figure 2: Energy Shares of the Employed and the Unemployed



Notes: The figure reports the average shares of energy-intensive consumption expenditure over time for employed and unemployed individuals interviewed in the CES, obtained in regressions of the form $e_{it} = \beta_0 + \beta_1\, une_{it} + X_{it}\,\gamma + \varepsilon_{it}$, where e_{it} is the energy share of individual i at time t, $une_{it} = 1$ if the individual is employed and zero if employed, and X_{it} controls for individual-specific characteristics (age, gender, birth abroad, education, presence of a partner). The vertical bars indicate 95% confidence intervals.

—before the surge in energy prices— the share was around 18% for the employed and about 3 percentage points higher for the unemployed. As energy prices surged, the share rose to nearly 20% for the employed and around 4 percentage points higher for the unemployed. Therefore, even if many countries have implemented fiscal measures targeted at mitigating the impact of the shock on vulnerable households (Arregui et al., 2022), the energy share for the unemployed has increased by around 1 percentage point more than for the employed.

To investigate in more depth the presence of different consumption patterns between the employed and the unemployed, in particular on energy-intensive consumption, I then focus on the period before the energy price surge (2020 Q3 to 2021 Q2) and look at average differences in the expenditure shares of the unemployed relative to that of the employed as well as at average percentage differences in overall consumption, utilities, and transport services expenditure.

As shown in Table 1, controlling for individual-specific observable characteristics (age, gender, birth abroad, education, presence of a partner), the employed devoted, on average, 11.2% of their overall consumption expenditure to utilities, while this share was 3.1 percentage points higher for the unemployed. This is due to the fact that while overall consumption expenditure was

Table 1: Energy-intensive Consumption and Unemployment

(a) Expenditure

	ln(*consumption*)	ln(*utilities*)	ln(*transports*)	ln(*energy*)
unemployed	-0.247	-0.043	-0.359	-0.124
	[-0.283,-0.212]	[-0.082,-0.005]	[-0.465,-0.252]	[-0.166,-0.082]

(b) Expenditure Shares

	utilities share	transports share	total energy share
unemployed	0.031	-0.002	0.029
	[0.025,0.037]	[-0.005,0.001]	[0.022,0.036]
employed (baseline)	0.112	0.070	0.182
	[0.111,0.113]	[0.069,0.071]	[0.180,0.183]

Notes: The table reports results of regressions of the form $y_{it} = \beta_0 + \beta_1\, une_{it} + X_{it}\,\gamma + \varepsilon_{it}$, run on 20899 observations from the CES for the period from 2020 Q3 to 2021 Q2. y_{it} is, alternatively, the share of energy-intensive items (utilities and transport services) in the overall consumption expenditure of individual i at time t, the log of their overall consumption expenditure, log expenditure on utilities, transport services, and their sum. $une_{it} = 1$ when individual i is unemployed at time t and 0 when employed. X_{it} controls for individual-specific characteristics (age, gender, birth abroad, education, presence of a partner). 95% confidence intervals are reported in brackets.

$\exp\{-0.247\} - 1 \simeq 22\%$ lower for the unemployed compared to the employed, expenditure on utilities was only around 4% lower. Moreover, despite the fact that expenditure on transport services was around 30% lower for the unemployed, their expenditure share of energy-intensive consumption (utilities plus transport services) was nevertheless around three percentage points higher than that of the employed, for whom it was slightly above 18%.

3 The Model

The model tracks a sticky-price economy with uninsurable unemployment risk, along the lines of Challe (2020), enriched by introducing energy as an input both in production and in the consumption bundle. Following Blanchard and Galí (2007) and most of the subsequent literature on the macroeconomic effects of oil and energy shocks, energy is regarded as a non-produced input whose real price evolves according to an exogenous stochastic process.

On the household side, there is a unit measure of workers who, due to their inability to borrow, cannot perfectly insure against the risk of becoming unemployed, and who derive utility from a basket of a core consumption good and energy. The latter is consumed above a subsistence level, implying that poorer households (i.e. the unemployed, in this setting) will devote a higher share of their overall consumption expenditure to energy. Additionally, there is a measure ν of risk-neutral firm owners who simply collect and consume hand-to-mouth the dividends arising in the production sector, net of fiscal transfers.

On the producer side, the core good is assembled by aggregating differentiated varieties from a monopolistically competitive wholesale sector. Each wholesaler uses, in turn, energy and labour services as inputs. These services are supplied competitively by intermediaries who hire labour from households in a market with standard S&M frictions à la Mortensen and Pissarides (1994).[7]

3.1 Households

Workers face idiosyncratic income risk from being either employed or unemployed: due to imperfect insurance, the employed and the unemployed will have different consumption levels at equilibrium.

The consumption basket of each worker type $i \in \{n, u\}$ (employed, unemployed), c_t^i, is defined as a Stone-Geary aggregator of a core consumption good, g_t^i, and energy, e_t^i, which is consumed above a subsistence level $\xi > 0$,

$$c_t^i = (g_t^i)^{(1-\omega_e)}(e_t^i - \xi)^{\omega_e} \tag{1}$$

where $\omega_e \in (0,1)$ is the quasi-share of energy. Allowing for the presence of the subsistence level ξ will imply, consistently with the empirical evidence, that energy consumption demand is inelastic and that poorer households (i.e. the unemployed, in this setting) will devote a higher share of their income to energy consumption.[8]

The expected lifetime utility of a currently employed (resp. unemployed) worker can be formulated recursively as follows

$$U_t^n = \ln(c_t^n) + \beta\,\mathbb{E}_t\left[(1-\lambda_{t+1})\,U_{t+1}^n + \lambda_{t+1}\,U_{t+1}^u\right]$$
$$U_t^u = \ln(c_t^u) + \beta\,\mathbb{E}_t\left[f_{t+1}\,U_{t+1}^n + (1-f_{t+1})\,U_{t+1}^u\right]$$

where λ is the transition rate from employment to unemployment, while f is the job-finding rate.

[7] Introducing labour market frictions in a separate, upstream sector is isomorphic to considering these frictions affecting wholesalers who directly hire labour from households.

[8] See Appendix B.2 for more details about the properties of consumer demand in the presence of the non-homotheticities of Stone-Geary preferences.

Each worker type maximises expected lifetime utility subject to (1) and

$$\begin{cases} P_{g,t}\, g_t^i + P_{e,t}\, e_t^i + B_t^i = Y_t^i + (1+i_{t-1})\, B_{t-1}^i \\ B_t^i \geq 0 \end{cases}$$

where $P_{e,t}$ is the price of energy and $P_{g,t}$ is the price of the core consumption good; B_t denotes the nominal amount of 1-period bonds held at the end of period t, and i_t is the nominal return on these assets. Y_t^i is equal either to W_t (the nominal wage income a worker receives when employed) or Δ_t (nominal home production when unemployed).

Demands for the core consumption good and energy are given by

$$g_t^i = (1-\omega_e)\left(\frac{P_t}{P_{g,t}}\right) c_t^i \tag{2}$$

$$e_t^i = \omega_e \left(\frac{P_t}{P_{e,t}}\right) c_t^i + \xi \tag{3}$$

where

$$P_t = \left(\frac{P_{g,t}}{1-\omega_e}\right)^{(1-\omega_e)} \left(\frac{P_{e,t}}{\omega_e}\right)^{\omega_e} \tag{4}$$

represents the overall consumer price index (CPI). Letting $\pi_{t+1} := \frac{P_{t+1}}{P_t} - 1$, to a first order

$$\pi_t \simeq (1-\omega_e)\, \pi_{g,t} + \omega_e\, \pi_{e,t}\,. \tag{5}$$

Euler conditions for employed and unemployed workers are, respectively,

$$\frac{1}{c_t^n} \geq \beta\, \mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}}\right) \left[(1-\lambda_{t+1})\, \frac{1}{c_{t+1}^n} + \lambda_{t+1}\, \frac{1}{c_{t+1}^u}\right]\right\} \tag{6}$$

$$\frac{1}{c_t^u} \geq \beta\, \mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}}\right) \left[f_{t+1}\, \frac{1}{c_{t+1}^n} + (1-f_{t+1})\, \frac{1}{c_{t+1}^u}\right]\right\} \tag{7}$$

each holding with strict inequality if the agent is liquidity-constrained (i.e. wishing to borrow) and with equality otherwise. Due to the presence of non-homothetic preferences, captured by $\xi > 0$, consumption does not equal real overall expenditure on goods and is instead given by

$$c_t^i = \frac{X_t^i}{P_t} - \frac{P_{e,t}}{P_t}\, \xi$$

where $X_t^i = P_{g,t}\, g_t^i + P_{e,t}\, e_t^i$. Intuitively, households derive utility only from what is left over after spending on energy for subsistence purposes. This form of non-homotheticity has the advantage of linking the presence of different consumption patterns directly to the inability to perfectly insure against job losses, which will force the unemployed to spend less on consumption at equilibrium.

Employment Dynamics. Worker transitions between employment and unemployment give rise to the following laws of motion for the stocks of employed (n_t) and unemployed (u_t) workers

$$n_{t+1} = (1 - \lambda_{t+1})\, n_t + f_{t+1}\, u_t$$
$$u_{t+1} = (1 - f_{t+1})\, u_t + \lambda_{t+1}\, n_t$$

where $u_t = 1 - n_t$. Being $s_{t+1} = u_t + \rho\, n_t$ the stock of effective searchers, $\lambda_{t+1} = \rho\,(1 - f_{t+1})$, where ρ is the separation rate. In other words, a worker employed in the current period faces a risk of being unemployed in the next period given by λ_{t+1}, the joint probability of undergoing separation, and not being able to find another job subsequently.

3.2 Producers

The core consumption good is produced under perfect competition by aggregating a continuum of wholesale goods with a constant elasticity of substitution technology

$$Y_t = \left(\int_0^1 y_t(k)^{\frac{\varepsilon-1}{\varepsilon}}\, dk \right)^{\frac{\varepsilon}{\varepsilon-1}} . \tag{8}$$

Therefore, demand for variety k is given by

$$y_t(k) = Y_t \left[\frac{P_t(k)}{P_{g,t}} \right]^{-\varepsilon} \tag{9}$$

where $P_{g,t} = \left(\int_0^1 P_t(k)^{1-\varepsilon} dk \right)^{1/(1-\varepsilon)}$, and $\varepsilon > 1$.

Wholesalers

Each wholesaler is a monopolistic supplier of the variety k it produces using labour services as well as energy, combined in fixed proportions according to the following production function

$$y_t(k) = \min \left\{ \frac{l_t(k)}{1 - \gamma_e}, \frac{e_t(k)}{\gamma_e} \right\}$$

where $y_t(k)$ is the amount of variety k produced, $e_t(k)$ and $l_t(k)$ are respectively the amounts of energy and intermediate inputs used in production by wholesaler k, and $\gamma_e \in (0,1)$ captures the factor proportion of energy relative to labour services.

Demand for inputs from wholesalers will then be given by

$$l_t(k) = (1-\gamma_e)\, y_t(k) \tag{10}$$

$$e_t(k) = \gamma_e\, y_t(k) \tag{11}$$

and the nominal marginal cost by

$$MC_t = (1-\gamma_e)\,\Phi_t + \gamma_e\, P_{e,t} \tag{12}$$

where Φ_t denotes the price of labour services and $P_{e,t}$ is the energy price.

Moreover, wholesale firms are assumed to face Calvo pricing frictions, with θ being the probability that a wholesale firm cannot reset its price. Therefore, the optimal reset price, P_t^*, satisfies

$$p_{g,t}^* = \frac{P_t^*}{P_{g,t}} = \frac{\mathcal{Y}_t}{\mathcal{Z}_t} \tag{13}$$

where $\mathcal{Y}_t$ and $\mathcal{Z}_t$ obey the following recursions

$$\mathcal{Y}_t = (1-\tau_y)\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{MC_t}{P_t}\,Y_t + \theta\,\beta\,\mathbb{E}_t\left[(1+\pi_{g,t+1})^{\varepsilon}\,\mathcal{Y}_{t+1}\right]$$
$$\mathcal{Z}_t = \frac{P_{g,t}}{P_t}\,Y_t + \theta\,\beta\,\mathbb{E}_t\left[(1+\pi_{g,t+1})^{(\varepsilon-1)}\,\mathcal{Z}_{t+1}\right]$$

and τ_y is a production subsidy, financed through a lump-sum tax on firm owners, aimed at offsetting the steady-state distortion due to monopolistic competition.

Given Calvo pricing frictions and symmetry in the wholesale sector, gross inflation in the core consumption good price, $1+\pi_{g,t} = P_{g,t}/P_{g,t-1}$, evolves according to

$$1+\pi_{g,t} = \left[\frac{1}{\theta} - \left(\frac{1-\theta}{\theta}\right)\left(p_{g,t}^*\right)^{(1-\varepsilon)}\right]^{1/(\varepsilon-1)}. \tag{14}$$

Real dividends collected by firm owners from wholesaler k are then given by

$$d_t^W(k) = \frac{1}{P_t}\left[P_t(k)\,y_t(k) - (1-\tau_y)\,\Phi_t\,l_t(k) - (1-\tau_y)\,P_{e,t}\,e_t(k)\right] \tag{15}$$

and aggregate real dividends from the wholesale sector by

$$d_t^W = \int_0^1 d_t^W(k)\, dk = Y_t \, \{p_{g,t} - (1-\tau_y)\, [(1-\gamma_e)\, \varphi_t + \gamma_e\, p_{e,t}]\, \mathcal{D}_t\} \tag{16}$$

where $p_{g,t} = P_{g,t}/P_t$, $\varphi_t = \Phi_t/P_t$, and

$$\mathcal{D}_t := \int_0^1 \left[\frac{P_t(k)}{P_{g,t}}\right]^{-\varepsilon} dk = (1-\theta)\, \left(p_{g,t}^*\right)^{-\varepsilon} + \theta\, \Pi_{g,t}^{\varepsilon}\, \mathcal{D}_{t-1} \tag{17}$$

is an index of price dispersion among wholesalers.

Labour Intermediaries

Intermediaries hire labour from households in a frictional market. These frictions are summarised by an aggregate matching function, which is assumed to take a Cobb-Douglas form with constant returns to scale,

$$m_t = s_t^{\alpha}\, v_t^{(1-\alpha)} \tag{18}$$

where m_t denotes the total amount of formed matches, v_t is the total amount of vacancies, $\alpha \in (0,1)$, and $s_t = u_{t-1} + \rho\, n_{t-1}$ is the total amount of searching workers, given by those workers who were unemployed plus those workers who were employed but experience separation (at rate ρ). The job-finding and vacancy-filling rates are given, respectively, by

$$f_t = \frac{m_t}{s_t}\,, \tag{19}$$

$$q_t = \frac{m_t}{v_t} = f_t^{\frac{\alpha}{\alpha-1}}\,. \tag{20}$$

Active matches produce one unit of output at each period. The value of a match is

$$J_t = (1-\tau_z)\, (\varphi_t - w_t + S) + \beta\, (1-\rho)\, \mathbb{E}_t\, (J_{t+1}) \tag{21}$$

where $w_t = W_t/P_t$ is the real wage rate, and $\varphi_t = \Phi_t/P_t$ is the price of labour services in terms of the final good. τ_z and S are fiscal instruments that will be aimed at offsetting the steady-state distortions arising from labour market frictions and imperfect unemployment insurance.

When matches break up, inactive intermediaries post vacancies at a cost of κ units of the final good per vacancy per period, and each vacancy is filled with probability q_t. Free entry into vacancy posting gives rise to the following job creation condition

$$f_t^{\frac{\alpha}{1-\alpha}} = \frac{1-\tau_z}{\kappa} \, (\varphi_t - w_t + S) + \beta \, (1-\rho) \, \mathbb{E}_t \left(f_{t+1}^{\frac{\alpha}{1-\alpha}} \right) . \tag{22}$$

Aggregate real dividends collected by firm owners from labour intermediaries are then given by

$$d_t^I = n_t \, (1-\tau_z) \, (\varphi_t - w_t + S) - \kappa \, v_t \, . \tag{23}$$

3.3 Equilibrium

3.3.1 Market Clearing

Energy Market. Following Blanchard and Galí (2007), energy is modelled as a non-produced input whose real price is assumed to evolve exogenously according to

$$\ln(p_{e,t}) = \rho_e \, \ln(p_{e,t-1}) + \epsilon_t^e \tag{24}$$

where $\rho_e \in [0,1)$ and $\epsilon_t^e \sim iid(0, \sigma_e^2)$. In other words, the aggregate market demand for energy from households and firms is assumed to be always cleared at the exogenous real price $p_{e,t}$.

Labour Market. The total supply of labour services is given by the measure of active matches, n_t, while total demand can be obtained by aggregating over wholesalers. Given (9) and (10), we have

$$\int_0^1 l_t(k) = \int_0^1 (1-\gamma_e) \, y_t(k) = (1-\gamma_e) \, Y_t \, \mathcal{D}_t \, . \tag{25}$$

Therefore, labour market clearing requires

$$n_t = (1-\gamma_e) \, Y_t \, \mathcal{D}_t \, . \tag{26}$$

Good Market. The supply of the core consumption good is given by Y_t. Its total demand, $n_t \, g_t^n + (1-n_t) \, g_t^u + \nu \, g_t^o$, is obtained by aggregating over employed and unemployed workers and firm owners. Given (2), market clearing then requires

$$(1-\omega_e) \, [n_t \, c_t^n + (1-n_t) \, c_t^u + \nu \, c_t^o] = p_{g,t} \, Y_t \tag{27}$$

where c_t^n , c_t^u , c_t^o are, respectively, the final consumption baskets of employed and unemployed workers and firm owners.

3.3.2 Household Consumption and Zero Liquidity Property

Workers. Given the zero-debt limit households face, the supply of assets is always zero at equilibrium, no asset trade actually takes place, and all households turn out to spend all their current income on consumption. Therefore, letting $\delta_t = \Delta_t / P_t$ denote real home production when unemployed, the consumption bundles of the employed and the unemployed are, respectively,

$$c_t^n = w_t - p_{e,t}\,\xi \tag{28}$$

$$c_t^u = \delta_t - p_{e,t}\,\xi \tag{29}$$

from which it is immediate to see the following,

Proposition 1. *If $\delta_t < w_t$, then $c_t^u < c_t^n$ and a rise in the real price of energy implies that the consumption of the unemployed falls proportionately more than the consumption of the employed:*

$$\frac{\partial\, c_t^u \,/\, c_t^u}{\partial\, p_{e,t} \,/\, p_{e,t}} \;<\; \frac{\partial\, c_t^n \,/\, c_t^n}{\partial\, p_{e,t} \,/\, p_{e,t}} \;<\; 0\,.$$

In light of (28) and (29), the Euler conditions of the employed and the unemployed are, respectively,

$$1 \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[(1-\lambda_{t+1}) \left(\frac{w_t - p_{e,t}\,\xi}{w_{t+1} - p_{e,t+1}\,\xi} \right) + \lambda_{t+1} \left(\frac{w_t - p_{e,t}\,\xi}{\delta_{t+1} - p_{e,t+1}\,\xi} \right) \right] \right\} \tag{30}$$

$$1 \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[f_{t+1} \left(\frac{\delta_t - p_{e,t}\,\xi}{w_{t+1} - p_{e,t+1}\,\xi} \right) + (1-f_{t+1}) \left(\frac{\delta_t - p_{e,t}\,\xi}{\delta_{t+1} - p_{e,t+1}\,\xi} \right) \right] \right\} \tag{31}$$

each holding with strict inequality if the household is liquidity-constrained (i.e. wishing to borrow) and with equality otherwise.

It can be shown that at equilibrium, in the steady state neighbourhood, the Euler condition of employed workers holds with equality while that of the unemployed holds with inequality.[9] Formally:

Proposition 2. *When $\delta < w$, the steady-state Euler conditions of employed and unemployed*

[9]See Ravn and Sterk (2021) and Challe (2020) for more details.

workers are, respectively,

$$1 = \beta \left(\frac{1+i}{1+\pi}\right) \left[(1-\lambda) + \lambda \left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right)\right]$$
$$1 > \beta \left(\frac{1+i}{1+\pi}\right) \left[f \left(\frac{\delta - p_e\,\xi}{w - p_e\,\xi}\right) + (1-f)\right]$$

Proof. See Appendix B.3. □

In other words, at equilibrium, employed workers wish to precautionarily save, but as the unemployed are impeded from borrowing, no one is able to issue the assets that would allow this precautionary saving desire to be actually satisfied. Hence, no asset trade eventually takes place at equilibrium. This zero-liquidity property allows to price precautionary saving *desires* without the need to track a full wealth distribution over time.

Firm Owners. The final consumption basket of firm owners equals their after-tax real income, net of subsistence energy needs, $c_t^o = (d_t^I + d_t^W + \tau_t) - \xi\, p_{e,t}$, where

$$\tau_t = -\tau_y \left[(1-\gamma_e)\,\varphi_t + \gamma_e\, p_{e,t}\right] \mathcal{D}_t\, Y_t + \tau_z\, n_t\,(\varphi_t - w_t) - n_t\,(1-\tau_z)\, S\,. \tag{32}$$

Therefore, at equilibrium, firm owner consumption is

$$c_t^o = \left(\frac{p_{g,t}\,\mathcal{D}_t^{-1} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t\right) n_t - \kappa\, v_t - \xi\, p_{e,t} \tag{33}$$

where, given the equilibrium law of motion of employment, $n_t = (1-\rho)\,n_{t-1} + [1-(1-\rho)\,n_{t-1}]^{\alpha}\; v_t^{1-\alpha}$, v_t can be expressed as

$$v_t = \left\{\frac{n_t - (1-\rho)\,n_{t-1}}{[1-(1-\rho)\,n_{t-1}]^{\alpha}}\right\}^{\frac{1}{1-\alpha}}\,. \tag{34}$$

4 Constrained-Efficient Allocation

The constrained-efficient allocation is the solution to the problem of maximising the aggregate welfare of the economy, taking into account the decentralised equilibrium reactions by consumers and producers. Considering a utilitarian planner who attaches equal weights to the utility of each household, the flow welfare of the economy is given by

$$U_t = n_t\, \ln(c_t^n) + (1-n_t)\, \ln(c_t^u) + \nu\, c_t^o \tag{35}$$

where c_t^n , c_t^u ,and c_t^o are given, respectively, by (28), (29), and (33). Therefore, the constrained-efficient allocation is the solution to

$$W(\mathcal{D}_{t-1}, n_{t-1}, p_{e,t}) = \max_{\{p_{g,t}^*, w_t, n_t\}} \{U_t + \beta \, \mathbb{E}_t[W(\mathcal{D}_t, n_t, p_{e,t+1})]\} \tag{36}$$

subject to (14) and (17).

Now, $p_{g,t}^* = 1$ at every period is optimal, as it implies $\pi_{g,t} = 0$, thereby ensuring that $\mathcal{D}_t = 1$: zero inflation eliminates the dispersion in wholesale prices, that would imply lower output from given inputs, as can be seen from (26). Therefore, the problem reduces to

$$W(n_{t-1}, p_{e,t}) = \max_{\{w_t, n_t\}} \{U_t + \beta \, \mathbb{E}_t[W(n_t, p_{e,t+1})]\} \; . \tag{37}$$

The first-order condition with respect to w_t gives the following constrained-efficient wage rate

$$w_t^* = \frac{1}{\nu} + \xi \, p_{e,t} \, . \tag{38}$$

In other words, the constrained-efficient allocation involves risk-neutral firm owners insuring workers against fluctuations in their equilibrium consumption bundle as given in (28). Due to the need to consume energy for subsistence purposes, a constant wage rate is not sufficient for the purpose in this setting, in contrast with Challe (2020). Insuring workers against consumption fluctuations involves, instead, the constrained-efficient wage rate adjusting upward (downward) in response to positive (negative) shocks to real energy prices.

The first-order condition with respect to n_t gives the following forward recursion for the constrained-efficient job-finding rate

$$f_t^{*\,\frac{\alpha}{1-\alpha}} = \frac{1-\alpha}{\kappa} \left[\frac{p_{g,t} - \gamma_e \, p_{e,t}}{1-\gamma_e} - w_t^* + \frac{1}{\nu} \ln\left(\frac{w_t^* - \xi \, p_{e,t}}{\delta_t - \xi \, p_{e,t}} \right) \right] + \beta \, (1-\rho) \, \mathbb{E}_t \left[f_{t+1}^{*\,\frac{\alpha}{1-\alpha}} \, (1 - \alpha \, f_{t+1}^*) \right] \tag{39}$$

from which the constrained-efficient level of employment can be derived by exploiting the law of motion $n_t^* = (1-\rho) \, n_{t-1}^* + f_t^* \, [1 - (1-\rho) \, n_{t-1}^*]$.

4.1 Steady State

First of all, the constrained-efficient steady state must entail zero inflation, requiring in turn $p_g^* = 1$, i.e. $\varphi = \frac{1}{1-\tau_y} \left(\frac{\varepsilon-1}{\varepsilon}\right) \left(\frac{1}{1-\gamma_e}\right) p_g - \left(\frac{\gamma_e}{1-\gamma_e}\right) p_e$. Comparing the steady state counterparts of (22) and

(39), which read respectively

$$f^{\frac{\alpha}{1-\alpha}} = \frac{1-\tau_z}{\kappa\,[1-\beta\,(1-\rho)]}\,(\varphi - w + S)$$
$$f^{*\,\frac{\alpha}{1-\alpha}} = \frac{1-\alpha}{\kappa\,[1-\beta\,(1-\rho)\,(1-\alpha\,f^*)]}\left[\frac{p_g-\gamma_e\,p_e}{1-\gamma_e} - w^* + \frac{1}{\nu}\ln\left(\frac{w^*-\xi\,p_e}{\delta-\xi\,p_e}\right)\right]$$

we can then see that the zero-inflation steady state is constrained-efficient provided that $w = w^* = \frac{1}{\nu} + \xi\,p_e$, and that the tax instruments are such that

$$\tau_y = \frac{1}{\varepsilon}\,, \quad S = \frac{1}{\nu}\ln\left(\frac{w^*-\xi\,p_e}{\delta-\xi\,p_e}\right)\,, \quad \tau_z = 1 - \frac{(1-\alpha)\,[1-\beta\,(1-\rho)]}{1-\beta\,(1-\rho)\,(1-\alpha\,f^*)} \tag{40}$$

where

$$f^* = \left[\frac{1-\tau_z}{\kappa\,[1-\beta\,(1-\rho)]}\left(\frac{p_g-\gamma_e\,p_e}{1-\gamma_e} - w^* + S\right)\right]^{\frac{1-\alpha}{\alpha}}\,. \tag{41}$$

Intuitively, the wholesale subsidy, τ_y, optimally offsets the steady-state markup due to monopolistic competition. The tax on labour intermediaries, τ_z, counteracts congestion externalities in their vacancy posting behaviour. The employment subsidy, S, is aimed at avoiding excess unemployment, that hurts households due to the lack of complete insurance.

Given that the tax instruments are assumed to be constant, the constrained-efficient allocation cannot be decentralised outside the zero-inflation steady state.[10] Even if this mix of fiscal instruments were allowed to be time-varying, it is evident from equations (38)–(39) that it would decentralise the constrained-efficient allocation also outside the steady state only when coupled with a rising real wage in response to surging energy prices, an implausible occurrence in reality.

5 The Linearised Model

In order to investigate how the model responds to fluctuations in energy prices, one needs to specify how the home production of the unemployed, δ_t, behaves in comparison to the income of the employed. Following Challe (2020) as a benchmark, I assume that δ_t fluctuates so that the replacement rate δ_t/w_t is constant. Letting the percentage income loss upon unemployment be $\zeta := 1 - \frac{\delta_t}{w_t} = 1 - \frac{\delta}{w}$, we have

$$\delta_t = (1-\zeta)\,w_t\,. \tag{42}$$

[10]The fact that these tax instruments cannot be adjusted in response to aggregate shocks reflects the idea that fiscal policy is slow to adjust (Acharya et al., 2023).

Now, letting hatted variables denote level deviations from the zero-inflation steady state, and tilde-variables log (or proportional) deviations, the first-order approximate dynamics of the economy are described by the following set of equations

$$\begin{aligned}
\widehat{n}_t &= \frac{\rho}{f+\rho\,(1-f)}\,\widehat{f}_t + (1-\rho)\,(1-f)\,\widehat{n}_{t-1} \\
\widehat{u}_t &= -\widehat{n}_t \\
\widehat{y}_t &= \widehat{n}_t \\
\frac{\kappa}{q\,f}\left(\frac{\alpha}{1-\alpha}\right)\widehat{f}_t &= (1-\tau_z)\,(\widehat{\varphi}_t - \widehat{w}_t) + \beta\,(1-\rho)\,\frac{\kappa}{q\,f}\left(\frac{\alpha}{1-\alpha}\right)\mathbb{E}_t(\widehat{f}_{t+1}) \\
\widehat{mc}_t &= (1-\gamma_e)\,\widehat{\varphi}_t \;+\; \gamma_e\,p_e\,\widetilde{p}_{e,t} \\
\widehat{mc}_{g,t} &= \frac{1}{p_g}\,\widehat{mc}_t + \frac{\omega_e}{1-\omega_e}\,\widetilde{p}_{e,t} \\
\pi_{g,t} &= \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t} \\
\pi_t &= \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\,(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}) \\
\widetilde{I}_t - \mathbb{E}_t(\pi_{t+1}) &= -\frac{\Lambda}{\lambda}\,\mathbb{E}_t(\widehat{\lambda}_{t+1}) + \frac{(1+\Xi_w\,\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{w}_{t+1}) - \widetilde{w}_t}{1-\Xi_w} - \Xi_w\,\frac{(1+\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{p}_{e,t+1}) - \widetilde{p}_{e,t}}{1-\Xi_w} \\
\widehat{\lambda}_t &= -\rho\,\widehat{f}_t \\
\widetilde{p}_{e,t} &= \rho_e\,\widetilde{p}_{e,t-1} + \epsilon^e_t
\end{aligned}$$

where $\Theta = \frac{1-\theta}{\theta}\,(1-\theta\,\beta)$, $\Lambda = \lambda\left[\lambda + \left(\frac{1-\zeta-\Xi_w}{\zeta}\right)\right]^{-1} \in (0,1)$, $\Psi = \frac{1-\Xi_w}{1-\zeta-\Xi_w} > 1$, and $\Xi_w = \frac{p_e\,\xi}{w} \in [0,1)$ is the steady-state share of subsistence energy expenditure in the income of the employed.

I let the decentralised wage rate evolve as follows[11]

$$\widetilde{w}_t = -\chi\,\widetilde{p}_{e,t} \tag{43}$$

where the elasticity with respect to the real price of energy is assumed to be negative, i.e. $\chi > 0$, unless otherwise noted.

The model can be closed, for positive analysis purposes, by specifying an interest rate Taylor-type rule. This is assumed to take the following form,

$$\widetilde{I}_t = \phi_\pi\,\pi_{g,t} + \phi_f\,\widehat{f}_t \tag{44}$$

[11] This reduced-form specification can be consistent with wage-setting schemes such as nominal wage indexation or Nash bargaining between workers and firm owners.

targeting core inflation as well as fluctuations in labour market slack, as captured by the job-finding rate, $\widehat{f}_t$. In the model, $\pi_{g,t}$ is the welfare-relevant measure of inflation, as it is the one that is associated with price dispersion, reducing, in turn, the quantity of output that can be obtained from a given amount of inputs.[12] Blanchard and Galí (2007) similarly use a Taylor rule targeting core inflation, a variable they argue that many central banks appear to focus on as the basis for their interest rate decisions.[13]

5.1 Energy Prices and Aggregate Demand

Focusing on the demand block of the model, as captured by the dynamic IS equation, the percentage deviation of the gross real interest rate from its steady-state level, $\widetilde{R}_t := \widetilde{I}_t - \mathbb{E}_t(\pi_{t+1})$, can be expressed as follows, in first-order approximation,

$$\widetilde{R}_t \simeq \underbrace{-\frac{\Lambda}{\lambda}\mathbb{E}_t(\widehat{\lambda}_{t+1})}_{\text{Unemployment Risk}} + \underbrace{\chi(1-\rho_e)\widetilde{p}_{e,t}}_{\substack{\text{Standard}\\ \text{Consumption Smoothing}}} + \underbrace{\frac{\Xi_w}{1-\Xi_w}(1+\chi)(1-\rho_e)\widetilde{p}_{e,t}}_{\substack{\text{Non-homothetic}\\ \text{(employed) workers}}} \underbrace{-\Lambda\Psi\frac{\Xi_w}{1-\Xi_w}(1+\chi)\rho_e\widetilde{p}_{e,t}}_{\substack{\text{Heterogeneous Exposure}\\ \text{to Energy Price Shocks}}} \quad (45)$$

First of all, we can notice that in the perfect insurance limit, $\zeta \to 0$, we have $\Lambda = 0$, implying

$$\lim_{\zeta\to 0} \widetilde{R}_t \simeq \chi(1-\rho_e)\widetilde{p}_{e,t} + \frac{\Xi_w}{1-\Xi_w}(1+\chi)(1-\rho_e)\widetilde{p}_{e,t}$$

hence, in this case, aggregate demand is always sustained by an increase in real energy prices, by both a standard consumption smoothing channel and the non-homotheticity of consumer preferences. The former channel induces a desire to borrow as future wages are expected to bounce back up. The latter channel induces a desire to borrow as well, in order to smooth consumption as future energy prices —and the consumption losses they directly entail due to non-homothetic preferences— are expected to be lower than in the current period.

In the presence of imperfect insurance, i.e. $\zeta, \Lambda \in (0,1)$, we can first notice that when preferences are homothetic, and hence $\Xi_w = 0$, energy prices do not have any direct impact on the demand

[12] Similarly, in Bodenstein et al. (2008), it is core inflation that matters for welfare since core prices are sticky while the price of energy is assumed to be completely flexible.

[13] In line with the ECB's medium-term orientation in pursuing its primary objective of price stability, core inflation can serve as a valuable indicator for understanding medium-term developments in headline inflation. In the present setting, given a high persistence of energy price shocks, π_t will settle to $\pi_{g,t}$ in the periods after the shock.

side of the model as captured by equation (45), which therefore reduces to the same considered by Challe (2020),

$$\widetilde{R}_t \;\simeq\; -\frac{\Lambda}{\lambda}\,\mathbb{E}_t(\widehat{\lambda}_{t+1}) \;+\; \mathbb{E}_t(\widetilde{w}_{t+1}-\widetilde{w}_t)\,.$$

Still, even if energy prices do not directly affect aggregate demand, they can *indirectly* affect it through unemployment risk and real wage fluctuations.

Non-homothetic preferences imply, along with the presence of imperfect unemployment insurance, that employed and unemployed workers are heterogeneously exposed to energy price shocks. This channel, captured by the last term in (45), induces a precautionary saving desire that depresses aggregate demand when real energy prices increase. Indeed, the consumption losses upon unemployment are increased as the shock weighs more on the unemployed, causing, in turn, an incentive to precautionarily save on the part of currently employed workers out of the fear of becoming unemployed and hence more exposed to the surge in energy prices.

Therefore, the net effect of non-homotheticity on aggregate demand depends on which channel prevails between the desire to borrow conditional on remaining employed, captured by the third term in (45), and the desire to save out of the fear of becoming unemployed and hence more exposed to the shock. One can easily check that the latter channel prevails when the shock is sufficiently persistent, namely when $\rho_e > 1 \big/ (1+\Lambda\,\Psi)$.

Assuming that a shock to the price of energy is quite persistent is not unreasonable. For instance, Blanchard and Galí (2007) argue that the real price of oil would be better characterised as non-stationary. In the present setting, when $\rho_e \to 1$, the *direct* impact of energy price increases on aggregate demand would undoubtedly be negative and amount to

$$-\Lambda\,\Psi = -\frac{\rho\,(1-f)\,\zeta}{\rho\,(1-f)\,\zeta + (1-\zeta-\Xi_w)}\left(\frac{1-\Xi_w}{1-\zeta-\Xi_w}\right)$$

which is larger, in absolute terms, the larger the proportional loss in income upon unemployment, ζ, as energy price increases end up weighing even more on the unemployed relative to the employed, and hence employed workers have more incentive to precautionarily save in direct response to these shocks.

5.2 Positive Analysis: an Illustrative Case

While a quantitative assessment of the behaviour of the linearised model will be performed later in Section 6 in comparison to normative analysis, to understand the various forces at work, it is useful to consider the following illustrative case, which allows to derive a simple and intuitive analytical solution for the unemployment response to energy price shocks.

I first assume constant real wages, implying in turn that only the two precautionary motives highlighted in equation (45) will be at work in determining saving desires. I also assume full worker reallocation ($\rho = 1$) implying $\widehat{n}_t = \widehat{f}_t$, and set the matching function parameter to $\alpha = 0.5$. Lastly, the policy rule parameters are set to $\phi_\pi = 1/\beta\,, \phi_f = 0$.

Under these assumptions, the first-order behaviour of the model is characterised by the following equations

$$\begin{aligned}
\widehat{mc}_{g,t} &= -\frac{2\,\kappa}{p_g}\,\widehat{u}_t + \left[\frac{p_e}{p_g}\left(\frac{\gamma_e}{1-\gamma_e}\right) + \left(\frac{1}{1-\gamma_e}\right)\frac{\omega_e}{1-\omega_e}\right]\widetilde{p}_{e,t} \\
\pi_{g,t} &= \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t} \\
\pi_t &= \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\,(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}) \\
\frac{1}{\beta}\,\pi_{g,t} - \mathbb{E}_t(\pi_{t+1}) &= -\frac{\Lambda}{1-f}\,\mathbb{E}_t(\widehat{u}_{t+1}) - \Xi_w\,\frac{(1+\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{p}_{e,t+1}) - \widetilde{p}_{e,t}}{1-\Xi_w}
\end{aligned}$$

Combining the last three equations, the dynamic IS curve becomes

$$\frac{\Theta}{\beta}\,\widehat{mc}_{g,t} + (1-\rho_e)\,\frac{\omega_e}{1-\omega_e}\,\widetilde{p}_{e,t} = -\frac{\Lambda}{1-f}\,\mathbb{E}_t(\widehat{u}_{t+1}) + (1-\rho_e)\,\frac{\Xi_w}{1-\Xi_w}\,\widetilde{p}_{e,t} - \rho_e\,\frac{\Xi_w}{1-\Xi_w}\,\Lambda\,\Psi\,\widetilde{p}_{e,t}\,.$$

When $\rho_e \to 1$, this can be approximated by the following linear expectational difference equation

$$\widehat{u}_t = \frac{p_g}{2\,\kappa}\,\mathcal{B}_n\,\mathbb{E}_t(\widehat{u}_{t+1}) + \frac{p_g}{2\,\kappa}\,\mathcal{C}_e\,\widetilde{p}_{e,t} \tag{46}$$

where $\mathcal{B}_n = \frac{\beta}{\Theta}\left(\frac{\Lambda}{1-f}\right)$, $\mathcal{C}_e = \frac{p_e}{p_g}\left(\frac{\gamma_e}{1-\gamma_e}\right) + \left(\frac{1}{1-\gamma_e}\right)\frac{\omega_e}{1-\omega_e} + \Lambda\,\Psi\,\frac{\beta}{\Theta}\left(\frac{\Xi_w}{1-\Xi_w}\right)$, and local determinacy requires $\frac{p_g}{2\,\kappa}\,\mathcal{B}_n < 1$.

5.2.1 Perfect Insurance Limit

In the perfect insurance limit where the income of the unemployed is equal to that of the employed, the first-order response of unemployment to an increase in real energy prices is given by

$$\lim_{\zeta \to 0} \widehat{u}_t \;=\; \mathcal{F}_e \, \widetilde{p}_{e,t} \tag{47}$$

where $\mathcal{F}_e = \frac{p_g}{2\,\kappa} \left[\frac{p_e}{p_g} \left(\frac{\gamma_e}{1-\gamma_e} \right) + \left(\frac{1}{1-\gamma_e} \right) \frac{\omega_e}{1-\omega_e} \right]$.

It is worth noticing that the perfect insurance limit corresponds, in the particular case considered here, to the natural (flexible-price) response, as the two precautionary motives shaping the response of the natural rate vanish in the perfect insurance limit, implying, in turn, a steady natural rate when real wages are assumed to remain constant. This perfect-insurance (and natural) response is, then, entirely driven by what happens on the producer side of the model: in response to rising real energy prices, firms demand less energy; given that this input is a complement with labour in production, and that real wages are assumed to remain fixed, also labour demand decreases, and hence unemployment rises at equilibrium.

5.2.2 Imperfect Insurance

Coming to the imperfect insurance case, and considering first a homothetic preferences situation, we have

$$\widehat{u}_t \;=\; \frac{\mathcal{F}_e}{1 - \frac{p_g}{2\,\kappa} \frac{\beta}{\Theta} \left(\frac{\zeta}{1-\zeta\, f} \right)} \widetilde{p}_{e,t} \tag{48}$$

where the denominator shall be positive under local determinacy. The presence of imperfect unemployment insurance then amplifies the increase in unemployment in response to a rise in real energy prices: this amplification channel is captured by the presence of the second term in the denominator of equation (48). Intuitively, the increase in unemployment risk that follows the reduction in activity induces a precautionary saving desire on the part of households, which depresses aggregate demand. Due to price rigidity, this drag on aggregate demand ends up increasing unemployment at equilibrium. Moreover, the magnitude of the response is larger the larger the degree of imperfect insurance, as captured by the proportional income loss upon unemployment, ζ.

In the non-homothetic case, we have

$$\widehat{u}_t \;=\; \frac{\mathcal{F}_e + \frac{p_g}{2\,\kappa}\,\frac{\beta}{\Theta}\,\left(\frac{\Xi_w}{1-\zeta-\Xi_w}\right)\,\left[\frac{(1-f)\,\zeta}{1-\zeta\,f-\Xi_w}\right]}{1-\frac{p_g}{2\,\kappa}\,\frac{\beta}{\Theta}\,\left(\frac{\zeta\,f}{1-\zeta\,f-\Xi_w}\right)}\,\widetilde{p}_{e,t} \tag{49}$$

where, again, the denominator shall be positive under local determinacy. The presence of heterogeneous exposure to the shock due to non-homotheticity then implies an additional source of amplification of the unemployment response to a rise in real energy prices. First, as before, greater unemployment risk depresses aggregate demand and employment. This channel is again captured by the second term in the denominator of equation (49). Second, due to the presence of non-homothetic preferences, as captured by $\Xi_w \in (0,1)$, the rise in real energy prices increases the consumption losses upon unemployment, thereby strengthening the drag on aggregate demand. This latter amplification channel is captured by the second term in the numerator of equation (49). As before, when ζ increases, the magnitude of the response increases as well, now through both the two channels.

6 Normative Analysis

In this section, I investigate the sources of the trade-off between stabilising core inflation and unemployment that arise in this economy. I first focus, in the spirit of Challe (2020), on a set of parametric restrictions that delivers a closed-form expression for the core inflation and unemployment rates under the optimal policy. This particular case with a closed-form analytical solution helps to gather the intuition about the joint role of imperfect unemployment insurance and non-homotheticity in determining a heterogeneous exposure to the shock, which stands as the primary source of the trade-off. The latter materialises as an endogenous wedge between natural unemployment (the level consistent with stable core inflation) and constrained-efficient unemployment. As a consequence, the monetary authority will not be able to replicate the constrained-efficient allocation by aiming to stabilise core inflation pressures and will need instead to seek a compromise between stabilising core inflation and unemployment at the constrained-efficient level.

I then move to a more general case, where the parameters will be disciplined from the data to calibrate the model and get a sense of its quantitative properties under numerical simulations.

6.1 Optimal Policy: an Illustrative Analytical Case

Assuming $\rho = 1$, we have $n_t = f_t = v_t^{1-\alpha}$. Above all, this assumption implies that employment does not act as a state variable in the welfare objective in (37), allowing, in turn, a full analytical treatment of the optimal policy problem. In this respect, also assuming $\alpha = 0.5$, the actual, natural, and constrained-efficient unemployment levels read, respectively,

$$u_t = 1 - \frac{1}{2\,\kappa}\,(\varphi_t - w_t + S) \tag{50}$$

$$u_t^n = 1 - \frac{1}{2\,\kappa}\left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t + S\right] \tag{51}$$

$$u_t^* = 1 - \frac{1}{2\,\kappa}\left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t^* + \frac{\ln(w_t^* - \xi\, p_{e,t}) - \ln(\delta_t - \xi\, p_{e,t})}{\nu}\right] \tag{52}$$

In terms of level deviations from the constrained-efficient steady state, we have, in first-order approximations,

$$\widehat{u}_t^n \simeq \mathcal{F}_e\,\widetilde{p}_{e,t} + \frac{w}{2\,\kappa}\,\widetilde{w}_t \tag{53}$$

$$\widehat{u}_t^* \simeq \mathcal{F}_e\,\widetilde{p}_{e,t} + \frac{w}{2\,\kappa}\,\widetilde{w}_t^* + \frac{w}{2\,\kappa}\,\Psi\,\left[(1-\zeta)\,\widetilde{\delta}_t - \Xi_w\,\widetilde{p}_{e,t}\right] \tag{54}$$

How differently the natural and constrained-efficient unemployment levels respond to energy price shocks then depends on how the decentralised wage w_t compares to the constrained-efficient wage $w_t^* = \frac{1}{\nu} + \xi\, p_{e,t}$, and on the cyclical behaviour of δ_t. In what follows, I assume again, as in Challe (2020) and as specified in (42), that the decentralised wage and home production fluctuate in such a way that the income loss upon unemployment is constant, implying, in turn, $\widetilde{\delta}_t = \widetilde{w}_t$, while the decentralised wage is assumed to evolve as in (43), i.e. $\widetilde{w}_t = -\chi\,\widetilde{p}_{e,t}$, with $\chi > 0$ unless otherwise noted. Therefore, the wedge between natural and constrained-efficient unemployment is

$$\widehat{u}_t^n - \widehat{u}_t^* = \frac{w}{2\,\kappa}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t}\,. \tag{55}$$

Now, given the New Keynesian Phillips Curve (NKPC)

$$\pi_{g,t} = \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t}$$

under the simplifying assumptions made in this section, we have $\widehat{mc}_{g,t} = -\frac{2\,\kappa}{p_g}\,(\widehat{u}_t - \widehat{u}_t^n)$. Letting

$\mathcal{U}_t := \widehat{u}_t - \widehat{u}_t^*$ denote the welfare-relevant unemployment gap,

$$\pi_{g,t} = \beta\,\mathbb{E}_t(\pi_{g,t+1}) - \Theta\,\frac{2\,\kappa}{p_g}\,\mathcal{U}_t + \underbrace{\Theta\,\frac{w}{p_g}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\text{cost-push term}} \tag{56}$$

and we can see that energy price shocks, by inducing a time-varying wedge between natural and constrained-efficient unemployment, act endogenously as a cost-push term in the NKPC. I now turn to discuss its primary sources.

Heterogeneous Exposure to Energy Price Shocks. If the constrained-efficient wage could be decentralised also outside the steady state, i.e. $\chi = -\Xi_w$, there would still be a wedge between natural and constrained-efficient unemployment, amounting to

$$\widehat{u}_t^n - \widehat{u}_t^* = \frac{w}{2\,\kappa}\,\zeta\,\Psi\,\Xi_w\,\widetilde{p}_{e,t}\,.$$

In other words, a wedge between natural and constrained-efficient unemployment arises endogenously, first of all, due to the presence of subsistence energy consumption and imperfect unemployment insurance, which jointly imply that the unemployed are more exposed to energy price shocks than the employed. Indeed, as a result, rising energy prices induce an increase in the consumption losses upon unemployment. These would be efficiently counteracted by larger employment subsidies, but as these remain constant at their steady-state amount S, a wedge between natural and constrained-efficient unemployment arises.

This, then, constitutes a novel source of a cost-push term in the NKPC compared to the benchmark setting of Challe (2020). Indeed, when $\Xi_w = 0$ and hence preferences are homothetic, we can see that the cost-push term vanishes. The term vanishes also in the perfect insurance limit $\zeta \to 0$, as in both these cases there is no heterogeneity between the employed and the unemployed in their direct exposure to energy price shocks.

Real Wage Rigidity. While in Challe (2020) constant real wages can be constrained-efficient (when they correspond to their constrained-efficient steady-state amount) here, as already mentioned, due to the presence of subsistence energy needs, a constant real wage rate cannot insure workers against fluctuations in their consumption bundle. As a result, real wage rigidity constitutes another novel source of endogenous cost-push terms, in contrast with Challe (2020). In particular, we can see that when constrained-efficient wage fluctuations cannot be decentralised and real wages remain constant (i.e. $\chi = 0$), the wedge becomes of larger magnitude

$$\widehat{u}_t^n - \widehat{u}_t^* = \frac{w}{2\,\kappa}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,\widetilde{p}_{e,t}\,.$$

Compared to the constant real wage case, on the one hand the downward adjustment (captured by $\chi > 0$) moderates the rise in natural unemployment. However, on the other hand, it also causes the home production of the unemployed to fall, calling for a larger employment subsidy on the constrained-efficient side. This latter effect dominates, driving a wedge between natural and constrained-efficient unemployment of larger magnitude compared to the case of constant real wages, as can be seen from (55).

6.1.1 Linear-quadratic Problem

In this setting, similarly to Challe (2020), one can derive the following expression for welfare losses from a second-order approximation of the welfare objective[14]

$$L_t = \frac{1}{2}\,\mathbb{E}_t\sum_{j=0}^{\infty}\beta^j\,\left(\mathcal{U}_{t+j}^2 + \Omega\,\pi_{g,t+j}^2\right) \tag{57}$$

where $\Omega = \frac{n\,\varepsilon}{(1-\gamma_e)}\,\frac{p_g}{2\,\kappa}\,/\Theta$. The loss function in (57) makes explicit that $\pi_{g,t}$ is the welfare-relevant measure of inflation in this economy, as mentioned before.

Optimal Discretionary Policy. The optimal discretionary policy minimises the period losses

$$\frac{1}{2}\,(\mathcal{U}_t^2 + \Omega\,\pi_{g,t}^2)$$

subject to (56) taking expected inflation as given. The solution to the problem (detailed in Appendix B.6) gives the following targets for core inflation and the welfare-relevant unemployment gap

$$\pi_{g,t} = \Upsilon\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{58}$$

$$\mathcal{U}_t = \left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\,\Upsilon\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{59}$$

where $\Upsilon = \Theta\,\frac{w}{p_g}\,\left(1-\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1}$. We can then see that, in response to an increase in real energy prices, some core inflation is optimally accommodated in order to achieve a smaller gap

[14] See Appendix B.5 for more details.

between actual and constrained-efficient unemployment. Indeed, one can easily check that if core inflation were fully stabilised, the welfare-relevant unemployment gap would amount to

$$\mathcal{U}_t = \frac{w}{2\,\kappa}\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}$$

therefore implying higher unemployment as $\frac{w}{2\,\kappa} > \left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\Upsilon$.

Intuitively, the optimal discretionary policy involves partly accommodating core inflation in order to indirectly sustain employment and, hence, avoid too many households flowing to unemployment, thereby becoming more exposed to the shock. We can also notice that when preferences are homothetic (corresponding to $\Xi_w = 0$), and hence the employed and the unemployed are homogeneously exposed to the shock, the trade-off vanishes, and the monetary authority can optimally stabilise core inflation and the welfare-relevant unemployment gap simultaneously.

Lastly, we can see that the targets imply, at equilibrium, that the nominal interest rate consistent with the optimal discretionary policy satisfies

$$\widetilde{I}_t \;\simeq\; \widetilde{R}^n_t \;+\; \frac{\Lambda}{1-f}\,\underbrace{\left(\frac{1-\beta}{\Theta}\right)\frac{p_g}{2\,\kappa}\,\Upsilon\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\mathbb{E}_t(\widehat{x}^n_{t+1})} \;+\; \underbrace{\Upsilon\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\mathbb{E}_t(\pi_{g,t+1})} \tag{60}$$

where we can notice that in the absence of a heterogeneous exposure to energy price shocks, when $\Xi_w = 0$, the optimal policy *outcome* is to align the nominal rate to the real natural rate, thereby sterilising core inflation pressures, just as in the textbook Representative-Agent New Keynesian (RANK) model. By contrast, when unemployed and employed households are heterogeneously exposed to these shocks, a wedge between natural and constrained-efficient unemployment arises endogenously, and the optimal policy prescription changes. From (60), we can see that the equilibrium real interest rate ends up being higher than the natural real rate, $\widetilde{I}_t - \mathbb{E}_t(\pi_{t+1}) > \widetilde{R}^n_t$: since unemployment is lower than its natural level, equilibrium demand is kept afloat as the precautionary saving motive due to unemployment risk is dampened.

Optimal Ramsey Policy. The optimal Ramsey policy minimises the lifetime welfare losses in (57) subject to the sequence of constraints posed by (56), giving the following targeted *paths* (formally derived in Appendix B.7) for core inflation and the welfare-relevant unemployment gap

$$\pi_{g,t+j} = \eta^j\,\overline{\Upsilon}\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t} \tag{61}$$

$$\mathcal{U}_{t+j} = \left(\frac{n\,\varepsilon}{1-\gamma_e} \right) \left(\frac{1-\eta^{j+1}}{1-\eta} \right) \overline{\Upsilon}\,\zeta\,\Psi \left(\frac{\Xi_w}{1-\Xi_w} \right) (1+\chi)\,\widetilde{p}_{e,t} \tag{62}$$

for $j \geq 0$, where $\eta := \frac{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}{2\,\beta} \left[1 - \sqrt{1 - 4\,\beta\,\left(1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-2}} \right] \in (0,1)$ and $\overline{\Upsilon} = \Theta\,\frac{w}{p_g}\,\left(1-\eta\,\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1} < \Upsilon$. Therefore, just as in the textbook RANK model, if the monetary authority is able to credibly commit to this policy plan, it can achieve a smaller welfare-relevant unemployment gap on impact and a smaller increase in core inflation relative to the optimal discretionary policy. The main intuition is, however, the same as in the discretionary case: core inflation is partly accommodated in order to indirectly sustain employment, thereby preventing too many workers from ending up being more exposed to the shock by becoming unemployed.

6.2 Optimal Policy: the General Case

This section considers the general case of $\rho \in (0,1)$, with the aim of giving a sense of the quantitative properties of the model. However, it should be stressed that the quantitative predictions of the model, given its stylised nature, should not be taken too literally.

When $\rho < 1$, employment acts as a state variable in the welfare objective since the existing employment stock impacts the current amount of hiring costs, as can be seen from (34). This requires resorting to a numerical solution of the Ramsey problem,[15] and hence choosing first of all an appropriate parametrisation for the model.

6.2.1 Parametrisation

The parametrisation, summarised in Table 2, is aimed to target an average continental European labour market, with each time period taken to correspond to a quarter.

Preferences. Given a quarterly frequency, the discount factor is set to $\beta = 0.98$, below its usual value of 0.99. Indeed, the presence of uninsured unemployment risk brings the average real rate of return below the value of $1/\beta$ that would prevail under complete markets.[16] Hence, β must be calibrated to a lower value to ensure that real rates of return are not unrealistically small.

The parameters governing the relative weight of energy and the degree of non-homotheticity in consumer preferences (ω_e and ξ, respectively) are calibrated by exploiting the following results

[15] The numerical solution is obtained using Dynare (www.dynare.org).

[16] Given the steady state Euler condition $1 = \beta\,R\,\left[(1-\lambda)+\lambda\,\left(\frac{w-p_e\,\xi}{\delta-p_e\,\xi}\right)\right]$, one can easily see that since $\left[(1-\lambda)+\lambda\,\left(\frac{w-p_e\,\xi}{\delta-p_e\,\xi}\right)\right] > 1$, then $R < 1/\beta$.

(formally derived in Appendix B.2)

$$\omega_e^n := \frac{p_e\, e^n}{w} = \omega_e + (1-\omega_e)\,\Xi_w \tag{63}$$

$$\omega_e^u := \frac{p_e\, e^u}{\delta} = \omega_e + (1-\omega_e)\,\frac{\Xi_w}{1-\zeta} \tag{64}$$

along with the empirical evidence from Table 1 on the energy shares of the employed and the unemployed —appearing on the LHS of equations (63) and (64)— and on the average percentage difference in overall consumption expenditure between them, which is accordingly set to $\zeta = 22\%$. Reassuringly, this is close to the 20% consumption loss *upon* unemployment documented by Chodorow-Reich and Karabarbounis (2016) and in the range of the estimates surveyed by Den Haan et al. (2018). Therefore, (63) and (64) jointly imply $\Xi_w = 0.11$ and $\omega_e = 0.08$, with the latter ending up being in line with the weight of energy in the euro-area Harmonised Index of Consumer Prices.

Labour Market. Following Blanchard and Galí (2010), I target an average job-finding rate $f = 0.25$ and an average unemployment rate $u = 0.10$, implying, in turn, a separation rate $\rho = 0.037$.

The matching function elasticity is set to $\alpha = 0.6$, which is the midpoint of the range of estimates surveyed by Petrongolo and Pissarides (2001). As in Challe (2020), flow vacancy costs κ are taken to be 4.5% of the real wage rate. Given the targets, the latter can be recovered from (41), and its negative elasticity with respect to the real price of energy is set to $\chi = 0.1$, which would imply a decline in real wages of 4% in light of a 40% increase in the real price of energy, roughly in line with what has been observed in the euro area during the current energy crisis.[17]

Producers. The energy share in production is set to $\gamma_e = 0.04$, as in Bachmann et al. (2022) and Pieroni (2023). The Calvo pricing parameter is set to $\theta = 3/4$, implying an average price duration of one year. The elasticity of substitution among core good varieties is set to $\varepsilon = 4$, as in Gagliardone and Gertler (2023), which would imply a steady-state markup of 1/3;[18] even when this is offset by the subsidy $\tau_y = 1/\varepsilon$, the parameter ε might still influence the relative welfare weight: for instance, a smaller ε implies less weight on inflation in the quadratic welfare loss function in (57).

Energy Price Shock. I set the shock as a 40% increase in the real price of energy,[19] hitting the economy with high persistence; namely, $\rho_e = 0.97$ as in Blanchard and Galí (2007), who choose their autoregressive parameter to have the real price of oil very close to a random walk, as it is in the data, while retaining stationarity.

[17] I check that the real wage remains always in the bargaining set along the simulated path.

[18] Kouvavas et al. (2021) document an average markup for the euro area of around 0.45 over the 1995—2018 period.

[19] This aligns with the recent peak rise of the eurozone real energy price index above its long-run average in 2022 Q4.

Table 2: PARAMETRISATION

	Targets		
	Description	Value	Source
f	Average job-finding rate	0.25	Blanchard and Galí (2010)
u	Average unemployment rate	0.10	Blanchard and Galí (2010)
κ/w	Flow vacancy cost	0.045	Challe (2020)
ζ	Expenditure loss upon unemployment	0.22	ECB Consumer Expectation Survey
ω_e^n	Energy share of the employed	0.182	ECB Consumer Expectation Survey
ω_e^u	Energy share of the unemployed	0.211	ECB Consumer Expectation Survey

	Parameters		
	Description	Value	Source/Targets
β	Discount factor	0.98	4% annual interest rate
ω_e	Quasi-share of energy in consumption	0.08	$\zeta, \omega_e^n, \omega_e^u$
ξ	Subsistence consumption of energy	0.109	$\zeta, \omega_e^n, \omega_e^u$
ρ	Separation rate	0.037	f, u
α	Matching function elasticity	0.6	Petrongolo and Pissarides (2001)
θ	Fraction of unchanged prices	0.75	1 year avg. price duration
ε	Elasticity of Substitution	4	1/3 steady-state markup
γ_e	Share of energy in production	0.05	Bachmann et al. (2022)
ρ_e	Energy price shock persistence	0.97	Blanchard and Galí (2007)

Even if the increase in the energy share of the unemployed relative to that of the employed is not explicitly targeted, the chosen parametrisation is able to match the one percentage point documented in the motivating evidence summarised in Figure 2. Indeed, given the energy shares formally derived in Appendix B.2, when the shock hits $\widehat{\omega}_{e,0}^u - \widehat{\omega}_{e,0}^n \simeq (\omega_e^u - \omega_e^n)(1+\chi)\,\widetilde{p}_{e,0} = 0.01$. This is reassuring, as the increased relative exposure to the shock on the part of the unemployed is suggested to be the key driver of the monetary policy trade-off uncovered analytically in Section 6.1, and the proposed parametrisation is able to quantitatively match this magnitude with the evidence.

6.2.2 Impulse Responses

Figure 3 shows the responses of core inflation and unemployment to a one-off increase in the real price of energy, comparing the behaviour of the linearised model under the optimal Ramsey policy with that under two alternative policy rules of the type of equation (44): a strict inflation targeting rule (with $\phi_\pi = 15\,, \phi_f = 0$), aimed at suppressing core inflationary pressures, and a rule aimed at stabilising the unemployment rate by also targeting fluctuations in labour market slack and reacting to inflation less aggressively (with $\phi_\pi = 1.5\,, \phi_f = 1.5$). For all these cases, I consider a scenario where the employed and the unemployed are heterogeneously exposed to the shock, as it is suggested by the evidence from Section 2, and a counterfactual scenario where instead preferences are homothetic (i.e. $\xi = \Xi_w = 0$) and hence the employed and the unemployed are homogeneous in their direct exposure to the shock. The top panel of Figure 3 reports the impulse responses from the former case, while the bottom panel those from the latter case.

In the heterogeneous exposure case, the surge in the real price of energy causes a sizeable rise in unemployment under strict inflation targeting. By contrast, under the optimal Ramsey policy, the monetary authority is able to achieve a notably smaller increase in unemployment at the cost of partial accommodation of core inflation, as suggested by the analytical results. Again, this is intuitively optimal as, in such case, fewer workers lose their jobs and hence avoid becoming more exposed to the surge in energy prices. However, a full stabilisation of the unemployment rate is also not optimal, as it implies an overshooting of core inflation compared to the optimal policy: even if unemployment barely increases on impact under the latter, a smaller rise in inflation is achieved by credibly committing to a future recession so as to dampen expectations about future price rises.

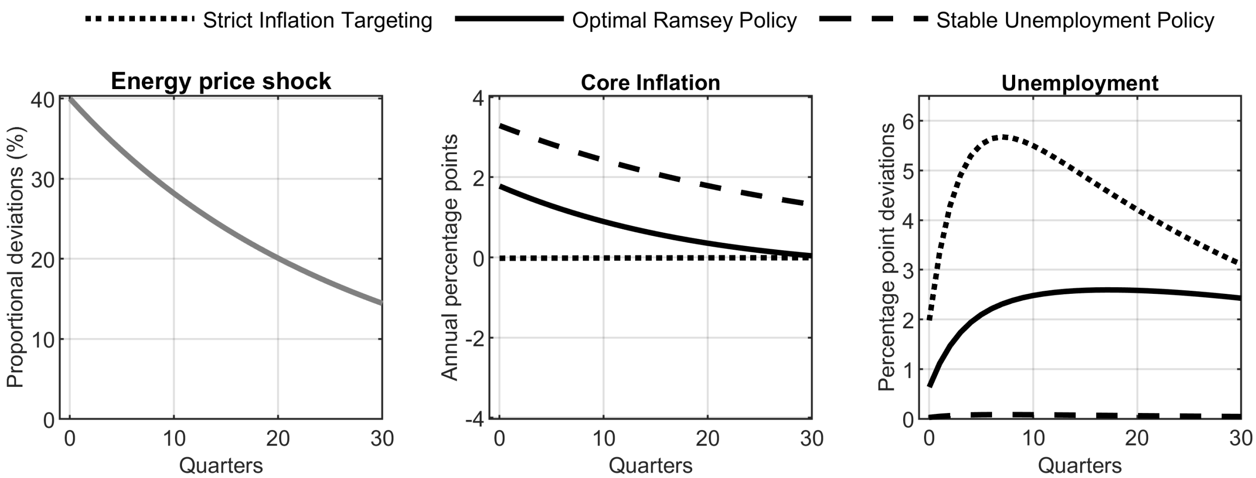
In the homogeneous exposure case, as also was suggested by the analytical results, the trade-off disappears, and the optimal Ramsey policy coincides with that achieved under a Taylor rule aimed at aggressively targeting core inflation.

It is also worth mentioning that the results would be substantially unchanged, over the medium term, if overall inflation instead of core inflation were the target of the Taylor rule: indeed, since $\pi_t = \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\left(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}\right)$, and ρ_e is large while ω_e is relatively small, we have that with the exception of the period when the shock hits, in subsequent periods $\frac{\omega_e}{1-\omega_e}\left(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}\right) \simeq 0$ and $\pi_t \simeq \pi_{g,t}$. However, if overall inflation were to be fully stabilised also in the period of the shock, this would require core deflation, implying a larger surge in unemployment than optimal also in the homogeneous exposure case, where instead stabilising only core inflation is optimal. In other words, in this case, the optimal policy is still accommodative towards the energy component of consumer inflation but hard on core inflation (similarly to Natal, 2012), as can be seen in Figure 4.

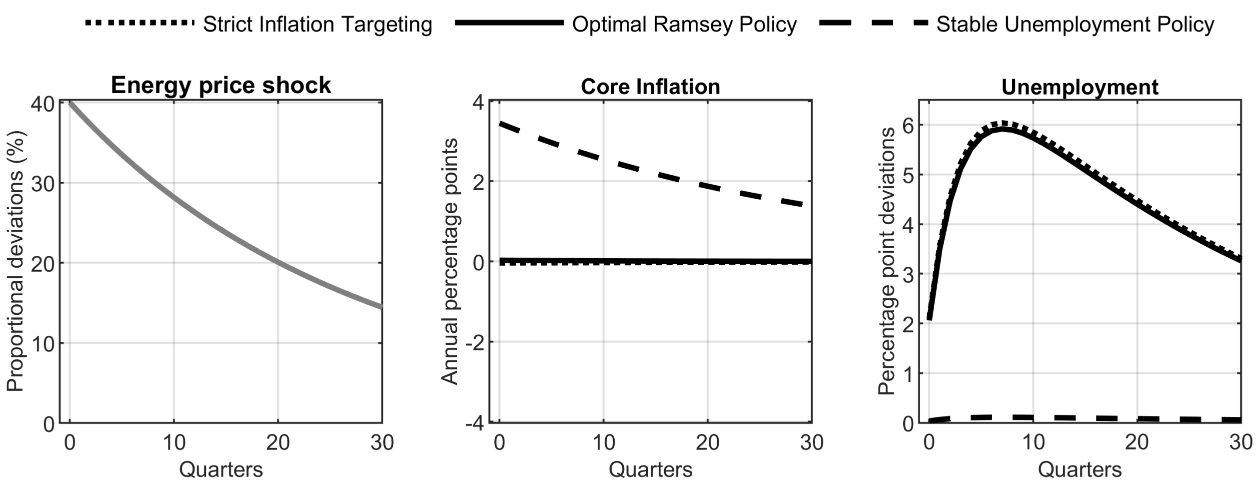
Taking stock, even if the model is highly stylised and its exact quantitative predictions should not be taken too literally, its qualitative predictions are likely to matter in practice and are in line with those suggested by the analytical results: compared to a policy rule aimed at fully stabilising

Figure 3: Unemployment and Inflation Effects of an Energy Price Shock

(a) Heterogeneous Exposure to the Shock

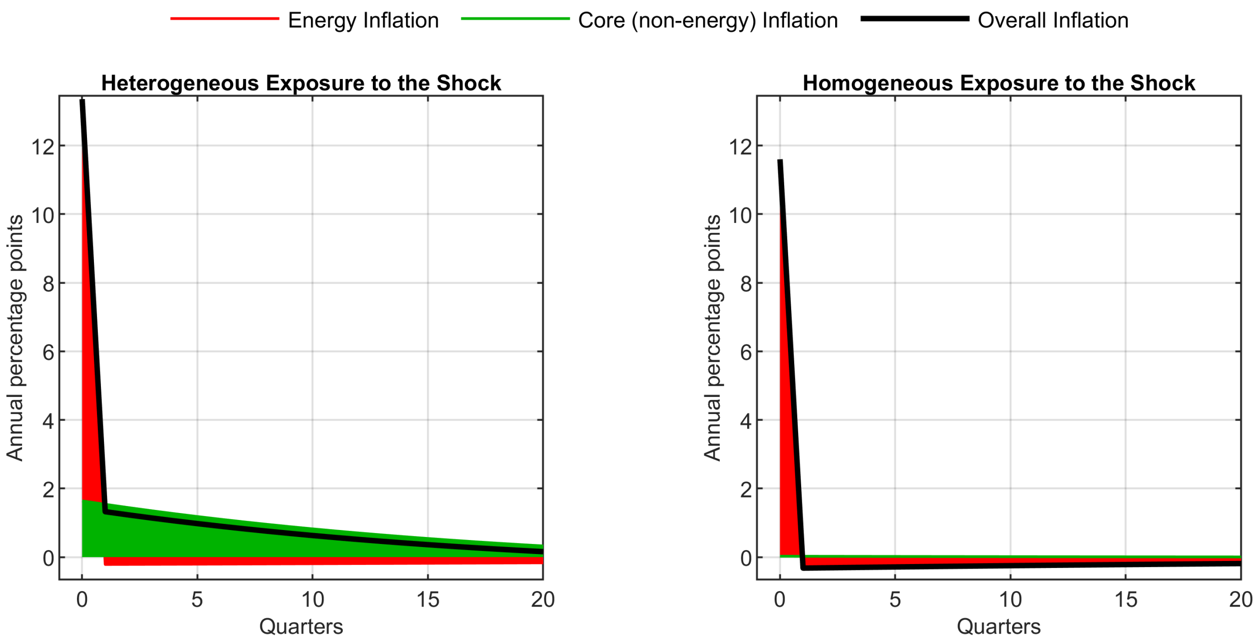


(b) Homogeneous Exposure to the Shock



Notes: The figure reports model-based impulse responses to a 40% real energy price shock of persistence $\rho_e = 0.97$ (left panels), under the parametrisation summarised in Table 2. Solid lines correspond to responses under the Ramsey-optimal monetary policy, dotted lines to those under strict inflation targeting ($\phi_\pi = 15\,, \phi_f = 0$), and dashed lines to those under a stable unemployment policy ($\phi_\pi = 1.5\,, \phi_f = 1.5$). Middle and right panels report, respectively, the responses of core inflation and unemployment under the alternative monetary policy regimes. The top panels refer to the heterogeneous exposure case ($\Xi_w = 0.11$) while the bottom panels to the homogeneous exposure case ($\xi = \Xi_w = 0$, other parameters unchanged).

Figure 4: Overall Inflation and its Components Under the Optimal Ramsey Policy



Notes: The figure reports model-based impulse responses, in the Ramsey-optimal monetary policy regime, to a 40% real energy price shock of persistence $\rho_e = 0.97$, under the parametrisation summarised in Table 2. Solid lines correspond to overall inflation, π_t, red areas to its energy component, $\omega_e\, \pi_{e,t}$, and green areas to its core (non-energy) component, $(1-\omega_e)\, \pi_{g,t}$. The left panel refers to the heterogeneous exposure case ($\Xi_w = 0.11$) while the right panel to the homogeneous exposure case ($\xi = \Xi_w = 0$, other parameters unchanged).

core inflation, the optimal policy is able to achieve a smaller increase in unemployment at the cost of partly accommodating core inflation; the rise in inflation is, however, smaller than if unemployment instead were to be fully stabilised. This trade-off arises due to the heterogeneous exposure to the increase in energy prices between the employed and the unemployed: in the case of homogeneous direct exposure to the shock, the optimal policy coincides with one aimed at fully stabilising core inflation.

7 Conclusions

This paper provides novel evidence on the heterogeneous exposure to energy price shocks between the employed and the unemployed, studying the implications of this fact for the (optimal) conduct of monetary policy in an analytically tractable HANK-S&M model, where non-homothetic preferences and imperfect unemployment insurance endogenously give rise to the heterogeneous exposure documented from the data. Rising energy prices induce a novel precautionary saving motive in this setting, as the consumption losses upon unemployment are increased, posing a drag on aggregate demand and employment. Moreover, the shock acts endogenously as a cost-push term, implying that the monetary authority optimally accommodates some core inflation so as to contain the rise in unemployment and, hence, avoid households becoming more exposed to the shock.

Even if this paper analyses a specific dimension of heterogeneity, it can lend potentially broader insights: some accommodation of core inflation might be needed to prevent households from becoming poorer and, hence, more hit by energy price shocks. Also, these insights can easily extend to similar situations when other subsistence goods besides energy receive sizeable relative price shocks. The monetary policy trade-off highlighted in this work can likely only be dampened, but not eliminated, by transfers from employed to unemployed workers. Even if, for the sake of exposure, this aspect was not explicitly modelled, these transfers would still provide only partial insurance to the unemployed. Indeed, the fiscal authority would be faced with yet another trade-off, between dampening the increased consumption losses upon unemployment with more generous insurance on the one hand and, on the other hand, the fact that this would depress job search incentives. Lastly, this paper considers a zero-liquidity economy, an assumption which allows analytical and numerical tractability of the optimal monetary policy problem. Therefore, the results focus on the effect of energy price shocks on consumption inequality between the employed and the unemployed, regardless of the potential wealth redistribution effects of inflation and monetary policy across households.

Appendix

A Data Description

The ECB Consumer Expectations Survey (CES) is a panel survey of consumers that has been carried out on a monthly basis starting effectively in April 2020. Its goal is to provide timely, high-frequency information on the perceptions and expectations of euro-area consumers about the economy, as well as their economic and financial behaviour. The microdata are collected in the six main euro-area countries: Belgium, Germany, Spain, France, Italy, and the Netherlands.[20] The collected sample aims to be representative of the surveyed population by age, gender, and residence region. Table A.1 summarises the main variables of interest for the analysis of Section 2, which are described in more detail in this appendix section.

Expenditure on goods and services. On a quarterly basis, interviewed individuals are asked how much their household spent during the last month on goods and services as listed in twelve major categories. These are: (1) Food, beverages, groceries, tobacco; (2) Restaurants (including take-out food and delivery), cafes/ canteens; (3) Housing (including rent, maintenance/repair costs, home owner/renter insurance, but excluding mortgage payments); (4) Utilities (including water, sewerage, electricity, gas, heating oil, phone, cable, internet); (5) Furnishings, household equipment, small appliances and routine maintenance of the house; (6) Debt repayments (installments in mortgage, consumer loans, car loans, credit cards, student loans, other loans); (7) Clothing, footwear; (8) Health (including personal care products and services), health insurance; (9) Transport (fuel, car maintenance, public transportation fares); (10) Travel, recreation, entertainment and culture; (11) Childcare and education (including tuition fees for child and adult education, costs of after school activities, care of children/ babysitting, but excluding installments on student loans); (12) Other expenditures not mentioned above.

Employment situation. Starting from the October 2020 wave of the survey, on a quarterly basis interviewed individuals are asked what best describes their current employment situation and can give one of the following answers: (1) Working full-time (self-employed or working for someone else); (2) Working part-time (self-employed or working for someone else); (3) Temporarily laid-off (expecting to return to the previous workplace); (4) On extended leave (disability, sick, parental or other leave); (5) Unemployed and actively looking for a job; (6) Unemployed, interested in having a

[20] In 2022, the sample was extended to cover five additional countries: Ireland, Greece, Austria, Portugal and Finland.

Table A.1: Variable Description

Expenditure on goods and services — food	
Expenditure on goods and services — restaurants	
Expenditure on goods and services — housing	
Expenditure on goods and services — utilities	
Expenditure on goods and services — furnishing	
Expenditure on goods and services — debt repayment	
Expenditure on goods and services — clothing	
Expenditure on goods and services — health	
Expenditure on goods and services — transport services	
Expenditure on goods and services — travel and recreation	
Expenditure on goods and services — childcare and education	
Expenditure on goods and services — other	
Employment situation:	employed/unemployed/inactive
Age at the start of the year:	18-34/35-49/50-70/71+
Gender:	male/female/other
Birth abroad:	yes/no
Education:	low/middle/high
Presence of a partner:	yes/no

job but not actively looking for a job; (7) Unable to work because of disability or other medical reasons; (8) In retirement or early retirement; (9) Studying, at school, or in training; (10) Looking after children or other persons, doing housework; (11) Other.

These 11 categories above are regrouped by the ECB into the following groups: Employed, if working full- or part-time, or if temporarily laid-off or on extended leave. Unemployed, if actively looking for a job, or interested in having a job but not actively looking for it.[21] Inactive, in the remaining cases.

Age at the start of the year. When recruited for the survey, individuals are asked about their birth month and year, and the derived age variable is updated at the beginning of each year based on the reported date of birth. The responses are then regrouped in the age brackets indicated in Table A.1.

[21] This category includes all respondents who consider themselves unemployed, including those who consider themselves unemployed but are not actively searching for a job. Thus, the definition of unemployment used by the ECB in the CES does not necessarily correspond to that of official EU labour market statistics.

Education. During recruitment, individuals are asked what is the highest level of school they have completed, or the highest degree they have received. Possible responses are: (1) Primary or no education; (2) Lower secondary education; (3) High school diploma (or equivalent professional degree); (4) Some college but no academic degree (for example: no BA, BS); (5) Bachelor's Degree (for example: BA, BS) or equivalent professional degree; (6) Master's Degree (for example: MA, MBA, MS, MSW) or equivalent; (7) Doctoral Degree (for example: PhD) or equivalent. These categories are then regrouped, before data dissemination, into three groups: low (no education, primary, or secondary), middle (high school diploma, or some college but no academic degree), and high (bachelor's or higher degree).

B Derivations and Proofs

B.1 Household Problem

Employed workers

$$V^n(B_{t-1}, p_{e,t}) = \max_{\{c_t^n, g_t^n, e_t^n, B_t\}} \{\ln(c_t^n) + \beta\,\mathbb{E}_t\,[(1-\lambda_{t+1})\,V^n(B_t, p_{e,t+1}) + \lambda_{t+1}\,V^u(B_t, p_{e,t+1})]\}$$

$$\text{s.t.} \begin{cases} P_{e,t}\,e_t^n + P_{g,t}\,g_t^n + B_t \leq W_t + (1+i_{t-1})\,B_{t-1} \\ c_t^n \leq (g_t^n)^{(1-\omega_e)}(e_t^n - \xi)^{\omega_e} \\ B_t \geq 0 \end{cases}$$

Unemployed workers

$$V^u(B_{t-1}, p_{e,t}) = \max_{\{c_t^u, g_t^u, e_t^u, B_t\}} \{\ln(c_t^u) + \beta\,\mathbb{E}_t\,[f_{t+1}\,V^n(B_t, p_{e,t+1}) + (1-f_{t+1})\,V^u(B_t, p_{e,t+1})]\}$$

$$\text{s.t.} \begin{cases} P_{e,t}\,e_t^u + P_{g,t}\,g_t^u + B_t \leq \Delta_t + (1+i_{t-1})\,B_{t-1} \\ c_t^u \leq (g_t^u)^{(1-\omega_e)}(e_t^u - \xi)^{\omega_e} \\ B_t \geq 0 \end{cases}$$

Letting Γ_t^i be the multiplier associated to the budget constraint of household $i \in \{n, u\}$, and μ_t^i that associated with the consumption aggregator, first-order conditions are

$$
\begin{aligned}
&\left[\frac{\partial}{\partial c_t^i}\right]: \quad \frac{1}{c_t^i} = \mu_t^i \\
&\left[\frac{\partial}{\partial g_t^i}\right]: \quad P_{g,t}\,\Gamma_t^i = (1-\omega_e)\,\frac{c_t^i}{g_t^i}\,\mu_t^i \\
&\left[\frac{\partial}{\partial g_t^i}\right]: \quad P_{e,t}\,\Gamma_t^i = \omega_e\,\frac{c_t^i}{e_t^i - \xi}\,\mu_t^i \\
&\left[\frac{\partial}{\partial B_t}\right]: \quad \begin{cases} \Gamma_t^n \geq \beta\,\mathbb{E}_t\left[(1-\lambda_{t+1})\,\frac{\partial V^n(B_t,p_{e,t+1})}{\partial B_t} + \lambda_{t+1}\,\frac{\partial V^u(B_t,p_{e,t+1})}{\partial B_t}\right] \\ \Gamma_t^u \geq \beta\,\mathbb{E}_t\left[f_{t+1}\,\frac{\partial V^n(B_t,p_{e,t+1})}{\partial B_t} + (1-f_{t+1})\,\frac{\partial V^u(B_t,p_{e,t+1})}{\partial B_t}\right] \end{cases}
\end{aligned}
$$

Combining the first three FOCs gives the following demand schedules

$$
g_t^i = (1-\omega_e)\,\frac{P_t}{P_{g,t}}\,c_t^i \tag{B.1}
$$

$$
e_t^i - \xi = \omega_e\,\frac{P_t}{P_{e,t}}\,c_t^i \tag{B.2}
$$

where

$$
P_t := \frac{\mu_t^i}{\Gamma_t^i} = \left(\frac{P_{g,t}}{1-\omega_e}\right)^{(1-\omega_e)} \left(\frac{P_{e,t}}{\omega_e}\right)^{\omega_e}\,. \tag{B.3}
$$

The envelope conditions are

$$
\frac{\partial V^i(B_{t-1},p_{e,t})}{\partial B_{t-1}} = (1+i_{t-1})\,\Gamma_t^i
$$

hence, since $\Gamma_t^i = 1/(P_t\,c_t^i)$, we get the following Euler conditions

$$
\frac{1}{c_t^n} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[(1-\lambda_{t+1})\,\frac{1}{c_{t+1}^n} + \lambda_{t+1}\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{B.4}
$$

$$
\frac{1}{c_t^u} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[f_{t+1}\,\frac{1}{c_{t+1}^n} + (1-f_{t+1})\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{B.5}
$$

Now, given the demand schedules for energy and non-energy goods, and the consumption aggregator and price index, total household expenditure is given by

$$
X_t^i := P_{g,t}\,g_t^i + P_{e,t}\,e_t^i = P_t\,c_t^i + P_{e,t}\,\xi \tag{B.6}
$$

giving in turn

$$
c_t^i = \frac{X_t^i}{P_t} - \frac{P_{e,t}}{P_t}\,\xi
$$

therefore, since at equilibrium $X_t^u = \Delta_t$ and $X_t^n = W_t$, Euler conditions become

$$\frac{1}{w_t - p_{e,t}\,\xi} \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[(1-\lambda_{t+1})\,\frac{1}{w_{t+1} - p_{e,t+1}\,\xi} + \lambda_{t+1}\,\frac{1}{\delta_t - p_{e,t+1}\,\xi} \right] \right\} \tag{B.7}$$

$$\frac{1}{\delta_t - p_{e,t}\,\xi} \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[f_{t+1}\,\frac{1}{w_{t+1} - p_{e,t+1}\,\xi} + (1-f_{t+1})\,\frac{1}{\delta_{t+1} - p_{e,t+1}\,\xi} \right] \right\} \tag{B.8}$$

where wage income, w_t, home production, δ_t, and the energy price, $p_{e,t}$, are expressed in real terms (i.e. in consumption basket units).

B.2 Implications of Non-homotheticity of Household Preferences

Own-price Elasticity of Energy Demand.

$$\varepsilon_{e,e} = \frac{\partial\, e}{\partial\, P_e}\frac{P_e}{e} = -\frac{\omega_e\, X}{\omega_e\, X + (1-\omega_e)\, P_e\, \xi} \tag{B.9}$$

demand for energy is inelastic, as $|\varepsilon_{e,e}| < 1$.

Cross-price Elasticity of Demand for Non-energy Goods.

$$\varepsilon_{g,e} = \frac{\partial\, g}{\partial\, P_e}\frac{P_e}{g} = -(1-\omega_e)\,\frac{P_e\,\xi}{P_g\, g} \tag{B.10}$$

since $\varepsilon_{g,e} < 0$, g is a gross complement of e.

Income Elasticity of Relative Demand.

$$\varepsilon_{X,e/g} = \frac{\partial\, e/g}{\partial\, X}\frac{X}{e/g} = -\frac{P_e\,\xi}{X}\left(\frac{P_e\,\xi}{X}\right)\left(\frac{X}{X - P_e\,\xi}\right)^2 \tag{B.11}$$

therefore, as X falls, households consume more e relative to g.

Income Share of Energy. At equilibrium, since households spend all their income, we have

$$P_{g,t}\, g_t^u + P_{e,t}\, e_t^u = \Delta_t \tag{B.12}$$

$$P_{g,t}\, g_t^n + P_{e,t}\, e_t^n = W_t \tag{B.13}$$

therefore, given (B.6),

$$P_t\, c_t^u = \Delta_t - P_{e,t}\,\xi \tag{B.14}$$

$$P_t\, c_t^n = W_t - P_{e,t}\,\xi \tag{B.15}$$

hence,

$$P_{e,t}\, e_t^n = \omega_e\, W_t + (1-\omega_e)\, P_{e,t}\, \xi \tag{B.16}$$

$$P_{e,t}\, e_t^u = \omega_e\, \Delta_t + (1-\omega_e)\, P_{e,t}\, \xi \tag{B.17}$$

and

$$\frac{P_{e,t}\, e_t^n}{W_t} = \omega_e + (1-\omega_e)\, \frac{P_{e,t}\, \xi}{W_t} \tag{B.18}$$

$$\frac{P_{e,t}\, e_t^u}{\Delta_t} = \omega_e + (1-\omega_e)\, \frac{P_{e,t}\, \xi}{\Delta_t} \tag{B.19}$$

from this, we see that since $\Delta_t < W_t$, the unemployed spend a higher share of their income on energy consumption.

B.3 Proof of Proposition 2

Since $\delta < w$,

$$\left(\frac{w - p_e\, \xi}{\delta - p_e\, \xi}\right) > 1 > \left(\frac{\delta - p_e\, \xi}{w - p_e\, \xi}\right)$$

therefore, since both $f < 1$ and $\lambda = \rho\,(1-f) < 1$,

$$(1-\lambda) + \lambda\, \left(\frac{w - p_e\, \xi}{\delta - p_e\, \xi}\right) > 1 > (1-f) + f\, \left(\frac{\delta - p_e\, \xi}{w - p_e\, \xi}\right)$$

implying, in turn, that

$$1 = \beta\, \left(\frac{1+i}{1+\pi}\right) \left[(1-\lambda) + \lambda\, \left(\frac{w - p_e\, \xi}{\delta - p_e\, \xi}\right)\right]$$

$$1 > \beta\, \left(\frac{1+i}{1+\pi}\right) \left[(1-f) + f\, \left(\frac{\delta - p_e\, \xi}{w - p_e\, \xi}\right)\right]$$

B.4 Constrained-Efficient Allocation

The first-order condition of (37) with respect to n_t gives

$$\ln\left(\frac{w_t^* - \xi\, p_{e,t}}{\delta_t - \xi\, p_{e,t}}\right) + \nu\, \left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t^* - \left(\frac{1}{1-\alpha}\right)\, \frac{\kappa}{q_t^*}\right] + \beta\, \mathbb{E}_t \left[\frac{\partial\, W(n_t, p_{e,t+1})}{\partial\, n_t}\right] = 0 \tag{B.20}$$

while the envelope condition reads

$$\frac{\partial W(n_{t-1}, p_{e,t})}{\partial n_{t-1}} = \nu \left(\frac{1}{1-\alpha}\right) (1-\rho) \frac{\kappa}{q_t^*} (1 - \alpha f_t^*) \,. \tag{B.21}$$

Combining (B.20) with (B.21) one period ahead and exploiting the fact that $q_t = f_t^{\frac{\alpha}{\alpha-1}}$, one can get the forward recursion for the constrained-efficient job-finding rate of equation (39).

B.5 Second-order Approximation of the Welfare Objective

When $\rho = 1$ and $\alpha = 0.5$,

$$U_t = \ln(\delta_t - \xi\, p_{e,t}) + n_t \left[\ln(w_t - \xi\, p_{e,t}) - \ln(\delta_t - \xi\, p_{e,t})\right] \\ + \nu \left\{ \left[\left(\frac{1}{1-\gamma_e}\right) \frac{p_{g,t}}{\mathcal{D}_t} - \left(\frac{\gamma_e}{1-\gamma_e}\right) p_{e,t} - w_t \right] n_t - \kappa\, n_t^2 - \xi\, p_{e,t} \right\}$$

and, at the constrained-efficient steady state,

$$\frac{\partial U}{\partial n} = \ln(w^* - \xi\, p_e) - \ln(\delta - \xi\, p_e) + \nu \left\{ \left[\left(\frac{1}{1-\gamma_e}\right) p_g - \left(\frac{\gamma_e}{1-\gamma_e}\right) p_e - w^* \right] - 2\kappa\, n^* \right\} = 0\,.$$

In second-order approximation, when $\widetilde{w}_t = -\chi\, \widetilde{p}_{e,t}$,

$$U_t \simeq -\nu\, \kappa\, \widehat{n}_t^2 + \frac{\partial^2 U}{\partial n\, \partial p_e} \widehat{p}_{e,t}\, \widehat{n}_t - \frac{\nu\, n\, p_g}{1-\gamma_e} (\mathcal{D}_t - 1) + \text{t.i.p.}$$

Noticing that, since derivatives are evaluated at the constrained-efficient steady state,

$$\frac{\partial^2 U}{\partial n\, \partial p_e} \widehat{p}_{e,t} = 2\, \kappa\, \nu\, \widehat{n}_t^*$$

and $\widehat{n}_t = -\mathcal{U}_t + \widehat{n}_t^*$, we have

$$U_t \simeq -\nu\, \kappa\, \mathcal{U}_t^2 - \frac{\nu\, n\, p_g}{1-\gamma_e} (\mathcal{D}_t - 1) + \text{t.i.p.}$$

where

$$\mathcal{D}_t - 1 \simeq \ln(\mathcal{D}_t) = \ln \int_0^1 \left[\frac{P_t(k)}{P_{g,t}}\right]^{-\varepsilon} dk \simeq \frac{\varepsilon}{2} \text{var}_k \left\{\ln(P_t(k))\right\}$$

Lemma 1.
$$\sum_{t=0}^{\infty} \beta^t \operatorname{var}_k \{\ln(P_t(k))\} = \frac{1}{\Theta} \sum_{t=0}^{\infty} \beta^t \pi_{g,t}^2 + t.i.p.$$

Proof. See Chapter 6 in Woodford (2003). □

Therefore,

$$\mathbb{E}_t \sum_{j=0}^{\infty} U_{t+j} \simeq -\kappa\, \nu\, \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\mathcal{U}_{t+j}^2 + \Omega\, \pi_{g,t+j}^2\right) + \text{t.i.p.} \tag{B.22}$$

with $\Omega := \frac{n\,\varepsilon}{(1-\gamma_e)} \frac{p_g}{2\,\kappa} / \Theta$.

B.6 Optimal Discretionary Policy

The optimality condition of the problem gives

$$\mathcal{U}_t = \left(\frac{n\,\varepsilon}{1-\gamma_e}\right) \pi_{g,t} \tag{B.23}$$

which, combined with (56), gives

$$\pi_{g,t} = \frac{\beta}{1 + \Theta\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}} \mathbb{E}_t(\pi_{g,t+1}) + \frac{\Theta\, \frac{w}{p_g}\, \zeta\, \Psi \left(\frac{\Xi_w}{1-\Xi_w}\right) (1+\chi)}{1 + \Theta\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}} \widetilde{p}_{e,t}\,. \tag{B.24}$$

Since $\beta \left(1 + \Theta\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1} < 1$, the linear expectational difference equation in (B.24) has a unique bounded solution giving, along with the optimality condition in (B.23), the targets for core inflation and the welfare-relevant unemployment gap of equations (58) and (59).

B.7 Optimal Ramsey Policy

The first-order conditions of the problem are, for $t \geq 0$,

$$\Omega\, \pi_{g,t} + \mu_t - \mu_{t-1} = 0 \tag{B.25}$$

$$\mathcal{U}_t + \Theta\, \frac{2\,\kappa}{p_g}\, \mu_t = 0 \tag{B.26}$$

where μ_t is the Lagrange multiplier associated with the period-t constraint and $\mu_{-1} = 0$. Approaching the problem from a *timeless perspective* (see Woodford, 2003), the first-order conditions give

$$\left(\frac{n\,\varepsilon}{1-\gamma_e}\right) \pi_{g,t} - \mathcal{U}_t + \mathcal{U}_{t-1} = 0\,. \tag{B.27}$$

Combined with the NKPC in (56), this gives

$$\mathcal{U}_t = \frac{\beta}{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}\,\mathbb{E}_t(\mathcal{U}_{t+1}) + \frac{1}{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}\,\mathcal{U}_{t-1} + \frac{\Theta\,\frac{w}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}{1+\beta+\Omega\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t}$$

whose unique stationary solution is

$$\mathcal{U}_t = \eta\,\mathcal{U}_{t-1} + \left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\,\overline{\Upsilon}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \qquad \text{(B.28)}$$

where $\eta := \frac{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}{2\,\beta}\left[1-\sqrt{1-4\,\beta\,\left(1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-2}}\,\right] \in (0,1)$ and $\overline{\Upsilon} = \Theta\,\frac{w}{p_g}\,\left(1-\eta\,\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1}$.

The paths for $\mathcal{U}_{t+j}$ and $\pi_{g,t+j}$ can then be recovered by exploiting (B.27) and (B.28), imposing a steady-state initial condition (i.e. $\mathcal{U}_{t-1} = 0$) and considering a one-off shock at t with persistence $\rho_e \to 1$.

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