Optimal Climate Policy in a Global Economy

Preliminary and incomplete

(**not for circulation**)

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Abstract

This paper studies the optimal climate policy in a global economy. Emissions impose a dynamic, *global*, negative externality, raising natural questions about international policy coordination. To understand the issues involved, I first build a simple dynamic multi-country model and study the optimal cooperative climate policy that corrects the global externality. Moreover, I move *beyond* cooperation and study the optimal climate policy for a *large* country, that faces a *passive* rest of the world. In such a setup, incentives for corrective taxation are intertwined with interest rate manipulation. Implications for optimal carbon taxes and capital controls are drawn.

Keywords: climate change, carbon taxation, international cooperation, non-cooperative policy, capital controls.

JEL classification: E61; H21; H23; Q43; Q54

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1 Introduction

One of the fundamental challenges for humankind is the risk of climate change and its repercussions for growth and welfare. A voluminous literature in climate science has developed over the last decades about global warming. Similarly, economists have started to think hard about aggregative models that incorporate climate issues, an effort that has lead to the environmental macroeconomics literature.[1] The stakes are high, necessitating the need for intergovernmental agreements and for a concerted effort among policymakers, industry leaders and international organizations.[2]

Integrated Assessment models (IAMs) have been built to study the interactions between the climate and the economy. One prominent model, especially for economists, is the DICE (Dynamic Integrated Climate Economy) model of Nordhaus (1994) and Nordhaus (2008). Climate-economic models typically integrate a climate module, which is usually a simplified summary of the climate system coming from the natural sciences, into a dynamic, aggregative model of the economy. Schematically: greenhouse gas emissions, a product of economic activity, lead into the rise of atmospheric temperature, which leads to damages to the environmental quality, rise of sea level, droughts, ice melting, extreme weather events, and ultimately to reduction in economic growth, maybe of catastrophic proportions.

New generations of models like Golosov et al. (2014) and Cai and Lontzek (2019) have introduced climate and growth risks in dynamic macroeconomic models and have analyzed the implications for the social cost of carbon (SCC), that is, the marginal cost of increasing by one unit atmospheric carbon. Given the fact that there is a *negative* externality, the social cost of carbon captures essentially the *Pigouvian* carbon tax.

There is a fundamental feature in the study of climate change coming from the nature of emissions. The effect of emissions is global, durable and "bad". So emissions act as a durable *public "bad"*. This feature necessitates international cooperation, a fact that policymakers have acknowledged for a long time and which led to the Paris Agreement in 2015. However, the agreement is non-binding and compliance to targets by large emitters as China (30% of global emissions in 2020) or the United States (14% of emissions in 2020) is crucial.

In this paper I want to understand deeper the global nature of the emission externality and its implications for international policy and coordination. In particular, I study environments that go beyond the *ideal* of cooperative policy and consider setups that may involve a large country that acts in a non-cooperative way.[3]

[1]See for example Hassler and Krusell (2018) for a detailed survey of environmental macroeconomics and climate change economics.

[2]The implications of climate change for the economy are topical also for monetary policy makers. See for example Hale et al. (2019) for a summary of a recent conference in November 2019 at the Federal Reserve Bank of San Francisco.

[3]See Hassler et al. (2021) for an exercise that goes beyond the standard Pigouvian taxation problem by

To understand the tradeoffs that emerge in a multiple-country model, I build a model based on a simplified version of Golosov et al. (2014). Consider first a deterministic closed economy, without physical capital and without exhaustible resources. There is a representative household that consumes in equilibrium a final good net of climate-induced damages. The final good is produced with labor and 'dirty' energy. Energy is produced with labor. Use of energy contributes to the stock of emissions over time, reducing net of damages output that is available for consumption. A benevolent social planner that maximizes the utility of the representative household chooses the optimal intersectoral allocation of labor, and the optimal use of energy, taking into account that the energy generates current and future damages. Instead, competitive firms in the market economy do not internalize how the production of energy generates damages, so energy generates a negative production externality. The social-planner allocation can be implemented by using a *corrective* carbon tax that is rebated lump-sum. Such a tax reduces the socially inefficient production of energy.

I extend this simple setup in a multiple-country model. Country-specific production of energy affects global emissions. And global emissions affect country-specific damages. I consider two distinct policy environments that capture several degrees of cooperation: a) a *cooperative* environment where a global social planner is maximizing the weighted average of the representative household's utilities in each country, subject to the global resource constraint, production technologies and and the law of motion of the stock of emissions, b) Following Costinot et al. (2014), a setup where a large country (that could be the US or China) trades intertemporally with the rest of the world and acts as a *dynamic monopolist*, treating the rest of the world as *passive*. In such a setup there are incentives for the manipulation of intertemporal terms of trade (interest rates) that may justify capital controls. Moreover, there is a non-trivial interaction between climate policies, consumption, and equilibrium prices that make it useful for the analysis of the joint capital-control and climate policy of a large country.

The cooperative equilibrium is characterized by three conditions: a) *consumption efficiency*, which posits that the ratio of marginal utilities across countries should be *constant* b) *Intersectoral labor efficiency in each country*, which determines the efficient allocation of labor between the two sectors in each country, taking into account the damages that an increase in the production of energy creates, b) *Intertemporal* climate efficiency, which posits that the shadow cost of the stock of emissions should reflect the *global* current and future marginal damages. A global carbon tax can implement the Pareto-efficient allocation.

I am currently working on the dynamic monopolist equilibrium. Costinot et al. (2014) have shown that in order to manipulate equilibrium interest rates, the large country has incentives to tax capital inflows when it grows faster than the rest of the world. Interestingly enough, in a setup with climate externalities, a dynamic monopolist cares for the damages of the rest

treating suboptimal policies.

of the world, even if there are *no* damages for its own country. The reason is interest rate manipulation. Foreign damages affect world interest rates, and are therefore taken into account by a large country. The interaction of these price manipulation incentives with the optimal climate policy is an open question.

Related literature. [To be completed.]

Organization. Section 2 lays down a simple closed economy that is the building block of the multiple-country economy. It derives the efficient allocation and it contrasts it to the competitive equilibrium allocation. Section 3 analyzes a multiple-country economy, the main setup of interest. Section 4 derives the optimal cooperative climate policy and shows how it can be implemented with a global carbon tax. Section 5 lays down the analysis for a large country that acts as a dynamic monopolist facing a passive rest of the world, and analyzes the interaction between climate policy and optimal capital flows. Section 6 concludes.

2 A simple closed economy

I start with a simple closed economy, to set the stage for the analysis in the multi-country environment. The setup is a simplified version of the closed economy of Golosov et al. (2014).

Time is discrete and the horizon is infinite. The economy is deterministic, without physical capital and without exhaustible resources. There is a final good that is produced by using labor n_t and the intermediate input energy E_t (in carbon content) (or the flow of "emissions"- we use the two terms interchangeably). Energy is 'dirty' and produced in an energy sector using labor $n_{E,t}$.

Resource constraint and production sectors. Climate enters the economy by creating *damages*. Damages are modeled as a reduction of total output that is available for consumption. The resource constraint in the economy reads

$$c_t = Y_t \equiv (1 - D(S_t))\hat{Y}_t \tag{1}$$

where c_t is consumption, Y_t net (of damages) output, $\hat{Y}_t$ gross output and S_t a climate variable. The damage function is increasing in S and differentiable, so $D_S > 0$. Gross output $\hat{Y}_t$ is given by

$$\hat{Y}_t = A_t F(n_t, E_t), \tag{2}$$

where A_t an exogenous technology process and F a constant returns to scale production function that is increasing in both arguments and concave. The production technology for energy is

$$E_t = z_t f(n_{E,t}), \tag{3}$$

where z_t an exogenous forcing process and f increasing and concave.[4]

Law of motion of emissions. The climate module in this economy is captured by a simple law of motions for the climate variable S_t, which we think of as the *stock* of emissions,

$$S_t = H(S_{t-1}, E_t), \tag{4}$$

where H is increasing and differentiable in both arguments, $H_E, H_S > 0$ and S_{-1} given.[5] The law of motion (4) captures the dependence on the past, so the *durability* of emissions (and the subsequent increases in temperature). Increases in E_t increase gross output through (2), but increase also S_t and damages through (4) and (1) respectively (so emissions are "bad"), reducing net output that is available for consumption. This is the basic tradeoff in the heart of the social planner's problem.

Preferences. Lastly, the economy is populated by a representative household that derives utility from streams of consumption $\{c_t\}$ and there is no disutility of labor. The preferences of the household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{5}$$

where u an increasing, concave and twice differentiable period utility function that satisfies the Inada condition, and $\beta \in (0, 1)$ the subjective discount factor. The household is endowed with one unit of time that is allocated as labor between the two sectors, so $n_t + n_{E,t} = 1$.

2.1 First-best

To understand the nature of the climate externality and the need for corrective taxation, consider first a benevolent social planner who maximizes the utility of the representative household, taking

[4] I allow full generality here for the production function f, but I assume a constant returns to scale production function when we proceed to the baseline example, so $E_t = z_t n_{E,t}$.

[5] The function H could also depend on time t. Any such dependence is implicit.

into account how energy increases emissions and damages in the economy. After eliminating labor in the energy sector $n_{E,t}$ by using $n_{E,t} = 1 - n_t$, we can state the social planner's problem.

Problem 1. *("First-best") The social planner chooses sequences $\{c_t, n_t, E_t, S_t\}$ to maximize the utility of the representative household (5), subject to*

$$c_t = (1 - D(S_t))A_t F(n_t, E_t) \tag{6}$$

$$E_t = z_t f(1 - n_t) \tag{7}$$

$$S_t = H(S_{t-1}, E_t), \tag{8}$$

non-negativity and feasibility constraints $c_t \geq 0, n_t \in [0, 1]$, with S_{-1} given.

Let $\{\beta^t \lambda_t, \beta^t \mu_t, \beta^t \xi_t\}$ denote the multipliers on (6), (7) and (8) respectively. Define also $\tilde{\mu}_t \equiv \mu_t/\lambda_t$ and $\tilde{\xi}_t \equiv \xi_t/\lambda_t$, that is, the respective *scaled* multipliers in terms of final good consumption, since $\lambda_t = u_c(c_t)$. The efficient allocation is characterized by the following optimality conditions.[6]

Efficient allocation of labor. The efficient allocation of labor between the final good sector and the energy sector satisfies[7]

$$\underbrace{(1 - D_t)A_t F_{n,t}}_{MP_n^{final}} = \underbrace{\tilde{\mu}_t}_{\text{shadow value of } E_t} \times \underbrace{z_t f_n(1 - n_t)}_{MP_n^{energy}} \tag{9}$$

Thus, at the optimal allocation, the marginal product of labor in the final good sector is *equalized* to the marginal product of labor (in terms of energy) in the energy sector, times the shadow value of energy E_t, as captured by $\tilde{\mu}_t$.

Shadow value of energy. The shadow value of energy $\tilde{\mu}_t$ is given by

$$\tilde{\mu}_t = \underbrace{(1 - D_t)A_t F_{E,t}}_{MP_E^{final}} \underbrace{- \tilde{\xi}_t H_{E,t}}_{\text{cost of emissions-externality}} . \tag{10}$$

The shadow value of energy has *two* components: an increase in E_t is beneficial, since it increases final good output, captured by the marginal product of energy. However, increases in energy increase the stock of emissions (through $H_{E,t}$), and therefore damages, which have

[6] Throughout the analysis, $F_{i,t}, i = n, E$, and $H_{i,t}, i = S, E$ is shorthand notation for the partial derivatives of F and H at t. Similarly, D_t and $D_{S,t}$ stand for the respective damages and marginal damages at t.

[7] I use the terms efficient and first-best interchangeably.

shadow cost $\tilde{\xi}_t$. This the second component in (10), and is exactly the production *externality* that is ignored by competitive firms in the market equilibrium, as we will see later.

Shadow cost of the stock of emissions. The shadow cost of S_t is given by

$$\tilde{\xi}_t = \underbrace{D_{S,t}\hat{Y}_t}_{\text{current marginal damages}} + \underbrace{\beta \frac{u_{c,t+1}}{u_{c,t}} H_{S,t+1}\tilde{\xi}_{t+1}}_{\text{future marginal damages}}, \tag{11}$$

which we can solve forward to get

$$\tilde{\xi}_t = \underbrace{\sum_{i=0}^{\infty} \beta^i \frac{u_{c,t+i}}{u_{c,t}} X_{t,i} D_{S,t+i}\hat{Y}_{t+i}}_{\propto \text{ social cost of carbon}} > 0, \tag{12}$$

where $X_{t,i} \equiv \Pi_{j=1}^{i} H_{S,t+i}$, with $X_{t,0} \equiv 1$, a multiplicative factor that captures the depreciation (if any) of the emission stock. Thus, the shadow cost of the stock of emissions consists of *current* marginal damages $D_S(S_t)\hat{Y}_t$, and of the present discounted value of *future* marginal damages, since emissions are durable. We can dub the product of $H_{E,t} \times \tilde{\xi}_t$ the *social cost of carbon*. This is the social cost of the production externality, that generates the rationale for corrective (Pigouvian) carbon taxation.

2.2 Market economy

Consider now the respective laissez-faire market economy (the "business as usual" scenario).

Household's problem. The household consumes, saves or borrows, gets labor income by providing its time endowment to the two sectors, and enjoys profits from the final good firm and the energy sector firm. The household's dynamic budget constraint is given by

$$c_t + p_t b_{t+1} = w_t + \Pi_t + \pi_t + b_t, \tag{13}$$

where p_t the price of a discount bond that promises one unit of consumption next period, b_t the bond holdings in the beginning of the period, w_t the wage rate, and Π_t and π_t the profits of the final good sector firm and the energy sector firm respectively. Initial bonds are set to zero, $b_0 = 0$. The problem of the household it to choose consumption $c_t \geq 0$ and bond holdings b_{t+1} to maximize its utility (5) subject to (13) and an appropriate no-Ponzi game condition. As usual, the household equalizes the price of bonds to its intertemporal marginal rate of substitution,

$$p_t = \beta \frac{u_{c,t+1}}{u_{c,t}}. \tag{14}$$

Final good sector. The competitive firm in the final good sector chooses labor n_t and energy E_t to maximize profits, taking the stock of emissions S_t, the wage rate w_t and the price of energy $p_{E,t}$ as given. Profits are given by

$$\Pi_t = (1 - D(S_t))A_t F(n_t, E_t) - w_t n_t - p_{E,t} E_t. \tag{15}$$

Profit maximization leads to the equalization of the marginal products to the respective prices,

$$(1 - D(S_t))A_t F_{n,t} = w_t \tag{16}$$
$$(1 - D(S_t))A_t F_{E,t} = p_{E,t} \tag{17}$$

These two conditions determine the demand for labor and energy in the final good sector.

Energy sector. Taking as given the wage rate and the price of energy, the energy sector firm chooses $n_{E,t}$ to mazimize its profits,

$$\pi_t = p_{E,t} z_t f(n_{E,t}) - w_t n_{E,t}. \tag{18}$$

Profit maximization leads to

$$p_{E,t} z_t f_n(n_{E,t}) = w_t, \tag{19}$$

which equalizes the marginal product labor in the energy sector to the relative wage $w_t/p_{E,t}$.

Equilibrium. A competitive equilibrium is a collection of prices $\{p, w, p_E\}$ and allocations, such that the household maximizes its utility, firms maximize profits, and markets clear, that is, the resource constraint (1) holds, bond markets clear, $b_{t+1} = 0, \forall t$, the labor market clears, $n_t + n_{E,t} = 1$, and the stock of emissions evolves according to (4).

Contrasting the market economy to the first-best. From the profit maximization conditions of the two firms (16), (17) and (19), and using $n_{E,t} = 1 - n_t$ we get the following relationship between the marginal products of labor and energy in the two sectors.

$$\underbrace{(1 - D(S_t))A_t F_n(n_t, E_t)}_{MP_n^{final}} = \underbrace{(1 - D(S_t))A_t F_E(n_t, E_t)}_{MP_E^{final}} \times \underbrace{z_t f_n(1 - n_t)}_{MP_n^{energy}} \tag{20}$$

Contrast (20) with the respective conditions in the first-best, (9). The shadow value of energy $\tilde{\mu}_t$ includes both the marginal product of energy and the costs associated with an increase in emissions, $\tilde{\xi}_t H_{E,t}$. These negative production externality is ignored by competitive firms, leading to the deviation of the market economy allocation from the efficient one.

We can rewrite (20) as

$$\frac{F_n(n_t, E_t)}{F_E(n_t, E_t)} = z_t f_n(1 - n_t), \tag{21}$$

which equalizes the marginal rate of technical substitution of n and E to the marginal product of labor in the energy sector. Instead, in the efficient allocation we have

$$\frac{F_n(n_t, E_t)}{F_E(n_t, E_t)} = \underbrace{[1 - \frac{\tilde{\xi}_t H_{E,t}}{(1 - D_t)A_t F_{E,t}}]}_{<1} z_t f_n(1 - n_t) < z_t f_n(1 - n_t), \tag{22}$$

so the marginal rate of technical substitution has to be *smaller* than the marginal product of labor in the energy sector. This leads, as we will see in our baseline example, to an amount of labor devoted to the final good sector that is larger in the first-best than in the competitive equilibrium. Analogously, the first-best production of energy is smaller than the competitive equilibrium amount of energy.

Market allocation. Using $E_t = z_t f(1 - n_t)$ in the *static* condition (21), we can solve for n_t *solely* as function of z_t, $n_t^{CE} = n(z_t)$. Thus, the market allocation of labor between the two sectors does not depend on the stock of emissions S_t and on A_t. The optimal allocation of labor determines the production of energy $E_t^{CE} = E(z_t)$, and therefore of gross output, $\hat{Y}_t^{CE} = \hat{Y}(z_t, A_t)$. Given energy E_t^{CE} we get from (4) the respective stock of emissions in the laissez-faire economy S_t^{CE}, which in the end determines net of damages output and therefore consumption through (1).

Decentralizing the first-best with carbon taxes on energy producers. A simple way to decentralize the first-best allocation is to internalize the externality by imposing a unit (carbon) tax on energy producers, τ_t, that is rebated lump-sum to consumers, $T_t \equiv \tau_t E_t$, and is equal to the *size* of the externality. To see that, consider the profits of the energy firm that pay the unit tax for any carbon unit E_t,

$$\pi_t^{\text{tax}} = \left[p_{E,t} - \tau_t\right] z_t f(n_{E,t}) - w_t n_{E,t}, \tag{23}$$

leading to the profit maximization condition $\left[p_{E,t} - \tau_t\right] z_t f_n(n_{E,t}) = w_t$. Using (16), (17) and setting $\tau_t = \tilde{\xi}_t H_{E,t}$, delivers the first-best allocation.

2.3 Baseline example

We make standard assumptions, following Golosov et al. (2014) and the macroeconomic literature.

Preferences and production functions. Consider a power period utility function of consumption, that implies a constant elasticity of intertemporal substitution,

$$u(c) = \frac{c^{1-\rho} - 1}{1-\rho}, \tag{24}$$

where $\rho > 0$. Let the production function in the final good sector be Cobb-Douglas and let the production function in the energy sector be linear, so

$$F(n, E) = n^{\alpha} E^{1-\alpha}, \quad f(n) = n, \tag{25}$$

with $\alpha \in (0, 1)$.

Damage function. Let the damage function have the exponential form of Golosov et al. (2014),

$$D(S_t) = 1 - \exp\left(-\gamma(S_t - \bar{S})\right), \tag{26}$$

where $\bar{S}$ the stock of emissions in pre-industrial times, and $\gamma > 0$ a positive parameter that captures the damage sensitivity. There would be no damages, $D(S_t) = 0$, and therefore no externality, if $\gamma = 0$, so the shadow cost of S_t. In that case, the social cost of carbon would collapse to zero, $\tilde{\xi}_t = 0$.

Law of motion of emissions. Use the Golosov et al. (2014) linear specification that combines transitory and permanent components.

[To be completed]

3 A multiple-country setup

Let's move now to the main environment of interest, which a multiple-country version of the economy in section 2. Assume that we have N countries and let the superscript l denote the country, $l = 1, ..., N$.

Production sectors and damages. Each country l has a unit of labor that is allocated in the production of the final good (n_t^l) and in the production of energy ($n_{E,t}^l$), so $n_t^l + n_{E,t}^l = 1, \forall l$. Gross final good output in country l is denoted by $\hat{Y}_t^l$. Net of damages output is given by

$$Y_t^l = (1 - D^l(S_t))\hat{Y}_t^l = (1 - D^l(S_t))A_t^l F^l(n_t^l, E_t^l), l = 1, .., N. \tag{27}$$

The production technology for energy E_t^l is given by

$$E_t^l = z_t^l f^l(n_{E,t}^l), l = 1, ..., N. \tag{28}$$

Note that production functions, technology processes and damage functions are indexed by country l, and have the same properties as in the one country model we previously considered. Country-specific damages though depend on the *global* stock of emissions S_t, capturing the global nature of the negative externality that emissions impose. Total flow emissions E_t are given by the sum of country-specific emissions, $E_t \equiv \sum_{l=1}^N E_t^l$. The global stock S_t follows the law of motion

$$S_t = H(S_{t-1}, \sum_{l=1}^{N} E_t^l), \tag{29}$$

with S_{-1} given, and H increasing in both arguments.

Preferences and resource constraint. The preferences of the representative household in each country l are given by

$$\sum_{t=0}^{\infty} \beta^t u^l(c_t^l), \tag{30}$$

where c_t^l the consumption of final good in country l and u^l a period utility function for country l, that is increasing, concave and satisfies the Inada condition. The global resource constraint in this setup is given by

$$\sum_{l=0}^{N} c_t^l = \sum_{l=0}^{N} Y_t^l. \tag{31}$$

4 Optimal cooperative policy

Assign on each country Pareto weights $\eta^l > 0, l = 1, ..., N$, with the normalization $\sum_{l=1}^{N} \eta^l = 1$. The problem of the global social planner is to choose sequences $\{c_t^l, n_t^l, E_t^l, S_t\}$ to maximize the weighted utility

$$\sum_{l=1}^{N} \eta^l \sum_{t=0}^{\infty} \beta^t u^l(c_t^l) \tag{32}$$

subject to

$$\sum_{l}^{N} c_t^l = \sum_{l=0}^{N} (1 - D^l(S_t)) A_t^l F^l(n_t^l, E_t^l), \forall t \tag{33}$$

$$E_t^l = z_t^l f^l(1 - n_t^l), \forall l, t \tag{34}$$

$$S_t = H(S_{t-1}, \sum_{l}^{N} E_t^l), \forall t \tag{35}$$

with S_{-1} given, and $c_t^l \geq 0, n_t^l \in [0, 1], \forall l, t$. Assign sequences of multipliers $\{\beta^t \lambda_t\}$ on (33), $\{\beta^t \mu_t^l\}$ on the energy production technology for each country (34), and $\{\beta^t \xi_t^{global}\}$ on the law of motion of S_t in (35).

4.1 Optimality conditions.

Consumption efficiency. The first-order condition with respect to consumption at country l delivers $\eta^l u_c^l(c_t^l) = \lambda_t, \forall l$, where λ_t the shadow value of world output. This implies the standard condition for Pareto-optimal allocations between countries k and l,

$$\frac{u_{c,t}^l}{u_{c,t}^k} = \frac{\eta^k}{\eta^l}, \forall k, l, t. \tag{36}$$

Thus, the ratio of marginal utilities is *constant* for each t and inversely related to the ratio of Pareto weights. Consequently, intertemporal marginal rates of substitution are equalized across countries, $\beta \frac{u_{c,t+1}^l}{u_{c,t}^l} = \beta \frac{u_{c,t+1}^k}{u_{c,t}^k} \forall k, l$. Condition (36), together with the global resource constraint (31) implies that consumption for each country is a function only of the world output $Y_t \equiv \sum_{l=1}^N Y_t^l$.

Efficient intersectoral allocation of labor and shadow value of energy in country l. Turning into labor n_t^l in country l, define for convenience the scaled multipliers $\tilde{\mu}_t^l \equiv \mu_t^l / \lambda_t, \forall l$ and $\tilde{\xi}_t^{global} \equiv \xi_t^{global} / \lambda_t$. The optimal allocation intersectoral allocation of labor satisfies

$$(1 - D_t^l) A_t^l F_{n,t}^l = \tilde{\mu}_t^l \times z_t^l f_n^l 1 - n_t^l), \tag{37}$$

where the shadow value of energy for country l is given by

$$\tilde{\mu}_t^l = \underbrace{(1 - D_t^l) A_t^l F_{E,t}^l}_{MP_E^{final} \text{in } l} \quad \underbrace{-\tilde{\xi}_t^{global} H_{E,t}}_{\text{global externality}}. \tag{38}$$

The shadow value of energy E_t^l is equal to the marginal product of energy in the final good sector, a component which is *country-specific*, minus the shadow cost of the stock of emissions, a component that is *global*, $\tilde{\xi}_t^{global}$. This feature necessitates a global carbon tax to correct the negative production externality.

Shadow cost of the global stock of emissions. The shadow cost of the stock of emissions in the multiple-country setup is given by

$$\tilde{\xi}_t^{global} = \underbrace{\sum_{l=1}^N D_{S,t}^l \hat{Y}_t^l}_{\text{global current marginal damages}} + \underbrace{\beta \frac{\lambda_{t+1}}{\lambda_t} H_{S,t+1} \tilde{\xi}_{t+1}^{global}}_{\text{future global marginal damages}}, \tag{39}$$

where $\lambda_{t+1}/\lambda_t = u^l_{c,t+1}/u^l_{c,t} = u^k_{c,t+1}/u^k_{c,t}, \forall k, l$. Thus, the global planner takes into account the global current and future marginal damages that emissions impose.

4.2 Market equilibrium

Household. The household in each country l chooses $c^l_t \geq 0, b^l_{t+1}$ to maximize (30) subject to the respective country budget constraint,

$$c^l_t + p_t b^l_{t+1} = w^l_t + \Pi^l_t + \pi^l_t + b^l_t, \tag{40}$$

and the no-Ponzi-game condition, where initial holdings are set to zero for each country, $b^l_0 = 0, \forall l$. The variables w^l_t, Π^l_t, π^l_t are the country specific wage rate, final good sector profits and energy sector profits respectively. The budget constraint (40) is the country-specific version of (13). Household optimality requires that

$$p_t = \beta \frac{u^l_{c,t+1}}{u^l_{c,t}}. \tag{41}$$

Final good sector and energy sector firms. The profits of the two firms in country l are respectively

$$\Pi^l_t = (1 - D^l(S_t))A^l_t F^l(n^l_t, E^l_t) - w^l_t n^l_t - p^l_{E,t} E^l_t \tag{42}$$
$$\pi^l_t = p^l_{E,t} z^l_t f^l(n^l_{E,t}) - w^l_t n^l_{E,t}. \tag{43}$$

As in the closed economy, profit maximization in the two sectors leads to equalization of marginal products to the respective country-specific prices,

$$(1 - D^l(S_t))A^l_t F^l_{n,t} = w^l_t \tag{44}$$
$$(1 - D^l(S_t))A^l_t F^l_{E,t} = p^l_{E,t} \tag{45}$$
$$p^l_{E,t} z^l_t f^l_n(n^l_{E,t}) = w^l_t. \tag{46}$$

Competitive equilibrium in the global economy with externalities. The competitive equilibrium is a collection of bond prices p, country-specific wage rates and energy prices $\{w^l, p^l_E\}^N_{l=1}$, allocations $\{c^l, n^l, E^l\}$, stock of emissions S and bond holdings b^l, such that the household in each country maximizes utility, firms in each country maximize profits, and markets clear, that is, the global resource constraint holds (31), bond markets clear $\sum_{l=1}^{N} b_{t+1}^l = 0$, labor markets clear $n_t^l + n_{E,t}^l = 1 \forall l$, and the global stock of emissions evolves as (29).

Analysis of the competitive equilibrium. Using (44), (45) and (46) we get

$$\frac{F_n^l(n_t^l, E_t^l)}{F_E^l(n_t^l, E_t^l)} = z_t^l f_n^l(1 - n_t^l), \forall l \tag{47}$$

which equalizes the marginal rate of technical substitution in the final good sector of country l to the marginal product of labor in the energy sector in l. Condition (47) extends the respective conditions we found in the closed economy in (20) and (21) and is contrasted to the efficiency conditions (37) and (38). Firms do not internalize the damages that are involved with the production of energy. Condition (47) allows us to solve for labor in the final good sector of country l as function only of z_t^l, $n_t^l = n(z_t^l)$, which makes also country-specific energy a function of z_t^l, $E_t^l = E(z_t^l)$.

Moreover, the equalization of intermporal marginal rates of substitution to the price of bonds p_t in (41), makes the ratio of marginal utilities in the competitive equilibrium *constant*, which (using (31)) implies that the consumption of country l is function solely of the global net output $\sum_l Y_t^l$. This property is the *same* as in the Pareto efficient allocation, *but* the level of the global output in the efficient allocation is obviously different.[8]

4.3 Decentralization of cooperative policy and baseline example

[To be completed.]

5 Beyond cooperation: a dynamic monopolist

Consider the previous environment and assume that we have two countries, $N = 2$, calling country 1 and 2, H (for Home) and F (for Foreign), respectively.[9] We may think of Home as a large emitter (China or the U.S.) and of Foreign the rest of the world. Home acts as a *dynamic monopolist*, understanding how prices are formed, following Costinot et al. (2014). The foreign country instead, is a *passive* price-taker.

I start with the problem of F. F is not implementing any climate policy, following a "business as usual" scenario. The household and firms in F acts as price-takers, so I can use the optimality

[8] Associate to the work of Hillebrand and Hillebrand (2019).

[9] To keep the notation comparable to the previous sections, I avoid using the customary asterisk notation of the international macroeconomic literature to denote the foreign country.

conditions we found in the analysis of the market economy in section 4.2.

The household's budget constraint in F is

$$c_t^2 + p_t b_{t+1}^2 = w_t^2 + \Pi_t^2 + \pi_t^2 + b_t^2, \tag{48}$$

with initial debt set to zero, $b_0^2 = 0$ (so we have also $b_0^1 = 0$). Maximization of utility of the household in F leads to

$$p_t = \beta \frac{u_c^2(c_{t+1}^2)}{u_c^2(c_t^2)}. \tag{49}$$

Moreover, recall that profit maximization in the final food sector and the energy sector in F, together with equilibrium in the labor market of F leads to

$$\frac{F_n^2(n_t^2, E_t^2)}{F_E^2(n_t^2, E_t^2)} = z_t^2 f_n^2(1 - n_t^2). \tag{50}$$

As previously noted, given that $E_t^2 = z_t^2 f(1 - n_t^2)$, equation (50) can be solved for labor n_t^2 as function solely of z_t^2, $n_t^2 = n^2(z_t^2)$. Similarly, foreign energy production is $E_t^2 = E^2(z_t^2)$.

Finally, the foreign resource constraint is

$$c_t^2 + p_t b_{t+1}^2 = Y_t^2 + b_t^2 \tag{51}$$

where $Y_t^2 = (1 - D^2(S_t))\hat{Y}_t^2$, with Y_t^2 the foreign *net* (of damages) output, and $\hat{Y}_t^2 = A_t^2 F^2(n_t^2, E_t^2)$, the *gross* foreign output. Given the determination of labor and energy from (50), the foreign gross output is only function of (A_t^2, z_t^2) and does *not* depend on the stock of emissions S_t.

5.1 Ramsey problem

Consider now H, that acts as a dynamic monopolist who faces a price-taker.[10] It maximizes at $t = 0$ the utility of the domestic representative household, by choosing consumption, the allocation of labor between the final good sector and the energy sector, and its emissions, taking into account the foreign optimality conditions, budget constraints and resource constraints, as well as the global resource constraint,

[10]Note the similarities to settings where a large firm faces a competitive fringe.

$$c_t^1 + c_t^2 = Y_t^1 + Y_t^2, \tag{52}$$

where $Y_t^1 = (1 - D^1(S_t))\hat{Y}_t^1$, the net output of H, and $\hat{Y}_t^1 = A_t^1 F^1(n_t^1, E_t^1)$, the respective gross output. The production function of energy at home is given by

$$E_t^1 = z_t^1 f^1(1 - n_t^1), \tag{53}$$

where I have already used the fact that in equilibrium the labor devoted to the energy sector $n_{F,t}^1 = 1 - n_t^1$.

H understands how $E_t^i, i = 1, 2$ affect the stock of emissions,

$$S_t = H(S_{t-1}, E_t^1 + E_t^2), \tag{54}$$

with S_{-1} given.

Implementability constraint. To set up the dynamic monopolist's problem, it is convenient to work with the $t = 0$ intertemporal budget constraint of F, which given (48) and (51), becomes

$$\sum_{t=0}^{\infty} q_t(c_t^2 - Y_t^2) = 0, \tag{55}$$

where $q_t \equiv \Pi_{j=1}^t p_j$, the respective time zero price of an Arrow-Debreu contract, with $q_0 \equiv 1$, so given (49), we have

$$q_t = \beta^t \frac{u_c^2(c_t^2)}{u_c^2(c_0^2)}. \tag{56}$$

H understands how equilibrium prices are related to foreign consumption c_t^2 through (49). As Costinot et al. (2014), I follow the primal approach of Lucas and Stokey (1983) and eliminate equilibrium prices q_t by using intertemporal marginal rates of substitution (56). Moreover, note that (n_t^2, E_t^2), are *not* controllable by the dynamic monopolist H, since they are functions of the exogenous process z_t^2. Same comment applies to the foreign gross output $\hat{Y}_t^2$, which is function of (A_t^2, z_t^2). Using these two facts, we can rewrite (55) as

$$\sum_{t=0}^{\infty}\beta^t u_c^2(c_t^2)\left[c_t^2-(1-D^2(S_t))\hat{Y}_t^2\right]=0. \tag{57}$$

Equation (57) acts as an implementability constraint for H. We are ready now to state the problem of H.

Problem 2. *('Dynamic monopolist') The problem of H is to choose $\{c_t^1, n_t^1, E_t^1, c_t^2, S_t\}$ to maximize*

$$\sum_{t=0}^{\infty}\beta^t u^1(c_t^1) \tag{58}$$

subject to the global resource constraint (31), the technology for producing energy at home (53), the law of motion for emissions (54), and the implementability constraint (57), as well as non-negativity and feasibility constraints $c_t^1, c_t^2 \geq 0$, $n_t^1 \in [0,1]$, where S_{-1} is given.

5.2 Optimality conditions

Assign multipliers $\{\beta^t\lambda_t^M, \beta^t\mu_t^M, \beta^t\xi_t^M\}$ and $\Phi \geq 0$ on (31), (53), (54) and (57) respectively, and form the Lagrangian,

$$\begin{aligned} L \;=\; & \sum_{t=0}^{\infty}\beta^t u^1(c_t^1)-\sum_{t=0}^{\infty}\beta^t\lambda_t^M\left[c_t^1+c_t^2-(1-D^1(S_t))A_t^1F^1(n_t^1,E_t^1)-(1-D^2(S_t)\hat{Y}_t^2)\right] \\ & -\sum_{t=0}^{\infty}\beta^t\mu_t^M\left[E_t^1-z_t^1f^1(1-n_t^1)\right]+\sum_{t=0}^{\infty}\beta^t\xi_t^M\left[S_t-H(S_{t-1},E_t^1+E_t^2)\right] \\ & +\Phi\sum_{t=0}^{\infty}\beta^t u_c^2(c_t^2)\left[c_t^2-(1-D^2(S_t))\hat{Y}_t^2\right]. \end{aligned} \tag{59}$$

First-order necessary conditions are

$$\begin{aligned} c_t^1: \quad & u_{c,t}^1=\lambda_t^M && (60)\\ n_t^1: \quad & \lambda_t^M(1-D_t^1)A_t^1F_{n,t}^1=\mu_t^M z_t^1 f_n^1(1-n_t^1) && (61)\\ E_t^1: \quad & \mu_t^M=\lambda_t^M(1-D_t^1)A_t^1F_{E,t}^1-\xi_t^M H_{E,t} && (62)\\ S_t: \quad & \xi_t^M=\lambda_t^M\left[D_{S,t}^1\hat{Y}_t^1+D_{S,t}^2\hat{Y}_t^2\right]-\Phi u_{c,t}^2 D_{S,t}^2\hat{Y}_t^2+\beta\xi_{t+1}^M H_{S,t+1} && (63)\\ c_t^2: \quad & \lambda_t^M=\Phi u_{cc,t}^2(c_t^2)(c_t^2-Y_t^2)+\Phi u_{c,t}^2 && (64) \end{aligned}$$

5.3 Analysis

Define the (scaled by the marginal utility of domestic consumption) multipliers $\tilde{\mu}_t^M \equiv \mu_t^M/\lambda_t^M$ and $\tilde{\xi}_t^M \equiv \xi_t^M/\lambda_t^M$. Using (60) and these definitions, we can rewrite (61) as

$$(1 - D_t^1)A_t^1 F_{n,t}^1 = \tilde{\mu}_t^M z_t^1 f_n^1(1 - n_t^1), \tag{65}$$

which determines the optimal intersectoral allocation of domestic labor, where $\tilde{\mu}_t$ captures the shadow value of energy,

$$\tilde{\mu}_t^M = (1 - D_t^1)A_t^1 F_{E,t}^1 \underbrace{-\tilde{\xi}_t^M H_{E,t}}_{\text{externality}}, \tag{66}$$

encompassing both the marginal product of energy and the externality coming from emissions, $\tilde{\xi}_t^M$. Rewriting (63), the shadow cost of S_t is given by

$$\tilde{\xi}_t^M = \underbrace{D_{S,t}^1 \hat{Y}_t^1}_{\text{current damage of H}} + \underbrace{\left(1 - \Phi\frac{u_{c,t}^2}{u_{c,t}^1}\right) D_{S,t}^2 \hat{Y}_t^2}_{\text{interest rate manipulation} \times \text{current damages of F}} + \beta\frac{u_{c,t+1}^1}{u_{c,t}^1}\tilde{\xi}_{t+1}^M H_{S,t+1} \tag{67}$$

Even if H cares only for its own utility, it takes into account how the stock of emissions generates *foreign* marginal damages. Why? Foreign damages affect foreign consumption and therefore equilibrium *prices*. As a result, besides the current domestic marginal damages, the shadow cost $\tilde{\xi}_t^M$ has a second component in (67), which captures the *interest rate* manipulation motives of H. To see this term clearer, rewrite (64) as

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \Phi\left[1 - \frac{u_{cc,t}^2}{u_{c,t}^2}(c_t^1 - Y_t^1)\right], \tag{68}$$

where I used the fact that $c_t^2 - Y_t^2 = Y_t^1 - c_t^1$. Define as the *price wedge*

$$\chi_t \equiv \frac{u_{c,t}^1}{\Phi u_{c,t}^2} - 1, \tag{69}$$

so (68) can be expressed as

$$\chi_t = -\frac{u^2_{cc,t}}{u^2_{c,t}}\left(c^1_t - Y^1_t\right). \tag{70}$$

Given the definition of the wedge, we can write (67) as

$$\tilde{\xi}^M_t = D^1_{S,t}\hat{Y}^1_t + \underbrace{\frac{\chi_t}{1+\chi_t}}_{+/-} D^2_{S,t}\hat{Y}^2_t + \beta \frac{u^1_{c,t+1}}{u^1_{c,t}}\tilde{\xi}^M_{t+1}H_{S,t+1}. \tag{71}$$

Discussion. Note that even in the extreme case where the home country has *no* damages, so $D^1(S_t) = 0\forall t$, and therefore $D^1_{S,t} \equiv 0\forall t$, the shadow cost of the stock of emissions for the monopolist is *not* zero, according to (71). In that case we have

$$\tilde{\xi}^M_t = \underbrace{\frac{\chi_t}{1+\chi_t}}_{+/-} D^2_{S,t}\hat{Y}^2_t + \beta \frac{u^1_{c,t+1}}{u^1_{c,t}}\tilde{\xi}^M_{t+1}H_{S,t+1}, \tag{72}$$

generating deviations for the shadow value of energy $\tilde{\mu}^M_t$ from the marginal product of energy in (66). The price wedge can be *positive* or *negative*, depending on the next exports of H, as we see in (70). So the negative production externality that emissions impose, can actually be *beneficial* to a policymaker that is not a price-taker.

5.4 Analysis of wedge, carbon taxes and capital controls

[To be completed].

6 Concluding remarks

In this paper, I study the optimal carbon policy in a global economy, taking into account the global nature of emissions externalities and the issues of (non)-cooperation. I analyze the cooperative solution and the respective carbon taxes, and move beyond cooperation by considering a large country/emitter that faces a passive rest of the world.

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