Optimal QE and QT*

Paolo Gelain[†a], **Junior Maih**[‡b], **Eric Sims**[§c], and **Jing Cynthia Wu**[¶d]

[a]Federal Reserve Bank of Cleveland
[b]Norges Bank
[c,d]Notre Dame and NBER

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Abstract

We analyze the set of optimal interest rate and quantitative easing policies for the Federal Reserve. Using an estimated New Keynesian dynamic stochastic general equilibrium model for the US with the zero lower bound and endogenous quantitative easing, we recover the counterfactual welfare-maximizing optimal policies and we assess their implications for the dynamic of the US economy.

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†Federal Reserve Bank of Cleveland, 1455 East 6th Street Cleveland, OH 44114, USA. Email: paolo.gelain@clev.frb.org

‡Norges Bank, Monetary Policy Department, P.O. Box 1179 Sentrum 0107 Oslo, and BI Norwegian Business School. Email: junior.maih@norges-bank.no.

§Department of Economics, 3060 Jenkins Nanovic Hall, Notre Dame, IN 46556, USA. Email: esims1@nd.edu

¶Department of Economics, 3060 Jenkins Nanovic Hall, Notre Dame, IN 46556, USA. Email: cynthia.wu@nd.edu

1 Introduction

Unconventional monetary policies have become part of the set of policies used by central banks to intervene in crises periods. Among others, quantitative easing and tightening have been used since the onset of the Great Recession.

Despite some studies have empirically investigated the effects of such policies on inflation and the real economy,[1] there has been little assessment on how they should be optimally conducted.

In this paper we want to analyze the set of optimal interest rate and quantitative easing policies that the Federal Reserve could have put in place. First, using a mix of US macroeconomic, financial, and central bank balance sheet data from 1998Q1 to 2023Q4, we estimate a New Keynesian dynamic stochastic general equilibrium model featuring all the standard bells and whistles, like for instance in Smets and Wouters (2007), and all the necessary ingredients to evaluate quantitative easing policies, like in Sims and Wu (2021). Government and firms issue public and private long term bonds respectively, and those can be purchased by financial intermediaries and by the central bank.

In the estimation, we explicitly take into account the zero lower bound on the interest rate by operating in a Markov Switching environment. We assume that there are two regimes, a normal one in which the interest rate follows a Taylor rule, and another one in which it is constrained to be zero. Moreover, we assume that quantitative easing is exogenous in normal times. In the zero lower bound regime, instead, we assume that the central bank actively manages its balance sheet by responding to the state of the economy, i.e., by following a Taylor-type rule for the purchase of long term private and public bonds expressed in terms of inflation and output

We then proceed to run a counterfactual scenario analysis under optimal policy. We study the model's optimal equilibrium, i.e., the welfare-maximizing equilibrium chosen by the central bank under commitment subject to the constraints represented by the behavior of private agents. More specifically, we use the solution of the model under Ramsey monetary policies to compute the counterfactual path of a relevant set of endogenous variables that would have emerged if policies had always been optimal over our sample and the economy had been perturbed by the series of shocks estimated in the baseline version of the model under the historical Taylor-type interest rate and quantitative easing rules (with the exception of the shocks entering the Taylor rules that do not affect the optimal equilibrium). We assume that the planner has both the interest rate and the balance sheet quantities available as instruments. Our analysis is similar in spirit to Justiniano, Primiceri and Tambalotti (2013) and Furlanetto, Gelain and Sanjani Taheri (2021), but it differs because we focus also on the optimality of unconventional policies.

We then make a further step forward. The optimal Ramsey policies are not necessarily neither achievable nor implementable. Therefore, we perform two extra exercises. First, we allow the central bank to have an ad-hoc loss function, more in line those taken as a reference in reality. Second, we look for optimized simple rules, namely rules whose coefficients are optimized to minimize the loss (or maximize the welfare) but that are simple, so easy to follow by the central bank and to communicate to the public.

[1] Mention some empirical papers.

All our analysis allows us to thoroughly asses how far the Federal Reserve policies have been from the optimal ones, and which costs and benefits the Fed and the US economy experienced from deviating from optimality. Moreover, given that we estimate the model using the most recent available data, we can provide a timely optimal exit strategy for the normalization of the Federal Reserve balance sheet.

Our paper contributes to the literature as follows. There are few papers studying optimal QE policy in New Keynesian models. Harrison (2017) and Karadi and Nakov (2021) focus on calibrated models, so they do not offer a retrospective analysis of the Federal Reserve monetary policy. Darracq Pariès and Köhl (2016) estimate a fully-fledged DSGE model for the Euro Area. They do not specifically account for the ZLB in the estimation, they do not include balance sheet variables among the observables, and they do not look at the welfare function. Kabaca et al. (2023) also estimate a large NK model for the Euro area, stressing its currency union nature and evaluating the optimality of the European Central Bank's policy of buying government bonds from the different countries in the union. De Groot et al. (2021) develop a toolkit for generating optimal policy projections. They provide QE projections for the period from 2009 onwards based on the calibrated model of Sims and Wu (2021) and on an ad-hoc quadratic loss function. Finally, Boehl, Goy and Strobel (2022) estimate a model similar in many respects to ours for the US, but they do not analyze optimal policy.

2 Model

There are several agents in the model – a representative household; a labor market that includes a competitive labor packer that transforms differentiated labor from unions into labor available for production, where unions in turn purchase labor from the household; a capital goods producing firm; a representative wholesale firm; a continuum of retail firms, who purchase and repackage wholesale output for sale to a final good firm; a fiscal authority; and a monetary authority. The subsections below lay out the problems and optimality conditions for each type of agent.

2.1 Household

There is a representative household with preferences over consumption and labor given by:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \varepsilon_t^b \beta^t \left\{ \ln\left(C_t - bC_{t-1}\right) - \chi \frac{L_t^{1+\eta}}{1+\eta} \right\}$$

Households consume and save through nominal deposits, D_t. They earn income from supply labor to labor unions at nominal wage W_t. As in Justiniano, Primiceri and Tambalotti (2013), their preferences are subject to exogenous time variation captured by the intertemporal preference shock ε_t^b such that:

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + u_t^b$$

with u_t^b independently and identically distributed $N(0, \sigma_b^2)$.

They receive dividends from ownership in non-financial firms as well as the equity leftover from remaining intermediaries. Each period, households make a *fixed* real equity transfusion to newly born intermediaries. This is given by X. They also pay a lump sum tax to the government. The flow budget constraint in nominal terms is:

$$P_t C_t + D_t \leq W_t L_t + R^D_{t-1} D_{t-1} + DIV_t - P_t X - P_t T_t \tag{1}$$

A Lagrangian is:

$$\mathbb{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \varepsilon^b_t \beta^t \Bigg\{ \ln\left(C_t - bC_{t-1}\right) - \chi \frac{L_t^{1+\eta}}{1+\eta} + \lambda^n_t \left[W_t L_t + R^D_{t-1} D_{t-1} + DIV_t - P_t X - P_t T_t - P_t C_t - D_t\right] \Bigg\}$$

The FOC are:

$$\frac{\partial \mathbb{L}}{\partial C_t} = \frac{\varepsilon^b_t}{(C_t - bC_{t-1})} - \lambda^n_t P_t - \beta b \mathbb{E}_t \frac{\varepsilon^b_{t+1}}{(C_{t+1} - bC_t)}$$

$$\frac{\partial \mathbb{L}}{\partial D_t} = -\lambda^n_t + \beta E_t \lambda^n_{t+1} R^d_t$$

Define $\mu_t = P_t \lambda^n_t$ as the *real* marginal utility of consumption. Further define the real stochastic discount factor as:

$$\Lambda_{t-1,t} = \frac{\beta \mu_t}{\mu_{t-1}} \tag{2}$$

Using this notation and setting the above to zero, we get:

$$\mu_t = \frac{\varepsilon^b_t}{(C_t - bC_{t-1})} - \beta b \mathbb{E}_t \frac{\varepsilon^b_{t+1}}{(C_{t+1} - bC_t)} \tag{3}$$

$$1 = R^d_t \mathbb{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \tag{4}$$

Where and $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate.

2.2 Labor Market

There are two layers to the labor market. There are a unit measure of labor unions, index by $h \in [0,1]$, who purchase labor from households and repackage for resale to a labor packer at $W_t(h)$. Then a competitive labor packer combines union labor into a final labor input.

Work backwards. The labor packer transforms union labor, $L_{d,t}(h)$, into final labor available for production via a CES technology:

$$L_{d,t} = \left(\int_0^1 L_{d,t}(h)^{\frac{1}{1+\Lambda_{w,t}}} dh \right)^{1+\Lambda_{w,t}} \tag{5}$$

The labor packer sells final labor input, $L_{d,t}$, to production firms at nominal wage W_t. It purchases union labor at $W_t(h)$. The elasticity of this aggregator $\Lambda_{w,t}$ corresponds to the desired markup of wages over households' marginal rate of substitution between consumption and leisure. As in Smets and Wouters (2007), it is modelled as follows:

$$\ln\left(1+\Lambda_{w,t}\right) \equiv \lambda_{w,t} = \left(1-\rho_w\right)\lambda_w + \rho_w \lambda_{w,t-1} + u_t^w - \mu_w u_{t-1}^w$$

with innovations u_t^w independently and identically distributed $N(0, \sigma_w^2)$. This is named a wage markup shock. The labor packer is competitive and earns no profit in equilibrium. Its problem is to pick each $L_{d,t}(h)$ to maximize:

$$\max_{L_{d,t}(h)} \; W_t \left(\int_0^1 L_{d,t}(h)^{\frac{1}{1+\Lambda_{w,t}}} dh \right)^{1+\Lambda_{w,t}} - W_t(h) L_{d,t}(h)$$

Optimization gives rise to a standard downward-sloping demand curve for labor and an aggregate wage index:

$$L_{d,t}(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\frac{1+\Lambda_{w,t}}{\Lambda_{w,t}}} L_{d,t} \tag{6}$$

$$W_t^{-\frac{1}{\Lambda_{w,t}}} = \int_0^1 W_t(h)^{-\frac{1}{\Lambda_{w,t}}} dh \tag{7}$$

The labor unions simply repackage labor purchases from households for sale to the labor packer: $L_{d,t}(h) = L_t(h)$. Labor is purchased from the household at W_t and sold to the packer at $W_t(h)$. Unions are subject to a Calvo wage rigidity: each period, there is a $1-\phi_w$ probability that a fraction of the, can adjust a nominal wage. For those that cannot adjust wages follow the indexation rule:

$$W_t(h) = W_{t-1}(h) \Pi_{t-1}^{\iota_w} \Pi^{(1-\iota_w)}$$

The remaining fraction of unions chooses instead an optimal wage $W_t^{\#}$ by maximizing:

$$\max_{W_t^{\#}} \; \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \phi_w^j \beta^j \left[-\varepsilon_{t+j}^b \chi \frac{L_{d,t+j}^{1+\eta}}{1+\eta} + \mu_{t+j} W_t(h) L_{d,t+j} \right] \right\}$$

subject to the labor demand function 6.

The FOC is:

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \phi_w^j \beta^j \mu_{t+j} L_{d,t+j} \left[W_t^{\#} \Pi_{t,t+j}^w - \left(1+\lambda_{w,t+j}\right) \varepsilon_{t+j}^b \chi \frac{L_{d,t+j}^{\eta}}{\mu_{t+j}} \right] \right\} = 0$$

with:

$$\Pi_{t,t+j}^w = \prod_{k=1}^{j} \left[\Pi^{(1-\iota_w)} \Pi_{t+k-1}^{\iota_w} \right]$$

That can be solved by $W_t^{\#}$ and written in the following recursive form (assuming $w_t = W_t/P_t$ is the real wage and $w_t^{\#} = W_t^{\#}/P_t$):

$$f_{1,t} = e^{\lambda_{w,t}} \chi \varepsilon_t^b L_{d,t}^{1+\eta} + \phi_w \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi} \right)^{\iota_w} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{-1} \left(\frac{w_{t+1}}{w_t} \right)^{-1} \right]^{-\frac{(1+\eta)(1+\lambda_w)}{\lambda_w}} f_{1,t+1} \right\}$$

$$f_{2,t} = \mu_t L_{d,t} w_t + \phi_w \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi}\right)^{\iota_w} \left(\frac{\Pi_{t+1}}{\Pi}\right)^{-1} \left(\frac{w_{t+1}}{w_t}\right)^{-1} \right]^{-\frac{1}{\lambda_w}} f_{2,t+1} \right\}$$

$$w_t^{\#} = w_t \left(\frac{f_{1,t}}{f_{2,t}}\right)^{\frac{\lambda_w}{\lambda_w + \eta(1+\lambda_w)}}$$

2.3 Investment Goods Producer

New capital, $\widehat{I}_t$, is produced using unconsumed output, I_t. It is sold to firms at P_t^K. The production function is:

$$\widehat{I}_t = \mu_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{8}$$

$S(\cdot)$ is an investment adjustment cost as in Christiano, Eichenbaum and Evans (2005), and μ_t^I is an investment shock, i.e. a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital, and thus into tomorrow's capital input. It is as follows:

$$\ln \mu_t^I = \rho_\mu \ln \mu_{t-1}^I + u_t^\mu$$

with u_t^μ independently and identically distributed $N(0, \sigma_\mu^2)$. Nominal profit is:

$$DIV_{k,t} = P_t^k \mu_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t I_t$$

Or, in real terms, with $p_t^k = P_t^k / P_t$:

$$div_{k,t} = p_t^k \mu_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t$$

The objective is to pick I_t to maximize the PDV of real profit, where discounting is by the stochastic discount factor. The problem is:

$$\max_{I_t} \ \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+1} \left\{ p_{t+j}^k \mu_{t+k}^I \left[1 - S\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right] I_{t+j} - I_{t+j} \right\}$$

The FOC is:

$$p_t^k \mu_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right] - 1 + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^k \mu_{t+1}^I S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 0$$

Or:

$$1 = p_t^k \mu_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^k \mu_{t+1}^I S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \tag{9}$$

2.4 Goods Production

There are three layers to production. A final good firm purchases retail outputs, where there are a continuum of retailers indexed by $f \in [0,1]$, at $P_t(f)$ and resells at P_t. The production technology is CES:

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{1}{1+\Lambda_{p.t}}} df \right)^{1+\Lambda_{p,t}}$$

The problem is:

$$\max_{Y_t(f)} \; P_t \left(\int_0^1 Y_t(f)^{\frac{1}{1+\Lambda_{p.t}}} df \right)^{1+\Lambda_{p,t}} - P_t(f) Y_t(f)$$

Optimization gives a standard downward-sloping demand for each retail output and an aggregate price index:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\frac{1+\Lambda_{p,t}}{\Lambda_{p,t}}} Y_t$$

$$P_t^{-\frac{1}{\Lambda_{p,t}}} = \int_0^1 P_t(f)^{-\frac{1}{\Lambda_{p,t}}} df$$

The curvature of the aggregator $\Lambda_{p,t}$ determines the degree of substitutability across intermediate goods in the production of each of these intermediates. It is modelled as an exogenous stochastic process:

$$\ln(1+\Lambda_{p,t}) \equiv \lambda_{p,t} = (1-\rho_p)\lambda_p + \rho_p \lambda_{p,t-1} + u_t^p - \mu_p u_{t-1}^p$$

driven by innovations u_t^p independently and identically distributed $N(0, \sigma_p^2)$. This is named a price markup shock. The final good firm earns no profit.

Retail firms purchase wholesale output at $P_{w,t}$. They simply repackage wholesale output: $Y_t(f) = Y_{w,t}(f)$, and then sell it to the final goods firm at $P_t(f)$. This is analogous to the labor union. Nominal profit for retailers is:

$$DIV_{R,t}(f) = P_t(f) Y_t(f) - P_{w,t} Y_{w,t}(f)$$

Every period a fraction of retailers cannot choose its price optimally with probability ϕ_p, but resets it according to the indexation rule:

$$P_t(f) = P_{t-1}(f)\, \Pi_{t-1}^{\iota_p} \Pi^{1-\iota_p}$$

The remaining fraction of retailers can only adjust their price with probability $1-\phi_p$. This makes their price-setting problem dynamic. A retailer with the opportunity to adjust will choose $P_t(f)$ to maximize the PDV of real profits, where discounting is the by stochastic discount factor as well as the probability that a price chosen today will remain in effect in the future. The problem is:

$$\max_{P_t(f)} \; \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \phi_p^j \frac{\beta^j \mu_{t+j}}{\mu_t} \left[P_t(f) \Pi_{t,t+j} - P_{w,t+j} \right] Y_{t+j}(f) \right\}$$

subject to equation:

$$Y_{t+j}(f) = \left(\frac{P_{t+j}(f)\Pi_{t,t+j}}{P_{t+j}} \right)^{-\frac{1+\Lambda_{p,t+j}}{\Lambda_{p,t+j}}} Y_{t+j}$$

with:

$$\Pi_{t,t+j} \equiv \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p} \right)$$

The FOC is:

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \phi_p^j \mu_{t+j} Y_{t+j} \left[P_t^{\#} \Pi_{t,t+j} - (1+\Lambda_{p,t+j}) \right] P_{w,t+j} \right\} = 0$$

Which may be written in real terms ($p_{w,t} = P_{w,t}/P_t$):

$$x_{1,t} = e^{\lambda_{p,t}} \mu_t p_{w,t} Y_t + \phi_p \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi} \right)^{\iota_p} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{-1} \right]^{-\frac{1+\lambda_p}{\lambda_p}} x_{1,t+1} \right\}$$

$$x_{2,t} = \mu_t Y_t + \phi_p \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi} \right)^{\iota_p} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{-1} \right]^{-\frac{1}{\lambda_p}} x_{2,t+1} \right\}$$

$$P_t^{\#} = \frac{x_{1,t}}{x_{2,t}}$$

There is a representative wholesale firm. It produces output using capital that it accumulates and labor purchased from the labor packer. Its production function is:

$$Y_{w,t} = (u_t K_t)^{\alpha} (A_t L_{d,t})^{1-\alpha} \tag{10}$$

where u_t is capital utilization, A_t represents exogenous labor-augmenting technological progress or, equivalently, a neutral technology factor. The level of neutral technology is non-stationary and its growth rate ($z_t \equiv \Delta \ln A_t$) follows an AR(1) process:

$$z_t = (1-\rho_z)\, z + \rho_z z_{t-1} + u_t^z$$

with u_t^z independently and identically distributed $N(0, \sigma_z^2)$. In nominal terms, the wholesaler's profit is:

$$DIV_{w,t} = P_{w,t}(u_t K_t)^{\alpha} (A_t L_{d,t})^{1-\alpha} - W_t L_{d,t} - P_t^k \widehat{I}_t - F_{w,t-1} + Q_t(F_{w,t} - \kappa F_{w,t-1})$$

The wholesaler has outstanding coupon liabilities on long bonds of $F_{w,t-1}$. It can issue new long bonds for Q_t, where $Q_t(F_{w,t} - \kappa F_{w,t-1})$ is the value of new bond issuance.

The wholesale firm is subject to a standard law of motion for physical capital:

$$K_{t+1} = \widehat{I}_t + (1 - \delta(u_t))K_t \tag{11}$$

where $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$ is utilization adjustment cost. The wholesale firm is also subject to an investment in advance constraint:

$$P_t^k \widehat{I}_t \leq Q_t(F_{w,t} - \kappa F_{w,t-1}) \tag{12}$$

In other words, (12) says that nominal expenditure on new investment cannot exceed issuance of new bonds.

Write dividends and the loan in advance constraint in real terms:

$$div_{w,t} = p_{w,t}(u_t K_t)^{\alpha} (A_t L_{d,t})^{1-\alpha} - w_t L_{d,t} - p_t^k \widehat{I}_t - \frac{F_{w,t-1}}{P_t} + Q_t \left(\frac{F_{w,t}}{P_t} - \kappa \frac{F_{w,t-1}}{P_t} \right)$$

$$\psi p_t^k \widehat{I}_t \leq Q_t \left(\frac{F_{w,t}}{P_t} - \kappa \frac{F_{w,t-1}}{P_t} \right)$$

where ψ is a fraction of a investment that must be financed by debt; $\psi = 1$ would correspond to Carlstrom, Fuerst and Paustian (2017). Profits are discounted by the household's real SDF. A Lagrangian is:

$$\mathbb{L}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \Bigg\{ p_{w,t}(u_t K_t)^{\alpha} (A_t L_{d,t})^{1-\alpha} - w_t L_{d,t} - p_t^k \widehat{I}_t - \frac{F_{w,t-1}}{P_t} + Q_t \left(\frac{F_{w,t}}{P_t} - \kappa \frac{F_{w,t-1}}{P_t} \right)$$
$$+ \nu_{1,t} \left(\widehat{I}_t + (1 - \delta(u_t)) K_t - K_{t+1} \right) + \nu_{2,t} \left(Q_t \left(\frac{F_{w,t}}{P_t} - \kappa \frac{F_{w,t-1}}{P_t} \right) - \psi p_t^k \widehat{I}_t \right) \Bigg\}$$

The derivatives of the Lagrangian are:

$$\frac{\partial \mathbb{L}}{\partial L_{d,t}} = (1-\alpha) p_{w,t} A_t (u_t K_t)^{\alpha} (A_t L_{d,t})^{-\alpha} - w_t$$

$$\frac{\partial \mathbb{L}}{\partial u_t} = \alpha p_{w,t} (u_t K_t)^{\alpha - 1} K_t (A_t L_{d,t})^{1-\alpha} - \nu_{1,t} \delta'(u_t) K_t$$

$$\frac{\partial \mathbb{L}}{\partial \widehat{I}_t} = -p_t^k + \nu_{1,t} - \nu_{2,t} \psi p_t^k$$

$$\frac{\partial \mathbb{L}}{\partial K_{t+1}} = -\nu_{1,t} + \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha p_{w,t+1} (u_{t+1} K_{t+1})^{\alpha-1} u_{t+1} (A_{t+1} L_{d,t+1})^{1-\alpha} + \nu_{1,t+1} (1 - \delta(u_{t+1})) \right]$$

$$\frac{\partial \mathbb{L}}{\partial F_{w,t}} = \frac{Q_t}{P_t} + \nu_{2,t} \frac{Q_t}{P_t} - \mathbb{E}_t \Lambda_{t,t+1} \left[\frac{1}{P_{t+1}} + \kappa \frac{Q_{t+1}}{P_{t+1}} + \nu_{2,t+1} \kappa \frac{Q_{t+1}}{P_{t+1}} \right]$$

Setting equal to zero, the first three become:

$$w_t = (1-\alpha) p_{w,t} A_t (u_t K_t)^{\alpha} (A_t L_{d,t})^{-\alpha} \tag{13}$$

$$\nu_{1,t} \delta'(u_t) = \alpha p_{w,t} (u_t K_t)^{\alpha-1} (A_t L_{d,t})^{1-\alpha} \tag{14}$$

$$(1 + \psi \nu_{2,t}) p_t^k = \nu_{1,t} \tag{15}$$

Then for the two dynamic Euler equations, we have:

$$\nu_{1,t} = \mathbb{E}_t\Lambda_{t,t+1}\left[\alpha p_{w,t+1}(u_{t+1}K_{t+1})^{\alpha-1}u_{t+1}\left(A_{t+1}L_{d,t+1}\right)^{1-\alpha} + \nu_{1,t+1}(1-\delta(u_{t+1}))\right] \tag{16}$$

$$(1+\nu_{2,t})Q_t = \mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left[1 + (1+\nu_{2,t+1})\kappa Q_{t+1}\right] \tag{17}$$

Note that discounting in (16) is by the *real* stochastic discount factor, whereas discounting in (17) is by the *nominal* stochastic discount factor, $\Lambda_{t,t+1}\Pi_{t+1}^{-1}$. This is because capital is a real asset whereas long-bonds are nominal.

Introduce two auxiliary variables. Let $M_{1,t} = 1 + \nu_{2,t}$ and $M_{2,t} = 1 + \psi\nu_{2,t}$. We can then write the FOC for investment as:

$$\nu_{1,t} = p_t^k M_{2,t} \tag{18}$$

We can then eliminate the multiplier in the utilization FOC:

$$p_t^k M_{2,t}\delta'(u_t) = \alpha p_{w,t}(u_t K_t)^{\alpha-1}\left(A_t L_{d,t}\right)^{1-\alpha} \tag{19}$$

We can then also write the dynamic Euler equations as:

$$p_t^k M_{2,t} = \mathbb{E}_t\Lambda_{t,t+1}\left[\alpha p_{w,t+1}(u_{t+1}K_{t+1})^{\alpha-1}u_{t+1}\left(A_{t+1}L_{d,t+1}\right)^{1-\alpha} + (1-\delta(u_{t+1}))p_{t+1}^k M_{2,t+1}\right] \tag{20}$$

$$Q_t M_{1,t} = \mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left[1 + \kappa Q_{t+1}M_{1,t+1}\right] \tag{21}$$

Where we have:

$$\frac{M_{1,t}-1}{M_{2,t}-1} = \psi^{-1} \tag{22}$$

Before proceeding, it is again useful to note the distortion we are introducing relative to a more standard model. This distortion is captured by $\nu_{2,t}$, where $\nu_{2,t} > 0$ means $M_{1,t} > 1$ and $M_{2,t} > 1$, and therefore distorts the FOC for utilization, the dynamic Euler equation for capital, and the dynamic Euler equation for long bonds. Without this distortion, these FOC would look standard.

2.5 Monetary Policy

The central bank sets the *notional* or *desired* interest rate on reserves, R_t^{tr}, according to a Taylor rule:

$$\ln R_t^{tr} = (1-\rho_r)\ln R^{tr} + \rho_r \ln R_{t-1}^{tr} + (1-\rho_r)\left[\frac{\phi_\pi}{4}(\sum_{j=0}^{3}\ln\Pi_{t-j} - 4\ln\Pi) + \frac{\phi_y}{4}(\ln Y_t - \ln Y_{t-4})\right] + u_t^r \tag{23}$$

with u_t^r is a monetary policy independently and identically distributed $N(0,\sigma_r^2)$. The actual interest rate on reserves is assumed be subject to a zero lower bound:

$$R_t^{re} = \max\left\{1, R_t^{tr}\right\} \tag{24}$$

The central bank has a balance sheet where the size is completely up to the central bank. We assume that the central bank can hold either private investment bonds (loosely, think about these as mortgage-backed securities, MBS) or long-term government bonds (loosely, long-term Treasuries). It finances this with reserves (the model is cashless, so there is no currency in circulation):

$$Q_t A_t F_{cb,t} + Q_{B,t} A_t B_{cb,t} = RE_t \tag{25}$$

Following Sims and Wu (2021), we assume that real bond holdings follow an exogenous process so long as the ZLB is not binding. But when the ZLB binds, an *endogenous* component to the QE rule kicks in. Therefore, bye defining $f^Y_{cb,t} = \frac{Q_t f_{cb,t}}{Y_t}$ and $b^Y_{cb,t} = \frac{Q_{B,t} b_{cb,t}}{Y_t}$, those obey the following rules:

$$f^Y_{cb,t} = (1-\rho_f) f^Y_{cb} + \rho_f f^Y_{cb,t-1} + u^f_t \quad \text{if } R^{tr}_t > 1$$

$$b^Y_{cb,t} = (1-\rho_B) b_{cb} + \rho_B b^Y_{cb,t-1} + u^B_t \quad \text{if } R^{tr}_t > 1$$

$$f^Y_{cb,t} = (1-\rho_f) f^Y_{cb} + \rho_f f^Y_{cb,t-1} - (1-\rho_f)\left[\frac{\phi_\pi}{4}(\sum_{j=0}^{3} \ln \Pi_{t-j} - 4\ln\Pi) + \frac{\phi_y}{4}(\ln Y_t - \ln Y_{t-4})\right] + u^f_t \quad \text{if } R^{tr}_t \leq 1$$

$$b^Y_{cb,t} = (1-\rho_B) b^Y_{cb} + \rho_B b^Y_{cb,t-1} - (1-\rho_B)\left[\frac{\phi_\pi}{4}(\sum_{j=0}^{3} \ln \Pi_{t-j} - 4\ln\Pi) + \frac{\phi_y}{4}(\ln Y_t - \ln Y_{t-4})\right] + u^B_t \quad \text{if } R^{tr}_t \leq 1$$

with u^f_t and u^{cb}_t independently and identically distributed $N(0,\sigma^2_f)$ and $N(0,\sigma^2_{cb})$ respectively.

The idea here is fairly simple. When the notional Taylor rule rate is zero or negative in net terms, so $R^{tr}_t \leq 1$, an endogenous component to QE "kicks on" that looks qualitatively like the reaction in the basic Taylor rule. This is given by the term $-\left[\frac{\phi_\pi}{4}(\sum_{j=0}^{3} \ln \Pi_{t-j} - 4\ln\Pi) + \frac{\phi_y}{4}(\ln Y_t - \ln Y\right.$ The target variables are the same as the Taylor rule, and the ϕ_π and ϕ_y are the same as well. There is a negative sign outside – this reflects that purchasing bonds is equivalent to cutting the policy rate, so the QE rule during the ZLB needs to react the opposite way from how the standard Taylor rule would.

The balance sheet constraint in real terms is:

$$Q_t A_t f_{cb,t} + Q_{B,t} A_t b_{cb,t} = re_t \tag{26}$$

Given $f_{cb,t}$ and $b_{cb,t}$, re_t automatically adjusts to make the balance sheet hold.

The central bank earns income on its assets and pays interest on its liabilities (reserves). In particular, it earns revenue:

$$P_t T_{cb,t} = (1+\kappa Q_t) A_{t-1} F_{cb,t-1} + (1+\kappa Q_{B,t}) A_{t-1} B_{cb,t-1} - R^{re}_{t-1} RE_{t-1}$$

This can be written:

$$P_t T_{cb,t} = \frac{1+\kappa Q_t}{Q_{t-1}} Q_{t-1} A_{t-1} F_{cb,t-1} + \frac{1+\kappa Q_{B,t}}{Q_{B,t-1}} Q_{B,t-1} A_{t-1} B_{cb,t-1} - R^{re}_{t-1} RE_{t-1}$$

But then using the balance sheet condition to sub out reserves, and defining $R^F_t = \frac{1+\kappa Q_t}{Q_{t-1}}$ and $R^B_t = \frac{1+\kappa Q_{B,t}}{Q_{B,t-1}}$, we have:

$$P_t T_{cb,t} = \left(R^F_t - R^{re}_{t-1}\right) Q_{t-1} A_{t-1} F_{cb,t-1} + \left(R^B_t - R^{re}_{t-1}\right) Q_{B,t-1} A_{t-1} B_{cb,t-1}$$

Or, in real terms:

$$T_{cb,t} = \left(R_t^F - R_{t-1}^{re}\right) \Pi_t^{-1} Q_{t-1} A_{t-1} f_{cb,t-1} + \left(R_t^B - R_{t-1}^{re}\right) \Pi_t^{-1} Q_{B,t-1} A_{t-1} b_{cb,t-1} \tag{27}$$

In other words, the central bank earns spreads over the of cost funds on its asset holdings. This is remitted to the government each period, so that the central bank maintains no equity.

2.6 Fiscal Policy

The fiscal authority consumes, G_t, taxes the household, T_t, and issues debt, $B_{G,t}$. It also receives a lump sum transfer each period from the central bank, $T_{cb,t}$. Its flow budget constraint is:

$$P_t G_t + A_{t-1} B_{G,t-1} = P_t T_t + P_t T_{cb,t} + Q_{B,t}(A_t B_{G,t} - \kappa A_{t-1} B_{G,t-1}) \tag{28}$$

Government debt has the same structure as private investment bonds, with coupon payouts decaying at κ. Government bonds trade at $Q_{B,t}$, which is not necessarily equal to Q_t (unlike in Carlstrom, Fuerst and Paustian, 2017). In this model, Ricardian Equivalence does not hold. So we have to make some assumptions on the path of government debt (i.e. it is not innocuous as in a standard model). We are going to assume that real government debt, where $b_{G,t} = \frac{B_t}{P_t}$, follows an exogenous AR(1) process. Lump sum taxes will then automatically adjust to make the government's budget constraint hold. We don't need to keep track of it.

2.7 Financial Intermediaries

There are a fixed mass of intermediaries indexed by i. Intermediaries hold long-term private issued bonds and government bonds as well as bank reserves; and they finance themselves with their own equity as well as deposits. The balance sheet of a typical intermediary in nominal terms is:

$$Q_t F_{i,t} + Q_{B,t} B_{i,t} + RE_{i,t} = D_{i,t} + N_{i,t} \tag{29}$$

Each period, an exogenous fraction σ of intermediaries stochastically die. Upon death, they simply return their net worth to the household. The household replaces the dying intermediaries with the same number of new intermediaries, given these new intermediaries start-up net worth of X (distributed among all the new intermediaries evenly).

As long as it can earn excess returns (which it will given the constraints we shall introduce), it behooves an intermediary to not pay any dividends – it just wants to accumulate net worth until it stochastically exits. Net worth can be shown to evolve according to:

$$N_{i,t} = \left(R_t^F - R_{t-1}^d\right) Q_{t-1} F_{i,t-1} + \left(R_t^B - R_{t-1}^d\right) Q_{B,t-1} B_{i,t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right) RE_{i,t-1} + R_{t-1}^d N_{i,t-1} + A_t X \tag{30}$$

If the intermediary earned no excess returns (i.e. none of the spreads were greater than zero), net worth would just grow at the cost of funds, the deposit rate. At this point the intermediary would be indifferent about accumulating net worth or paying it back to its owners (i.e. the household). But with excess returns, the intermediary is better off accumulating net worth to take advantage of lending spreads. As we shall see, the stochastic death assumption effectively makes intermediaries extra impatient and prevents them from accumulating enough net worth to overcome the limited enforcement constraint that we shall introduce below.

Consider an intermediary in period t. It needs to choose its balance sheet variables. Its objective is to maximize the expected value of *terminal net worth* – as noted above, the intermediary is just going to keep accumulating until it dies. Conditional on know it will survive from t into $t+1$, there is $1-\sigma$ probability that it dies in $t+1$. There is a $\sigma(1-\sigma)$ probability of exit in $t+2$ (i.e. a $1-\sigma$ probability of surviving past $t+1$, and a σ probability of exist in $t+2$. And so on. Accordingly, an intermediary's value function is:

$$V_{i,t} = \max \ \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+j} n_{i,t+j} \tag{31}$$

where $\Lambda_{t,t+j}$ is the household's stochastic discount factor and $n_{i,t} = N_{i,t}/P_t$ is real net worth. At the end of period t, before $t+1$, an intermediary can abscond with some of its assets and default. In particular, an intermediary can take $\theta_t Q_t f_{i,t}$ and $\theta_t \Delta Q_{B,t} b_{i,t}$, where $0 \leq \Delta \leq 1$. θ_t is also between zero and one, but is time-varying. We will consider this to be a credit shock. It evolves as follows

$$\ln \theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta \ln\theta_{t-1} + \varepsilon_t^\theta \tag{32}$$

with ε_t^θ independently and identically distributed $N(0,\sigma_\theta^2)$.[2] Equivalently, depositors (i.e. the household) can recover $1-\theta_t$ of private bonds and $1-\theta_t\Delta$ of government bonds. This enforcement constraint is relatively "tighter" for private bonds – it is "easier" for an intermediary to abscond with these relative to government bonds as long as $\Delta < 1$. The intermediary cannot abscond with reserves; these are perfectly recoverable by creditors in the event of default. The left hand side of (31) is the enterprise value of being an intermediary (i.e. the value of continuing). This enforcement constraint can be written:

$$V_{i,t} \geq \theta_t (Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}) \tag{33}$$

Constraint (33) says that creditors will only allow intermediaries to borrow up until the point where it is not optimal for them to default.

Letting $\lambda_{i,t}$ denote the multiplier on the enforcement constraint, we have a Lagrangian in the recursive formulation of the value function:

$$\mathbb{L} = (1+\lambda_{i,t})\left[(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}n_{i,t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}V_{i,t+1}\right] - \lambda_{i,t}\theta_t\left(Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}\right)$$

Plugging in the evolution of real net worth, (30) divided through by P_t and writing all

[2]Other papers that make a similar assumption about the time-varying nature of this parameter are: Gelain and Lorusso (2022), Boehl, Goy and Strobel (2022), Sims and Wu (2021), Gelain and Ilbas (2017), Dedola, Karadi and Lombardo (2013), and Bean et al. (2010)

quantities in real terms, we have:

$$\mathbb{L} = (1+\lambda_{i,t})\mathbb{E}_t\Bigg\{(1-\sigma)\Lambda_{t,t+1}\Big[\left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t f_{i,t} + \left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_{B,t}b_{i,t}+$$

$$\left(R^{re}_t - R^d_t\right)\Pi^{-1}_{t+1} re_{i,t} + R^d_t\Pi^{-1}_{t+1}n_{i,t}\Big] + \sigma\mathbb{E}_t\Lambda_{t,t+1}V_{i,t+1}\Bigg\} - \lambda_{i,t}\theta_t\left(Q_t f_{i,t} + \Delta Q_{B,t}b_{i,t}\right)$$

The derivatives of the Lagrangian are:

$$\frac{\partial \mathbb{L}}{\partial f_{i,t}} = (1+\lambda_{i,t})\left\{\mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t + \sigma\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\frac{\partial n_{i,t+1}}{\partial f_{i,t}}\right\} - \lambda_{i,t}\theta_t Q_t$$

$$\frac{\partial \mathbb{L}}{\partial b_{i,t}} = (1+\lambda_{i,t})\left\{\mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t + \sigma\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\frac{\partial n_{i,t+1}}{\partial b_{i,t}}\right\} - \lambda_{i,t}\theta_t\Delta Q_{B,t}$$

$$\frac{\partial \mathbb{L}}{\partial re_{i,t}} = (1+\lambda_{i,t})\left\{\mathbb{E}_t(1-\sigma_{t+1})\Lambda_{t,t+1}\left(R^{re}_t-R^d_t\right)\Pi^{-1}_{t+1} + \sigma_{t+1}\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\frac{\partial n_{i,t+1}}{\partial re_{i,t}}\right\}$$

Note that

$$\frac{\partial n_{i,t+1}}{\partial f_{i,t}} = \left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t$$

$$\frac{\partial n_{i,t+1}}{\partial b_{i,t}} = \left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_{B,t}$$

$$\frac{\partial n_{i,t+1}}{\partial re_{i,t}} = \left(R^{re}_t-R^d_t\right)\Pi^{-1}_{t+1}$$

Plug these in and set to zero. We get:

$$(1+\lambda_{i,t})\mathbb{E}_t\left\{(1-\sigma)\Lambda_{t,t+1}\left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t + \sigma\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_t\right\} = \lambda_{i,t}\theta_t Q_t$$

$$(1+\lambda_{i,t})\mathbb{E}_t\left\{(1-\sigma)\Lambda_{t,t+1}\left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_{B,t} + \sigma\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}Q_{B,t}\right\} = \lambda_{i,t}\theta_t\Delta Q_{B,t}$$

$$(1+\lambda_{i,t})\mathbb{E}_t\left\{(1-\sigma)\Lambda_{t,t+1}\left(R^{re}_t-R^d_t\right)\Pi^{-1}_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}}\left(R^{re}_t-R^d_t\right)\Pi^{-1}_{t+1}\right\} = 0$$

Define:

$$\Omega_{i,t+1} = 1-\sigma+\sigma\frac{\partial V_{i,t+1}}{\partial n_{i,t+1}} \tag{34}$$

We can then write these FOC as:

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R^F_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}\Omega_{i,t+1} = \frac{\lambda_{i,t}}{1+\lambda_{i,t}}\theta_t$$

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}\Omega_{i,t+1}=\frac{\lambda_{i,t}}{1+\lambda_{i,t}}\Delta\theta_t$$
$$\mathbb{E}_t\Lambda_{t,t+1}\left(R^{re}_t-R^d_t\right)\Pi^{-1}_{t+1}\Omega_{i,t+1}=0$$

Since R^{re}_t and R^d_t are known at the time expectations are formed, we can pull them out of the expectations operator and conclude:

$$R^{re}_t=R^d_t \tag{35}$$

For other part, we need to show that nothing depends on i for aggregation. *Guess* that the value function is linear in net worth:

$$V_{i,t}=a_t n_{i,t} \tag{36}$$

When the constraint binds, given this guess we have:

$$a_t n_{i,t}=\theta_t\left(Q_t f_{i,t}+\Delta Q_{B,t}b_{i,t}\right)$$

Define $\phi_{i,t}$ as a modified leverage ratio:

$$\phi_{i,t}=\frac{Q_t f_{i,t}+\Delta Q_{b,t}b_{i,t}}{n_{i,t}}$$

But this would imply: $a_t=\theta_t\phi_t$. Since we are guessing a_t doesn't vary with i, then neither can ϕ_t. So we have:

$$a_t=\phi_t\theta_t$$

If this is the case, then:

$$\Omega_t=1-\sigma+\sigma\phi_t\theta_t \tag{37}$$

Now write down the law of motion for net worth, led forward one period:

$$n_{i,t+1}=\Pi^{-1}_{t+1}\left[\left(R^F_{t+1}-R^d_t\right)Q_t f_{i,t}+\left(R^B_{t+1}-R^d_t\right)Q_{B,t}b_{i,t}+\left(R^{re}_t-R^d_t\right)re_{i,t}+R^d_t n_{i,t}\right]$$

Multiply both sides by $\Lambda_{t,t+1}\Omega_{t+1}$:

$$\Lambda_{t,t+1}\Omega_{t+1}n_{i,t+1}=\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left[\left(R^F_{t+1}-R^d_t\right)Q_t f_{i,t}+\left(R^B_{t+1}-R^d_t\right)Q_{B,t}b_{i,t}+\left(R^{re}_t-R^d_t\right)re_{i,t}+R^d_t n_{i,t}\right]$$

Now take expectations of both sides:

$$\begin{aligned}\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{i,t+1}=&\,\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^F_{t+1}-R^d_t\right)Q_t f_{i,t}+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^B_{t+1}-R^d_t\right)Q_{B,t}b_{i,t}\\&+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^{re}_t-R^d_t\right)re_{i,t}+\mathbb{E}_t\Lambda_{t,t+1}\Pi^{-1}_{t+1}\Omega_{t+1}R^d_t n_{i,t}\end{aligned}$$

Now where is this getting us? From above, we have:

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R^B_{t+1}-R^d_t\right)\Pi^{-1}_{t+1}\Omega_{i,t+1}=\Delta\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^F_{t+1}-R^d_t\right)$$

So plug this in. We get:

$$\begin{aligned}\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{i,t+1}=&\,\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^F_{t+1}-R^d_t\right)Q_t f_{i,t}+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^F_{t+1}-R^d_t\right)\Delta Q_{B,t}b_{i,t}\\&+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\left(R^{re}_t-R^d_t\right)re_{i,t}+\mathbb{E}_t\Lambda_{t,t+1}\Pi^{-1}_{t+1}\Omega_{t+1}R^d_t n_{i,t}\end{aligned}$$

We also know that $R_t^{re} = R_t^d$ from the FOC. Hence, we can write:

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} n_{i,t+1} = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) \phi_t n_{i,t} + \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d n_{i,t}$$

Now go back to the value function. We have:

$$V_{i,t} = (1-\sigma) \mathbb{E}_t \Lambda_{t,t+1} n_{i,t+1} + \sigma \mathbb{E}_t \Lambda_{t,t+1} V_{i,t+1}$$

Now plug in our guess of the value function:

$$a_t n_{i,t} = (1-\sigma) \mathbb{E}_t \Lambda_{t,t+1} n_{i,t+1} + \sigma \mathbb{E}_t \Lambda_{t,t+1} a_t n_{i,t+1}$$

But this is:

$$a_t n_{i,t} = \mathbb{E}_t \Lambda_{t,t+1} n_{i,t+1} (1 - \sigma + \sigma a_{t+1})$$

Which is:

$$a_t n_{i,t} = \mathbb{E}_t \Lambda_{t,t+1} n_{i,t+1} \Omega_{t+1}$$

But from above we know what $\mathbb{E}_t \Lambda_{t,t+1} n_{i,t+1} \Omega_{t+1}$ is:

$$a_t n_{i,t} = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) \phi_t n_{i,t} + \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} R_t^d n_{i,t}$$

The $n_{i,t}$ cancel out:

$$a_t = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) \phi_t + \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d$$

But given our guess, we have $a_t = \phi_t \theta_t$. So we have:

$$\phi_t \theta_t = \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) \phi_t + \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d$$

Therefore:

$$\phi_t \left[\theta_t - \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) \right] = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d$$

So:

$$\phi_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d}{\theta_t - \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right)} \tag{38}$$

(38) is consistent with our guess – ϕ_t does not depend on anything firm specific, and hence neither does a_t. But then this means Ω_t really does not depend on anything firm specific, which then from the FOC means that $\lambda_{i,t} = \lambda_t$ and is the same across firms. The FOC taking all this into account may be written:

$$\mathbb{E}_t \Lambda_{t,t+1} \left(R_{t+1}^F - R_t^d \right) \Pi_{t+1}^{-1} \Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t} \theta_t \tag{39}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(R_{t+1}^B - R_t^d \right) \Pi_{t+1}^{-1} \Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t} \Delta \theta_t \tag{40}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(R_t^{re} - R_t^d \right) \Pi_{t+1}^{-1} \Omega_{t+1} = 0 \tag{41}$$

Before proceeding, note that we can combine (39) with (38) to write:

$$\phi_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Omega_{t+1} R_t^d}{\theta_t - \frac{\lambda_t}{1+\lambda_t}\theta_t}$$

But this is:

$$\theta_t \phi_t = (1+\lambda_t) \mathbb{E}_t \Lambda_{t,t+1} R_t^d \Pi_{t+1}^{-1} \Omega_{t+1} \tag{42}$$

Suppose that the enforcement constraint were never binding. Then we would have $\lambda_t = 0$. So we could write (42) as:

$$\theta_t \phi_t = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_t^d \left(1 - \sigma + \sigma \theta_{t+1} \phi_{t+1}\right)$$

But then we can guess and verify that $\theta_t \phi_t = 1$ is a solution at all times, because:

$$1 = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} R_t^d$$

Which is just the household's first order condition for bonds. So, if $\lambda_t = 0$, then we have $a_t = \phi_t \theta_t = 1$. This makes sense – if the intermediaries are not constrained, then there are no excess returns to holding long bonds. Then net worth is just as valuable "inside" the firm as outside of it, i.e. $\frac{\partial V_{i,t}}{\partial n_{i,t}} = 1$. But if $\lambda_t > 0$, then we know that $\theta_t \phi_t > 1$ – i.e. net worth is worth more inside the firm than outside of it, because only inside the FI can long bonds be held and excess returns achieved.

2.8 Aggregation

Aggregate inflation evolves according to:

$$1 = \Bigg\{ (1-\phi_p) \left(P_t^{\#}\right)^{-\frac{1}{\lambda_p}} + \phi_p \left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{i_p} \left(\frac{\Pi_t}{\Pi}\right)^{-1} \right]^{-\frac{1}{\lambda_p}} \Bigg[^{-\lambda_p} \tag{43}$$

Similarly, the aggregate real wage obeys:

$$1 = (1-\phi_w) \left(\frac{w_t^{\#}}{w_t}\right)^{-\frac{1}{\lambda_w}} + \phi_w \left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\iota_w} \left(\frac{\Pi_t}{\Pi}\right)^{-1} \frac{w_{t-1}}{w_t} \right]^{-\frac{1}{\lambda_w}} \tag{44}$$

To get the aggregate production function, integrate across retailers:

$$\int_0^1 Y_t(f) df = Y_t \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\Lambda_{p,t}}{\Lambda_{p,t}}} df$$

Recall that retailers just repackage wholesale output. Hence, aggregate demand for retail output, $\int_0^1 Y_t(f) df$, just equals wholesale output, $Y_{w,t}$. So we have:

$$Y_{w,t} = Y_t v_t^p \tag{45}$$

$v_t^p = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\Lambda_{p,t}}{\Lambda_{p,t}}} df$ is a measure of price dispersion that may be written recursively using properties of Calvo pricing:

$$v_t^p = (1-\phi_p)\left(P_t^{\#}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \phi_p \left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\iota_p}\left(\frac{\Pi_t}{\Pi}\right)^{-1}\right]^{-\frac{1+\lambda_p}{\lambda_p}} v_{t-1}^p \tag{46}$$

Similarly, integrate demand the demand for union labor across unions:

$$\int_0^1 L_{d,t}(h)dh = L_{d,t}\int_0^1 \left(\frac{w_t(h)}{w_t}\right)^{-\frac{1+\Lambda_{w,t}}{\Lambda_{w,t}}} dh$$

Unions purchase labor from the household. Aggregate union labor demand, $\int_0^1 L_{d,t}(h)dh$, equals household labor supply, L_t. So we have:

$$L_t = L_{d,t}v_t^w \tag{47}$$

$v_t^w = \int_0^1 \left(\frac{w_t(h)}{w_t}\right)^{-\frac{1+\Lambda_{w,t}}{\Lambda_{w,t}}} dh$ is a measure of wage dispersion. It throws a wedge between household labor supply and labor that gets used in production. Using properties of Calvo wage-setting, this satisfies:

$$v_t^w = (1-\phi_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\frac{(1+\lambda_w)(1+\eta)}{\lambda_w}} + \phi_w \left[\left(\frac{w_{t-1}}{w_t}\right)\left(\frac{\Pi_t}{\Pi}\right)^{-1}\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\iota_w}\right]^{-\frac{(1+\lambda_w)(1+\eta)}{\lambda_w}} v_{t-1}^w \tag{48}$$

The FI balance sheet condition is linear in FI-specific variables. So it simply sums up to the same aggregate condition. Market-clearing for long-bonds requires that bonds issued by the wholesale firm and government, respectively, are either held by the central bank or the financial intermediaries:

$$f_{w,t} = f_t + A_t f_{cb,t} \tag{49}$$

$$A_t b_{G,t} = b_t + A_t b_{cb,t} \tag{50}$$

Aggregate net worth evolves as follows. A fraction σ of intermediaries survive from $t-1$ to t. The typical such intermediary has real net worth:

$$n_{i,t} = P_t^{-1}\left[\begin{array}{c} \left(R_t^F - R_{t-1}^d\right)Q_{t-1}F_{i,t-1} + \left(R_t^B - R_{t-1}^d\right)Q_{B,t-1}B_{i,t-1} + \\ \left(R_{t-1}^{re} - R_{t-1}^d\right)RE_{i,t-1} + R_{t-1}^d N_{i,t-1} \end{array}\right]$$

Each of the bond/net worth terms inside brackets needs to be divided by P_{t-1} to put in real terms. So, multiplying and dividing by P_{t-1}, we get:

$$n_{i,t} = \Pi_t^{-1}\left[\begin{array}{c} \left(R_t^F - R_{t-1}^d\right)Q_{t-1}f_{i,t-1} + \left(R_t^B - R_{t-1}^d\right)Q_{B,t-1}b_{i,t-1} + \\ \left(R_{t-1}^{re} - R_{t-1}^d\right)re_{i,t-1} + R_{t-1}^d n_{i,t-1} \end{array}\right]$$

This is just linear in all variables the FI can choose. So we can sum this across FIs. Because those that die are randomly chosen, via a law of large numbers, the sum of surviving-FI variables (e.g. $n_{i,t}$) is just proportional the aggregate via σ. Newly borne intermediaries are given, in aggregate, X of real start-up net worth. Hence, aggregate real net worth evolves as a convex-combination of these:

$$n_t = \sigma \Pi_t^{-1} \left[\begin{array}{c} \left(R_t^F - R_{t-1}^d\right) Q_{t-1} f_{t-1} + \left(R_t^B - R_{t-1}^d\right) Q_{B,t-1} b_{t-1} + \\ \left(R_{t-1}^{re} - R_{t-1}^d\right) re_{t-1} + R_{t-1}^d n_{t-1} \end{array} \right] + A_t X \tag{51}$$

Before proceeding, it is worth pointing something out about a sort of unmodeled friction here. This unmodeled friction is that we are not allowing the household to choose the new equity transferred to intermediaries, X. Given that the intermediaries earn excess returns on long bonds that the household cannot directly access, it would be optimal for the households to transfer more equity to intermediaries each period than they do. We are assuming, implicitly, that something stops this from happening.

The limited enforcement constraint will bind in the region of the steady state we are interest in. This requires that $V_{i,t} = a_t n_{i,t} = \phi_t \theta_t \geq \theta \left(Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}\right)$. Summing across intermediaries gives the aggregate version of the constraint:

$$\phi_t = \frac{Q_t f_t + \Delta Q_{B,t} b_t}{n_t} \tag{52}$$

Finally, the aggregate resource constraint:

$$Y_t = C_t + I_t + G_t \tag{53}$$

Public spending is a time-varying fraction of output

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t$$

where we assume that government spending follows an AR(1) in the log:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + u_t^g \tag{54}$$

with u_t^g independently and identically distributed $N(0, \sigma_g^2)$. Therefore:

$$\frac{1}{g_t} Y_t = C_t + I_t$$

3 Estimation

In this section, we discuss the data we use to estimate our model and we provide some details of the estimation procedure. Then, we describe how we calibrate some of the model parameters and how we estimate the remainder.

3.1 Data

3.2 Calibrated Parameters and Prior Distributions

I calibrate the model loosely following Sims and Wu (2020). I describe the parameterization here. I set $\beta = 0.995$, which implies a steady state real interest rate of 2 percent annualized.

$\kappa = 1 - 40^{-1}$, implying a 10 year duration on corporate and government bonds. I set $\psi = 0.81$ – firms must finance 80 percent of their investment via issuing debt. $\epsilon_p = \epsilon_w = 11$, implying steady state price and wage markups of 10 percent. I set $\alpha = 1/3$. I set $\delta_0 = 0.025$ (this is steady state capital depreciation) and $\delta_2 = 0.01$, which implies rather volatile capital utilization. δ_1 is fixed to be consistent with the normalization of $u = 1$. I set the government spending share of output to $g = 0.2$ in steady state. The habit formation parameter is $b = 0.8$. The inverse Frisch elasticity, η, is 1. χ is chosen to be consistent with the normalization that $L = 1$ in steady state.

For financial variables, I set $\sigma = 0.95$. I target a total leverage ratio (the ratio of all assets to net worth in steady state, not the modified leverage ratio ϕ) to be 5. I assume that the central bank's steady state balance sheet is 10 percent of output, and that 90 percent of its assets are government bonds (so only a small fraction are corporate bonds). I assume that the steady state debt-GDP ratio for the fiscal authority is 50 percent. I target a corporate bond spread of 3 percent annualized, and a government bond spread of 1 percent annualized. Altogether, these targets imply values of X, θ, and Δ. In particular, I get $X = 0.0442$, $\theta = 0.6555$, and $\Delta = 0.33$. Concretely, this means that in default an intermediary may abscond with about two-thirds of its private assets and a little more than 20 percent of its government bonds. To put X into perspective, steady state net worth of intermediaries is $n = 3.75$. So the new equity infusion to new intermediaries is only about 1 percent of total equity.

I set the price and wage stickiness parameters to $\phi_p = \phi_w = 0.75$. This implies average four quarter durations between price/wage changes. The investment adjustment cost function is: $S\left(I_t/I_{t-1} - 1\right) = \frac{\psi_k}{2}\left(I_t/I_{t-1} - 1\right)^2$. I set $\psi_k = 2$. The parameters of the Taylor rule are $\rho_r = 0.8$, $\phi_\pi = 1.5$, and $\phi_y = 0.15$.

It remains to parameterize the shock prices. The shock standard deviations matter for unconditional moments but impulse responses are just scaled versions of the shock sizes. Consequently, I'm not going to focus here on trying to get the shock sizes correct to match any particular unconditional moments; rather I'm going to focus on impulse responses and how the model works. To be transparent, I just set all the shock standard deviations to 0.01. I'm going to set the AR(1) terms on government spending, government bonds, and productivity to be $\rho_G = \rho_B = \rho_A = 0.90$. I'm going to set the AR(1) on the credit shock variable to $\rho_\theta = 0.95$. Finally, I set the AR(1) parameter on exogenous private and government bond purchases by the central bank to $\rho_f = \rho_b = 0.97$. This is a good bit higher than what Sims and Wu (2020) use. The impulse responses I show are of logs of variables – so we can interpret units of things like output, consumption, and investment in percentage tersm. For inflation and interest, plotting the responses of logged gross rates gives the net rates (hence lowercase letters). Interest rates and inflation rate responses are multiplied by 400, to express them in annualized percentage terms.

4 Results

5 Robustness

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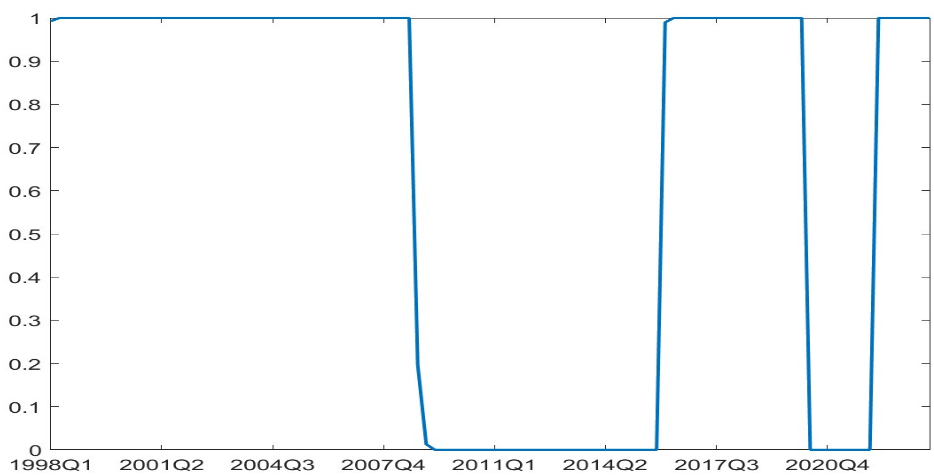
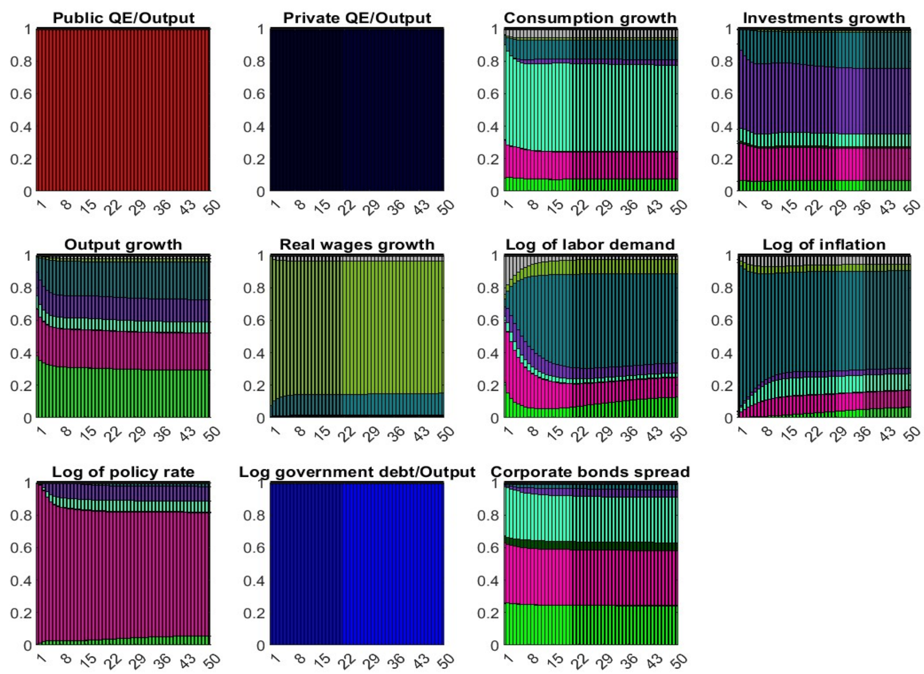
Tables and Figures

Figure 1: **Smoothed probabilities**



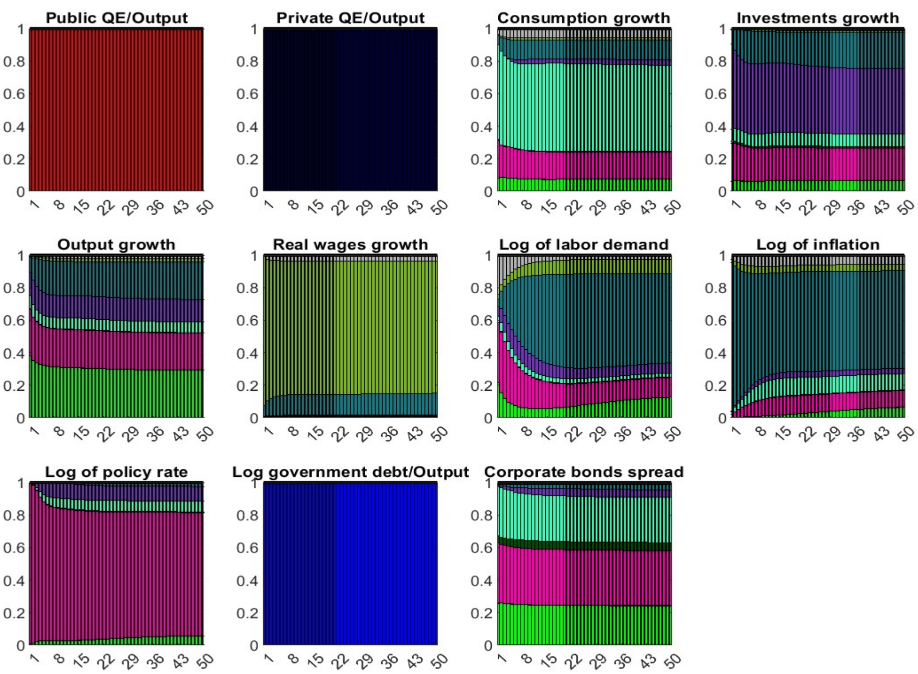
Notes: The figure shows the smoothed probabilities of being in normal times.

Figure 2: **Variance decomposition in normal times**



Notes: The figure shows the variance decomposition in normal time. Blue: government debt shock, very light green: government spending shock, red: public QE shock, dark blue: private QE shock, magenta: monetary policy shock, dark green: credit shock, turquoise: preference shock, purple: investment specific shock, teal: price markup shock, light green: wage markup shock.

Figure 3: **Variance decomposition in ZLB times**



Notes: The figure shows the variance decomposition in ZLB time. Blue: government debt shock, very light green: government spending shock, red: public QE shock, dark blue: private QE shock, magenta: monetary policy shock, dark green: credit shock, turquoise: preference shock, purple: investment specific shock, teal: price markup shock, light green: wage markup shock.

Appendices - All the material from this point onward can be for an online Appendix

A Data

As we described in the main body of the paper, the data are quarterly and the model is estimated for the sample period 1998:Q1-2023:Q4. In this Appendix we provide the original sources and construction methods of the observed series.

Real GDP is released by the US BEA (Real Gross Domestic Product [GDPC1], downloaded from https://fred.stlouisfed.org/series/GDPC1). The series of nominal personal consumption expenditures is the sum of personal consumption expenditures of nondurable goods released by the US BEA (Personal Consumption Expenditures: Non-durable Goods [PCND], downloaded from https://fred.stlouisfed.org/series/PCND) and personal consumption expenditures of services released by the US BEA (Personal Consumption Expenditures: Services [PCESV], downloaded from https://fred.stlouisfed.org/series/PCESV). The series of nominal private investment is the sum of personal consumption expenditures of durable goods released by the US BEA (Personal Consumption Expenditures: durable Goods [PCDG], downloaded from https://fred.stlouisfed.org/series/PCDG) and gross private domestic investment released by the US BEA (Gross Private Domestic Investment [GPDI], downloaded from https://fred.stlouisfed.org/series/GPDI). The civilian non-institutional population is released by the US BLS (Population Level [CNP16OV], downloaded from https://fred.stlouisfed.org/series/CNP16OV) and is transformed in LNSINDEX. GZ is the spread derived in Gilchrist and Zakrajšek (2012), downloaded from https://www.federalreserve.gov/econres/notes/feds-notes/updating-the-rece html. The GDP deflator is released by the US BEA (Gross Domestic Product: Implicit Price Deflator [GDPDEF], downloaded from https://fred.stlouisfed.org/series/GDPDEF). The quarter average Federal funds rate [DFF], downloaded from https://fred.stlouisfed.org/series/DFF, the quarterly Personal Consumption Expenditures: Chain-type Price Index [PCEPI], download at https://fred.stlouisfed.org/series/PCEPI, the average quarterly hours of production and nonsupervisory employees for total private industries [AWHNONAG], downloaded at https://fred.stlouisfed.org/series/AWHNONAG, and the quarterly compensation per hour for the non-farm business sector [COMPNFB], downloaded from https://fred.stlouisfed.org/series/COMPNFB. The nominal corporate bonds held by the central bank are the sum of (Securities Held Outright: Mortgage-Backed Securities [WSHOMCB], downloaded from https://fred.stlouisfed.org/series/WSHOMCB) and (Securities Held Outright: Federal Agency Debt Securities [FEDDT], downloaded from https://fred.stlouisfed.org/series/FEDDT). We divide that by the nominal GDP (Gross Domestic Product [GDP], downloaded from https://fred.stlouisfed.org/series/GDP). The long term government bonds held by the central bank are measured as the SOMA Domestic Securities Holdings in Ten-Year Equivalents, downloaded from https://www.newyorkfed.org/markets/annual_reports. We divide that by the nominal GDP. Finally, the total long term government bonds are measured as the sum of the previous item plus the bonds held by all commercial banks (Treasury and Agency Securities: Non-MBS, All Commercial Banks [TNMACBM027NBOG]), downloaded from https://fred.

stlouisfed.org/series/TNMACBM027NBOG.

Those variables are transformed as follows: Let Δ denote the temporal difference operator. Then the variables are transformed as follows:

- Output growth $= \Delta LN(GDPC1/LNSINDEX)$
- Consumption growth $= \Delta LN(((PCND + PCESV)/GDPDEF)/LNSINDEX)$
- Investment growth $= \Delta LN(((PCDG + GPDI)/GDPDEF)/LNSINDEX)$
- Spread $= GZ/4$
- Federal funds rate $= DFF/4$
- Inflation $= \Delta LN(PCEPI)$
- Hours worked $= LN((AWHNONAG * CE16OV/100)/LNSINDEX)$
- Real wage growth $= \Delta LN(COMPNFB/GDPDEF)$
- Central Banks holding of private bonds $= (WSHOMCB + FEDDT)/GDP$
- Central Banks holding of government bonds $=$ Treasury Securities$/GDP$
- Long terms government bonds $= LN(($Treasury Securities$+TNMACBM027NBOG)/GDPDEF)$

where [CE16OV] is the employment level, downloaded from https://fred.stlouisfed.org/series/CE16OV.

B Stationary Equilibrium Conditions

To get a stationary system we use the following variable transformations: $\widetilde{\mu}_t = \mu_t A_t, c_t = \frac{C_t}{A_t}, \widehat{i}_t = \frac{\widehat{I}_t}{A_t}, k_t = \frac{K_t}{A_t}, i_t = \frac{I_t}{A_t}, \widetilde{w}_t = \frac{w_t}{A_t}, y_{w,t} = \frac{Y_{w,t}}{A_t}, \widetilde{n}_t = \frac{n_t}{A_t}, y_t = \frac{Y_t}{A_t}, \widetilde{re}_t = \frac{re_t}{A_t}, \widetilde{d}_t = \frac{d_t}{A_t}, \widetilde{w}_t^{\#} = \frac{w_t^{\#}}{A_t}, \widetilde{f}_{w,t} = \frac{f_{w,t}}{A_t}, \widetilde{f}_t = \frac{f_t}{A_t}, \widetilde{b}_t = \frac{b_t}{A_t}$

- Household:

$$\Lambda_{t-1,t} = \frac{\beta \widetilde{\mu}_t}{\widetilde{\mu}_{t-1} e^{z_t}} \tag{55}$$

$$\widetilde{\mu}_t = \frac{\varepsilon_t^b}{(c_t - bc_{t-1}e^{-z_t})} - \beta b \mathbb{E}_t \frac{\varepsilon_{t+1}^b}{(c_{t+1}e^{z_{t+1}} - bc_t)} \tag{56}$$

$$1 = R_t^d \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} \tag{57}$$

- Labor union:

$$\widetilde{w}_t^{\#} = \widetilde{w}_t \left(\frac{f_{1,t}}{f_{2,t}} \right)^{\frac{\lambda_w}{\lambda_w + \eta(1+\lambda_w)}} \tag{58}$$

$$f_{1,t} = e^{\lambda_{w,t}} \chi \varepsilon_t^b L_{d,t}^{1+\eta} + \phi_w \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t e^{z_t}}{\Pi e^z} \right)^{\iota_w} \left(\frac{\Pi_{t+1} e^{z_{t+1}}}{\Pi e^z} \right)^{-1} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t} \right)^{-1} \right]^{-\frac{(1+\eta)(1+\lambda_w)}{\lambda_w}} f_{1,t+1} \right\} \tag{59}$$

$$f_{2,t} = \mu_t L_{d,t} \widetilde{w}_t + \phi_w \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t e^{z_t}}{\Pi e^z} \right)^{\iota_w} \left(\frac{\Pi_{t+1} e^{z_{t+1}}}{\Pi e^z} \right)^{-1} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t} \right)^{-1} \right]^{-\frac{1}{\lambda_w}} f_{2,t+1} \right\} \tag{60}$$

- Investment Firm:

$$\widehat{i}_t = \mu_t^I \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \right] i_t \tag{61}$$

$$1 = p_t^k \mu_t^I \left[1 - S\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) - S'\left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \frac{i_t}{i_{t-1}} e^{z_t} \right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^k \mu_{t+1}^I S'\left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right) \left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right)^2 \tag{62}$$

- Retail firm:

$$P_t^{\#} = \frac{x_{1,t}}{x_{2,t}} \tag{63}$$

$$x_{1,t} = e^{\lambda_{p,t}} \mu_t p_{w,t} y_t + \phi_p \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi} \right)^{\iota_p} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{-1} \right]^{-\frac{1+\lambda_p}{\lambda_p}} x_{1,t+1} \right\} \tag{64}$$

$$x_{2,t} = \mu_t y_t + \phi_p \beta \mathbb{E}_t \left\{ \left[\left(\frac{\Pi_t}{\Pi} \right)^{\iota_p} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{-1} \right]^{-\frac{1}{\lambda_p}} x_{2,t+1} \right\} \tag{65}$$

- Wholesale firm:

$$\widetilde{w}_t = (1-\alpha)p_{w,t}(u_t k_t)^{\alpha}\left(L_{d,t}\right)^{-\alpha} \tag{66}$$

$$p_t^k M_{2,t}\delta'(u_t) = \alpha p_{w,t}(u_t k_t)^{\alpha-1}\left(L_{d,t}\right)^{1-\alpha} \tag{67}$$

$$p_t^k M_{2,t} = \mathbb{E}_t\Lambda_{t,t+1}\left[\alpha p_{w,t+1}(u_{t+1}k_{t+1})^{\alpha-1}u_{t+1}\left(L_{d,t+1}\right)^{1-\alpha} + (1-\delta(u_{t+1}))p_{t+1}^k M_{2,t+1}\right] \tag{68}$$

$$Q_t M_{1,t} = \mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left[1+\kappa Q_{t+1}M_{1,t+1}\right] \tag{69}$$

$$\frac{M_{1,t}-1}{M_{2,t}-1} = \psi^{-1} \tag{70}$$

$$y_{w,t} = (u_t k_t)^{\alpha}\left(L_{d,t}\right)^{1-\alpha} \tag{71}$$

$$k_{t+1}e^{z_{t+1}} = \widehat{i}_t + (1-\delta(u_t))k_t \tag{72}$$

$$\psi p_t^k \widehat{i}_t = Q_t\left(\widetilde{f}_{w,t} - \kappa\Pi_t^{-1}\widetilde{f}_{w,t-1}e^{-z_t}\right) \tag{73}$$

- Financial intermediary:

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R_{t+1}^F - R_t^d\right)\Pi_{t+1}^{-1}\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\theta_t \tag{74}$$

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R_{t+1}^B - R_t^d\right)\Pi_{t+1}^{-1}\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Delta\theta_t \tag{75}$$

$$\mathbb{E}_t\Lambda_{t,t+1}\left(R_t^{re} - R_t^d\right)\Pi_{t+1}^{-1}\Omega_{t+1} = 0 \tag{76}$$

$$\Omega_t = 1-\sigma+\sigma\phi_t\theta_t \tag{77}$$

$$\phi_t = \frac{\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Omega_{t+1}R_t^d}{\theta_t - \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\left(R_{t+1}^F - R_t^d\right)} \tag{78}$$

$$Q_t\widetilde{f}_t + Q_{B,t}\widetilde{b}_t + \widetilde{re}_t = \widetilde{d}_t + \widetilde{n}_t \tag{79}$$

$$\phi_t = \frac{Q_t\widetilde{f}_t + \Delta Q_{B,t}\widetilde{b}_t}{\widetilde{n}_t} \tag{80}$$

$$\widetilde{n}_t = \sigma\Pi_t^{-1}\left[\left(R_t^F - R_{t-1}^d\right)Q_{t-1}\widetilde{f}_{t-1} + \left(R_t^B - R_{t-1}^d\right)Q_{B,t-1}\widetilde{b}_{t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right)\widetilde{re}_{t-1} + R_{t-1}^d\widetilde{n}_{t-1}\right]e^{-z_t} + X \tag{81}$$

- Central Bank

$$\begin{aligned}
\ln R_t^{tr} &= (1-\rho_r)\ln R^{tr} + \rho_r \ln R_{t-1}^{tr} && (82)\\
&+(1-\rho_r)\left[\frac{\phi_\pi}{4}\left(\sum_{j=0}^{3}\ln\Pi_{t-j} - 4\ln\Pi\right) + \frac{\phi_y}{4}\left(\ln y_t - \ln y_{t-4} + z_t + z_{t-1} + z_{t-2} + z_3 - 4z\right)\right] && (83)\\
&+u_t^r && (84)
\end{aligned}$$

$$R_t^{re} = \max\left\{1, R_t^{tr}\right\} \tag{85}$$

$$f_{cb,t}^Y = (1-\rho_f)f_{cb}^Y + \rho_f f_{cb,t-1}^Y + u_t^f \quad \text{if } R_t^{tr} > 1 \tag{86}$$

$$b_{cb,t}^Y = (1-\rho_B)b_{cb} + \rho_B b_{cb,t-1}^Y + u_t^B \quad \text{if } R_t^{tr} > 1 \tag{87}$$

$$\begin{aligned}
f_{cb,t}^Y &= (1-\rho_f) f_{cb}^Y + \rho_f f_{cb,t-1}^Y \\
&\quad -(1-\rho_f)\left[\frac{\phi_\pi}{4}\left(\sum_{j=0}^{3}\ln \Pi_{t-j} - 4\ln \Pi\right) + \frac{\phi_y}{4}\left(\ln y_t - \ln y_{t-4} + z_t + z_{t-1} + z_{t-2} + z_3 - 4z\right)\right] \\
&\quad + u_t^f \quad \text{if } R_t^{tr} \leq 1 \\
b_{cb,t}^Y &= (1-\rho_B) b_{cb}^Y + \rho_B b_{cb,t-1}^Y \\
&\quad -(1-\rho_B)\left[\frac{\phi_\pi}{4}\left(\sum_{j=0}^{3}\ln \Pi_{t-j} - 4\ln \Pi\right) + \frac{\phi_y}{4}\left(\ln y_t - \ln y_{t-4} + z_t + z_{t-1} + z_{t-2} + z_3 - 4z\right)\right] \\
&\quad + u_t^B \quad \text{if } R_t^{tr} \leq 1
\end{aligned}$$

$$Q_t f_{cb,t} + Q_{B,t} b_{cb,t} = \widetilde{re}_t \tag{88}$$

- Aggregate conditions:

$$1 = \left\{(1-\phi_p)\left(P_t^{\#}\right)^{-\frac{1}{\lambda_p}} + \phi_p\left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{i_p}\left(\frac{\Pi_t}{\Pi}\right)^{-1}\right]^{-\frac{1}{\lambda_p}}\right\}^{-\lambda_p} \tag{89}$$

$$1 = (1-\phi_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\frac{1}{\lambda_w}} + \phi_w\left[\left(\frac{\Pi_{t-1}e^{z_{t-1}}}{\Pi e^z}\right)^{\iota_w}\left(\frac{\Pi_t e^{z_t}}{\Pi e^z}\right)^{-1}\frac{w_{t-1}}{w_t}\right]^{-\frac{1}{\lambda_w}} \tag{90}$$

$$y_{w,t} = y_t v_t^p \tag{91}$$

$$v_t^p = (1-\phi_p)\left(P_t^{\#}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \phi_p\left[\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\iota_p}\left(\frac{\Pi_t}{\Pi}\right)^{-1}\right]^{-\frac{1+\lambda_p}{\lambda_p}} v_{t-1}^p \tag{92}$$

$$L_t = L_{d,t} v_t^w \tag{93}$$

$$v_t^w = (1-\phi_w)\left(\frac{w_t^{\#}}{w_t}\right)^{-\frac{(1+\lambda_w)(1+\eta)}{\lambda_w}} + \phi_w\left[\left(\frac{w_{t-1}}{w_t}\right)\left(\frac{\Pi_t e^{z_t}}{\Pi e^z}\right)^{-1}\left(\frac{\Pi_{t-1}e^{z_{t-1}}}{\Pi e^z}\right)^{\iota_w}\right]^{-\frac{(1+\lambda_w)(1+\eta)}{\lambda_w}} v_{t-1}^w \tag{94}$$

$$\widetilde{f}_{w,t} = \widetilde{f}_t + f_{cb,t} \tag{95}$$

$$b_{G,t} = \widetilde{b}_t + b_{cb,t} \tag{96}$$

$$y_t \frac{1}{g_t} = c_t + i_t \tag{97}$$

$$R_t^F = \frac{1+\kappa Q_t}{Q_{t-1}} \tag{98}$$

$$R_t^B = \frac{1+\kappa Q_{B,t}}{Q_{B,t-1}} \tag{99}$$

- Exogenous processes:

$$z_t \equiv \Delta \ln A_t \tag{100}$$

$$\ln \theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta \ln \theta_{t-1} + u_t^\theta \tag{101}$$

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + u_t^g \tag{102}$$

$$\ln b_{G,t} = (1-\rho_B)\ln b_G + \rho_B \ln b_{G,t-1} + u_t^{bg} \tag{103}$$

$$z_t = (1-\rho_z)\, z + \rho_z z_{t-1} + u_t^z \tag{104}$$

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + u_t^b \tag{105}$$

$$\lambda_{w,t} = (1-\rho_w)\,\lambda_w + \rho_w \lambda_{w,t-1} + u_t^w \tag{106}$$

$$\ln \mu_t^I = \rho_\mu \ln \mu_{t-1}^I + u_t^\mu \tag{107}$$

$$\lambda_{p,t} = (1-\rho_p)\,\lambda_p + \rho_p \lambda_{p,t-1} + u_t^p \tag{108}$$

C Steady State of the Stationary Model

We are going to focus on a zero inflation steady state. This means that $\Pi = 1$, so $P^{\#} = 1$, $v^p = 1$, $v^w = 1$, and $\widetilde{w}^{\#} = \widetilde{w}$. $v^w = 1$ means that $L = L_d$ and $y_w = y$. Similarly, since the investment adjustment cost is irrelevant in the steady state, we have $\widehat{i} = i$. I will also normalize the model such that $L = 1$. I am also going to pick parameters to have steady state utilization be 1. The utilization adjustment cost is:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2 \tag{109}$$

Focusing first on the household block, we get:

$$\Lambda = \frac{\beta}{e^z} \tag{110}$$

Which implies:

$$R^d = R^{re} = R^{tr} = \frac{e^z \Pi}{\beta} \tag{111}$$

I am going to target two spreads: sp_F is the private lending spread, $R^F - R^d$¡ and $sp_B = R^B - R^d$ is the government lending spread. I will choose $sp_F = 1.03^{1/4}$ and $sp_B = 1.01^{1/4}$, so that I am targeting steady state spreads of 300 and 100 basis points, respectively, at an annual frequency. This then gives me:

$$R^F = sp_F R^d \tag{112}$$

$$R^B = sp_B R^d \tag{113}$$

But this then gives us steady state long bond prices as functions of κ, which I set to be $1 - 40^{-1}$:

$$Q = \left(R^F - \kappa\right)^{-1} \tag{114}$$

$$Q_B = \left(R^B - \kappa\right)^{-1} \tag{115}$$

But now I can solve for M_1 since:

$$QM_1 = \frac{\beta}{e^z}\Pi^{-1}(1 + \kappa Q M_1)$$

Which implies:

$$M_1 = \frac{\frac{\beta}{e^z}\Pi^{-1}}{Q(1 - \frac{\beta}{e^z}\Pi^{-1}\kappa)} \tag{116}$$

But then we have M_2:

$$M_2 = 1 + \psi(M_1 - 1) \tag{117}$$

As in Sims and Wu (2020), I pick $\psi = 0.81$.

For both price and wage-setting, I am going to assume that $\lambda_p = \lambda_w = 0.1$. From the price-setting and wage setting conditions, I then get:

$$p_w = \frac{1}{e^{\lambda_p}} \tag{118}$$

(**??**) is steady state real marginal cost; equivalently, this is the inverse steady state markup of price over marginal cost. (**??**) tells us the wage the household receives is a markdown over the wage charged to the wholesale firm; the difference is captured by unions.

Given that I am normalizing $L_d = u = 1$, I can now solve for steady state capital from the capital Euler equation. First, note from the FOC for investment that $p^k = 1$. We then have:

$$p^k M_2 = \frac{\beta}{e^z}\left[\alpha p_w \left(uk\right)^{\alpha-1}\left(uL_d\right)^{1-\alpha} + (1-\delta_0)p^k M_2\right]$$

Which implies:

$$k = \left(\frac{\alpha p_w}{M_2\left(\frac{e^z}{\beta} - (1-\delta_0)\right)}\right)^{\frac{1}{1-\alpha}} \tag{119}$$

It is useful to look at (**??**) and point out different distortions matter. First, $p_w < 1$, owing to monopoly power in price-setting, lowers steady state capital. Second, $M_2 > 1$, which comes about because of positive interest rate spreads making the loan in advance constraint binding for the wholesale firm, also results in too little steady state capital relative to what would be efficient.

Once I have k, I have $y = y_w$ as well as $i = \widehat{i}$ and $\widetilde{w} = \widetilde{w}^{\#}$:

$$y = k^{\alpha} \tag{120}$$

$$i = (e^z - 1 + \delta_0)\, k \tag{121}$$

$$w = (1-\alpha) p_w k^\alpha \tag{122}$$

Note that $\delta'(u) = \delta_1$. We need to pick δ_1 to be consistent with our normalization; δ_0 and δ_2 are free parameters. In particular, from the FOC for utilization, we must have:

$$\delta_1 = \frac{\alpha p_w k^{\alpha-1}}{M_2} \tag{123}$$

Let's assume that in steady state $G/Y = g$ (e.g. $g = 0.2$). Then we can solve for steady state consumption as:

$$c = (1-g)y - i \tag{124}$$

But then we can solve for $\widetilde{\mu}$:

$$\widetilde{\mu} = \frac{1}{c}\left[\frac{1}{(1-be^{-z})} - \beta b \frac{1}{(e^z - b)}\right] \tag{125}$$

To derive the steady state value for χ we exploit equations **??**, **??**, **??**, **??**, **??**, $\widetilde{w}^{\#} = \widetilde{w}$, and the normalization of $L = 1$. That gives

$$\chi = \frac{\widetilde{\mu}\widetilde{w}}{e^{\lambda_w}} \tag{126}$$

We can now figure out how much debt the wholesale firm must float in steady state:

$$\widetilde{f}_w = \frac{\psi i}{Q(1-\kappa \Pi^{-1} e^{-z})} \tag{127}$$

Let's suppose that the total size of the central bank's balance sheet is some fraction of output, say $bcs = 0.1Y$. This tells us steady state reserves, since that is the steady state balance sheet size. Suppose that some other fraction, $bcbGs$ of the central bank's balance sheet is held in government bonds. Let $bcbGs = 0.9$. This then gives us steady state central bank government debt holdings:

$$b_{cb} = \frac{bcbGs \times re}{Q_B} \tag{128}$$

But then we can determine central bank holdings of private bonds via the central bank budget constraint:

$$f_{cb} = \frac{re - Q_B b_{cb}}{Q} \tag{129}$$

Which then from the adding up constraint tells us how much private debt FIs must hold:

$$\widetilde{f} = \widetilde{f}_w - f_{cb} \tag{130}$$

Now suppose that the outstanding value of government debt is some fraction of GDP, by (e.g. 0.5). So we have:

$$b_G = \frac{by \times y}{Q_B} \tag{131}$$

But then from the market-clearing constraint, we have government bonds held by the FI:

$$\widetilde{b} = b_G - b_{cb} \tag{132}$$

Let's then target a total leverage ratio, lev, where lev is the ratio of total assets to net worth. I will use $lev = 5$. This implies a steady state value of net worth:

$$\widetilde{n} = \frac{Q\widetilde{f} + Q_B\widetilde{b} + \widetilde{re}}{lev} \tag{133}$$

We can then get steady state deposits from the FIs balance sheet condition:

$$\widetilde{d} = Q\widetilde{f} + Q_B\widetilde{b} + \widetilde{re} - \widetilde{n} \tag{134}$$

Note that there is a restriction implied on Δ, which is the relative recoverability of government bonds to private bonds. From the FOC from the FI problem, in steady state we have:

$$\Delta = \frac{R^B - R^d}{R^F - R^d} \tag{135}$$

In other words, (**??**) tells us that Δ governs the relative spread between private and government bonds.

But now we can also get the steady state value of the modified leverage ratio, ϕ, given Δ:

$$\phi = \frac{Q\widetilde{f} + \Delta Q_B\widetilde{b}}{\widetilde{n}} \tag{136}$$

Now we can get θ from the FOC giving us. This is more complicated than it looks because ϕ and θ show up in Ω. We have:

$$\Omega = 1 - \sigma + \sigma\phi\theta$$

$$\phi = \frac{\Lambda\Pi^{-1}\Omega R^d}{\theta - \Lambda\Omega\Pi^{-1}\left(R^F - R^d\right)}$$

$$\phi = \frac{\frac{\beta}{e^z}\Pi^{-1}\Omega R^d}{\theta - \frac{\beta}{e^z}\Omega\Pi^{-1}\left(R^F - R^d\right)}$$

Since $\frac{e^z\Pi}{\beta} = R^d$

$$\phi\left(\theta - \frac{\beta}{e^z}(1 - \sigma + \sigma\phi\theta)\Pi^{-1}(R^F - R^d)\right) = 1 - \sigma + \sigma\phi\theta$$

I'm going to set $\sigma = 0.95$. This parameter governs how long FIs are expected to live. The above is now one equation in one unknown, θ. Multiply the LHS through:

$$\phi\theta - \phi\frac{\beta}{e^z}(1-\sigma)(R^F - R^d) - \frac{\beta}{e^z}\sigma\phi^2\theta(R^F - R^d) = 1 - \sigma + \sigma\phi\theta$$

Isolate the terms involving θ on the LHS:

$$\phi\theta - \frac{\beta}{e^z}\sigma\phi^2\theta(R^F - R^d) - \sigma\phi\theta = 1 - \sigma + \phi\frac{\beta}{e^z}(1-\sigma)(R^F - R^d)$$

Solving for θ:

$$\theta = \frac{1 - \sigma + \phi\frac{\beta}{e^z}(1-\sigma)(R^F - R^d)}{(1-\sigma)\phi - \frac{\beta}{e^z}\sigma\phi^2(R^F - R^d)} \tag{137}$$

Now, there is something useful to notice here. In particular, if there is no lending spread, then we get $\theta = 1/\phi$. This is useful because we know the firm's value function is proportional to net worth via $a = \theta\phi$. With no spread, then $a = 1$, which tells us that net worth is as valuable inside an FI as not. But with spreads, net worth is more valuable inside the firm than out.

We can then solve for the equity transfer given everything else we have found:

$$X = \widetilde{n} - \sigma\left[(R^F - R^d)Q\widetilde{f} + (R^B - R^d)Q_B\widetilde{b} + R^d\widetilde{n}\right]$$

We can finally solve for the steady state value of the multiplier on the limited enforcement constraint for the FIs:

$$\frac{\beta}{e^z}(R^F - R^d)(1 - \sigma + \sigma\phi\theta) = \frac{\lambda}{1+\lambda}\theta$$

So:

$$\lambda = \left(\frac{\theta}{\frac{\beta}{e^z}(R^F - R^d)(1 - \sigma + \sigma\phi\theta)} - 1\right)^{-1} \tag{138}$$

Note if there is no spread, then the first term inside the parentheses goes to infinity, so $\lambda \to 0$. The bigger is the spread, the bigger is λ (i.e. the tighter is the constraint).