Sovereign debt sustainability, the carbon budget and climate damages*

Caterina Seghini [1]

[1]Swiss Finance Institute, Université de Genève.

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Abstract

This paper investigates the trade-offs between managing the financial sustainability of public debt and addressing climate change. Mitigation efforts and increasing temperatures imply economic costs that reduce countries' growth rates, respectively in the short and in the long term. This can make the repayment of outstanding debt more difficult. I explore and quantify the evolution of debt limits –maximum sustainable debt-to-GDP– for advanced economies, under various scenarios, which respect, or not, the carbon budget constraints of the Paris Agreement. Various scenarios are analysed according to the costs of emissions' abatement and the political coordination among countries in the transition. The evidence shows that failing to enforce a slowdown in emissions at a global level, and to stabilize climate damages, generate plunging debt limits in the medium-long term and shrinking fiscal spaces for all countries, even for the few ones actuating the transition. On the contrary, if the green transition is coordinated globally, debt limits converge to stable and higher levels, despite an initial and temporary decrease, given by the negative impact of emission reductions on GDP growth rates. From the evidence presented, it results as significantly more beneficial for countries to collaboratively and promptly transition towards mitigating climate impacts on growth and fiscal spaces. This will support sustainable public debt and the potential to finance the green evolution of our economies.

Keywords: Sovereign Debt, Fiscal Limits, Climate Change, Mitigation Policies.

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1 Introduction

Ensuring the financial sustainability of substantial amounts of public debt and addressing the climate change emergency are two primary concerns most governments are simultaneously facing. The Intergovernmental Panel on Climate Change (IPCC) has been continuously recalling the need of respecting a global carbon budget that keeps global warming below 2, or even 1.5, degrees Celsius over preindustrial levels, requiring immediate and effective action. The green transition involves moderate to significant economic costs in the short-medium term, which might cause, for highly indebted countries, difficulties in ensuring the repayment of currently outstanding debts. On the other side, failing to successfully perform, at a global level, this structural change to a green production system will imply mounting climate damages and continuously diminishing growth rates in the medium and long term. In that case, default on current debt levels will become certain, sooner or later, for most advanced economies.

This study addresses this critical issue by estimating national *fiscal* (or *debt*) *limits*, while taking into account the economic costs of reducing carbon emissions and of climate damages. A *fiscal limit* refers to the maximum amount of debt-to-GDP that a government is allowed to accumulate without impairing the credibility of honoring its repayment. This concept is fundamental for determining the so-called *fiscal space* (that is the distance between the fiscal limit and the actual debt-to-GDP ratio, Ghosh et al., 2013) and the probability of defaulting on its current debt.

Traditionally in the literature, fiscal limits depend on economic growth, risk-free interest rates and primary surpluses. Thus, the challenge for public debt sustainability created by the transition and/or climate damages, which is analysed in this paper, is more technically defined as the reduction of fiscal limits generated by their negative impacts on economic growth.

In this paper, fiscal limits and probabilities of defaults are estimated by extending the model proposed by Collard, Habib, and Rochet (2015) with a reduced-form growth rate function in terms of carbon emissions, where abatement costs are defined as the impact of reducing emissions on GDP growth rates. For this purpose, the cost of emissions' reduction in each country is calibrated, utilizing the results of the OECD study constructing the "Environmentally Adjusted Multifactor Productivity" indicator (Rodríguez et al., 2018), around which several sensitivity analysis are conducted.

Debt limits are studied here for 31 advanced economies, which both signed the Paris Agreement, and submitted their plans for climate action, known as "Nationally Determined Contributions" (NDCs), and are analysed in the OECD empirical work by Rodríguez et al. (2018), from which we take estimates of countries' abatement cost and green growth rates.

This paper relates to and combines two strands of literature: the macro-financial literature on fiscal limits and debt sustainability, and the macro-climate literature on environmental sustainability, aiming to estimate the economic costs of transition policies and climate change.

Regarding the first strand, it is worth mentioning that the current model for fiscal limits, as in Collard, Habib, and Rochet (2015) and Collard et al. (2022), is conceived for advanced economies, for which monetary finance is in principle excluded, public debt is denominated in the local currency, and the assumption of involuntary/excusable default (Grossman and Huyck, 1988) is appropriate. This type of default refers to the situation where a country is not able to borrow enough on the market in order to service its due debt repayments. It is then "forced" to fail by market investors, in line with the evidence on sovereign crises in advanced economies (Yeyati and Panizza, 2011, Bolton et al., 2022). This modeling approach juxtaposes most of the macroeconomic literature on public debt sustainability, which assumes, on the contrary, a strategic type of default, by following Eaton and Gersovitz (1981).[1] Furthermore, the current model, by following Collard, Habib, and Rochet (2015), distinguishes itself by defining the fiscal limit as the debt-to-GDP ratio which maximizes government borrowing available for repayment of outstanding debt. A great part of this literature otherwise estimates fiscal limits as the present value of future maximum primary surpluses (Bi 2012, Bi and Leeper 2013, Tanner, 2013, Pallara and Renne, 2023 and forthcoming). Finally the concept of fiscal limit, which is central to this paper, is also used to determine the probability of default on sovereign debt in the dynamic stochastic general equilibrium literature. This subset of the literature studies in particular the macroeconomic consequences of high-risk public debt: Corsetti et al. (2013), Batini et al. (2016), Darracq et al. (2016).

The second strand of literature - the macro-climate literature - intends to quantify the impact of climate and transition risks on economic growth rates, which is fundamental to estimate fiscal limits and the probability of repaying the outstanding debt-to-GDP. This literature is characterized, given the novelty and complexity of the subject, by great uncertainty, especially on the economic costs of carbon policies. In general, these costs are estimated to be modest in the empirical literature (e.g. Metcalf and Stock, 2020 and forthcoming), with the great caveat, though, of taking as reference a period where mitigation efforts have been quite low, and surely not at the levels necessary to respect the Paris agreements.[2] Given this uncertainty, and the willingness to stay general regarding the transition policy mix, the model is calibrated using the estimation results of Rodríguez et al. (2018). Regarding climate damages, I resort to the literature on analytical Integrated Assessment Models (IAM),[3] and choose the exponential damage function of Dietz and Venmans (2019), for its good fit with temperature-emissions data.[4] The first contribution of this paper is to connect these two strands of the literature. Whereas some research has been conducted about the impact of adaption to climate change on primary surpluses (e.g. Barrage 2020, 2023), there has been no literature about the implications of the transition and climate damages for public debt sustainability, at least at the quantitative

[1] Some recent examples are Aguiar et al. (2022), Hatchondo et al. (2022).

[2] More recently, higher values have been estimated, e.g. Dees (2020).

[3] Such as Golosov et al. (2014), and the following literature.

[4] Alternative notable damage functions are defined in Nordhaus (2014) and Weitzman (2012)

level, until very recently (IMF Fiscal Monitor October 2023, Seghini and Dees 2024, and empirical analyses by Beirne et al. 2021, Boehm 2022 and Collender et al. 2023). Among this new literature this paper is the first, to my knowledge, to propose a theoretical connection between climate-related costs and fiscal limits and to quantify this phenomenon.

The second contribution of this paper is to propose various scenarios according to the short- and long-term costs of emissions' abatement and the political coordination among countries in the transition. In the coordinated and successful transition scenarios, as expected, fiscal limits at the beginning of the transition are found to be lower than their respective long-term stationary values. Some countries, such as Greece, Japan, and Portugal, face general debt sustainability issues, independently from the transition. For other nations, like Italy and France, these problems emerge specifically in the context of the transition. Furthermore, higher short-term abatement costs can imply a shift from a sustainable to an unsustainable current debt-to-GDP, e.g. for Italy, France and the United States. Finally, the most notable contribution of this paper is to show the merit of coordinated policies. The results show that, at the beginning of the transition, due to the adverse effects of mitigation efforts on GDP growth rates, the fiscal limit in the coordinated policy scenario is lower than in a business-as-usual scenario (where emissions uncontrollably increase and cause worsening climate damages). Nevertheless, by 2080 or even sooner, the trade-off leans positively, for all countries, towards a coordinated green transition, as it avoids plunging economic growth rates and, thus, plunging fiscal limits. Indeed, failing to enforce a slow-down in emissions at a global level would generate serious debt sustainability problems in the long term for all advanced countries, even for the few ones actuating the transition. On the contrary, a coordinated successful transition would stabilize climate damages[5] and fiscal limits in the long-term. From the evidence that will be presented, it results as beneficial for countries to collaboratively and promptly transition towards mitigating climate impacts on growth and fiscal spaces. This will support sustainable public debt and the potential to finance the green evolution of our economies.

Section 2 of the paper introduces the general model for fiscal limits with abatement costs in the growth function. Section 3 details the growth function employed in the simulations, the fiscal limit in the green net-zero economy, and the maximization problems. Section 4 describes the data used to calibrate the country-specific parameters in our analysis and Section 5 shows the results. Section 6 introduces climate damages and the relevance of globally coordinating the transition, in order to guarantee public debt sustainability and ample fiscal spaces. Section 7 concludes.

[5]Climate damages would still be present in a scenario of 2° or even 1.5°C, but they will be stabilized to a given level, which would mean a lower GDP in levels, but a stable growth rate

2 Modeling debt sustainability, growth and carbon emissions

This section introduces the model used in this paper to study how environmental and fiscal sustainability interact with one another. The concept of fiscal limit employed here is the debt-to-GDP ratio which maximizes government borrowing available for repayment of outstanding debt. This value represents the maximum sustainable debt (MSD). The framework delivering the measures of maximum sustainable borrowing (MSB) and maximum sustainable debt (MSD), proposed by Collard, Habib, and Rochet (2015), is extended in order to capture their dependence from the path of present and future carbon emissions.

As in their model, the government issues one-period bonds $B_t \equiv b_t Y_t$ every period, with a face value $D_{t+1} \equiv d_t Y_t$ to be repaid at the beginning of the next period.[6] Default at t occurs if:

$$D_t - \alpha Y_t > b_t^M Y_t \Longleftrightarrow \frac{D_t}{Y_t} > \alpha + b_t^M \tag{1}$$

where α represents the maximum primary surplus (MPS) with respect to GDP, that the government can extract from the private sector.[7] $B_t^M \equiv b_t^M Y_t$ denotes the MSB, defined by:

$$b_t^M = \max_{d_t} b(d_t) = \max_{d_t} \frac{d_t}{R(d_t)} \tag{2}$$

where $b(d_t)$ are the borrowing proceeds with respect to GDP, from issuing one-period debt with a face value d_t with respect to GDP.[8] For simplicity, as in Collard, Habib, and Rochet (2015), it will be assumed zero recovery in default, and a constant gross risk-free rate R.[9] Assuming competitive financial markets and risk-neutral investors, the return promised on government bonds $R(d_t)$ satisfies:

$$R(d_t)\,(1 - \mathsf{PD}(d_t)) = R \tag{3}$$

[6]For the sake of tractability, debt is assumed to be completely rolled over every period. This is in particular needed when the fiscal limit is defined as the value maximizing government borrowing, by balancing the face value of debt and its probability of default. Some notable papers which address the issue of debt sustainability with long-term debt are Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), Bacchetta et al. (2018), Lorenzoni and Werning (2019).

[7]The MPS α can be thought as the level of primary surplus (net taxation) which maximizes the Laffer curve. It is assumed to be constant, as in Tanner (2013), where it is defined as the level of "maximum feasible primary surplus that citizens can tolerate." Tanner (2013) then recovers, from the level of MPS, a value for the maximum sustainable debt (MSD), which satisfies a stabilization rule. The approach adopted by Collard, Habib, and Rochet (2015) is similar: they look for the maximum debt level for which sustainability and stability are guaranteed.

[8]Collard, Habib, and Rochet (2015) show how this maximization is equivalent to finding the debt level which minimizes the average interest rate on government bonds: $d_M = \arg\min_d \frac{R_t(d)}{d}$. The solution to the problem is the one which equates the marginal interest rate with the average interest rate: $R'(d_M) = \frac{R(d_M)}{d_M}$.

[9]Together with neither renegotiation nor bail-out, and an independent central bank that resists possible government demand to inflate debt away.

What follows extends the standard model by Collard, Habib, and Rochet (2015), by taking into account the existence for the government of an additional constraint on carbon emissions, and the necessity of respecting this "carbon budget". It is straightforward to recognize that the probability of default PD will be indirectly affected, through the impact of these constraints on the growth rate. In order to observe this channel, let rewrite the condition of default (1), at $t+1$, in terms of the gross growth rate:

$$D_{t+1} \equiv d_t Y_t > (\alpha + b^M_{t+1}) Y_{t+1} \iff g_{t+1} = \frac{Y_{t+1}}{Y_t} < \frac{d_t}{\alpha + b^M_{t+1}} \tag{4}$$

We express the stochastic growth rate, in terms of carbon emissions, as:

$$g_{t+1} = \eta_{t+1}(\{E_{t|t-1}\}) e^{\mu_0 + \varepsilon_{t+1}}, \text{ where } \ \varepsilon_j \sim_{i.i.d.} N(0, \sigma_0^2) \quad \forall j \tag{5}$$

The component $\eta_{t+1}(\{E_{t|t-1}\})$ represents the novelty of this paper, and the cost function which depends on the transition path. μ_0 is reinterpreted as the growth rate of a net-zero economy, which does not rely on emissions. Carbon compensation in the short-term is ruled out ($E_{t|t-1} \geq 0$, $\forall t$) and the time subscript indicates that the amount of carbon emissions $E_{t|t-1}$, exploited at t, is indeed decided at $t-1$, capturing the so called "carbon lock-in" phenomenon, and the commitment of countries in the Paris Agreement of updating their National Determined Contributions (NDCs) every five years. Indeed, in the following, I will assume a period of 5 years, which also corresponds to the average maturity of US outstanding debt, and for the sake of simplicity, I will refer to $E_{t|t-1}$ as E_t.

Rewriting the default condition as follows, by also making explicit the dependence from the emissions' path $\{E_t\}$:

$$g_{t+1} \equiv e^{\mu_0 + \varepsilon_{t+1}} \eta_{t+1}(\{E_t\}) < \frac{d_t}{\alpha + b^M_{t+1}(\{E_t\})} \iff e^{\varepsilon_{t+1}} < \frac{d_t}{[\alpha + b^M_{t+1}(\{E_t\})] e^{\mu_0} \eta_{t+1}(\{E_t\})}$$

and by making use of equivalence (3), the MSB (2) can be redifined, given a path for carbon emissions $\{E_t\}$, as:

$$b^M_t(\{E_t\}) = \max_{d_t} \frac{d_t}{R}[1 - \text{PD}(d_t, \{E_t\})] = \max_{d_t} \frac{d_t}{R}\left[1 - F\left(\frac{d_t}{[\alpha + b^M_{t+1}(\{E_t\})] e^{\mu_0} \eta_{t+1}(\{E_t\})}\right)\right] \tag{6}$$

where $F(\cdot)$ is the c.d.f. of the lognormally i.i.d. random shock $\exp(\varepsilon)$.

Calling the critical shock (the minimum shock's realization necessary to avoid default),

$$x_{t+1} \equiv \frac{d_t}{[\alpha + b^M_{t+1}(\{E_t\})] e^{\mu_0} \eta_{t+1}(\{E_t\})}, \tag{7}$$

and the constant borrowing factor (net of growth)

$$\gamma \equiv \max_x x[1-F(x)] = x_M[1-F(x_M)],$$

the MSB can be rewritten as:

$$b_t^M(\{E_t\}) = \frac{\gamma e^{\mu_0}}{R}[\alpha + b_{t+1}^M(\{E_t\})]\eta_{t+1}(\{E_t\}) \tag{8}$$

In the following section this most general version of the function η, in terms of the emission path, will take a more particular specification, where it will only depend on current and next-period emissions.

2.1 Historical GDP growth, green growth and the GDP-emissions elasticity

The growth function assumed in (5) is now motivated, by showing its consistency with the existing empirical literature. I employed in particular the results of the OECD paper (Rodríguez et al., 2018), which constructs the novel Environmentally Adjusted Multifactor Productivity indicator indicator. I show first the relation between the present model and their estimation equation, from whose results our function and the abatement parameter are calibrated for different countries.

From the transformation function

$$H(Y,E,L,K,S,t) \geq 1,$$

where Y denotes the desirable output of the economy (GDP) and E the undesirable output (carbon emissions), and L, K and S, labour, produced capital and natural capital, respectively, they define the following linear regression equation:

$$\dot{Y}_{i,t} = a_i + \delta_t + \rho_i \dot{X}_{it} + \sum \beta_{ji} \dot{E}_{jit} + \varepsilon_{it}.$$

$\dot{Y}_{i,t}$ is the GDP net growth rate of country i, $\dot{X}_{i,t}$ its elasticity-weighted growth rate of inputs and $\dot{E}_{jit}$ the growth rate of each type j of GHG emissions.[10] δ_t are time dummies, and ε_{it} a normally distributed error term. a_i represents environmentally adjusted productivity growth. The coefficients relate to the elasticities of the transformation function with respect to desirable and undesirable outputs as follows:

$$\rho_i = -\frac{1}{\varepsilon_{HYi}}; \quad \beta_{ji} = \rho_i \varepsilon_{HEji} = -\frac{\varepsilon_{HEji}}{\varepsilon_{HYi}} \equiv \varepsilon_{YEji}$$

where the latter is the elasticity of output with respect to emissions of type j, for country i.
I use the historical average (for the data range in the OECD anaylsis, 1990-2013) of equation (9), for

[10] We define: $\dot{x}_{i,t} = \ln(x_{i,t}) - \ln(x_{i,t-1})$, for every variable x.

calibrating our function $\eta(\cdot)$ and the parameters μ, σ, μ_0, and σ_0, for each country:

$$\underbrace{\mathbb{E}[\dot{Y}_{i,t}]}_{\mu_i} = \underbrace{a_i + \mathbb{E}[\delta_t] + \rho_i \mathbb{E}[\dot{X}_{it}]}_{\mu_{0,i}} + \sum \beta_{ji} \mathbb{E}[\dot{E}_{jit}]$$

For simplicity, it is assumed that the total contribution of the different elements in $\mu_{0,i}$ would not be affected by changes in the growth rate of carbon emissions. Therefore, the historical net growth rate μ_i can be split into a "green" growth rate $\mu_{0,i}$ and the contribution of emissions. I also rewrite this latter term as: $\sum_j \beta_{ji} \dot{E}_{jit} = \sum_j \beta_{ji} w_{ji} \dot{E}_{it} = \beta_i \dot{E}_{it}$, where w_{ji} represents the weight (assumed constant) of emissions of type j of country i over its total emissions. How these weights are calibrated in the present analysis will be explained in Section 4. Notice that, by following the OECD results, $\beta_i < 1$ for every country. By recalling the expression for the gross growth rate in Collard, Habib, and Rochet (2015) (for country i), we can write:

$$g_{i,t+1} = e^{\mu_i + \varepsilon_{i,t+1}} = e^{\mu_{0,i} + \beta_i \dot{E}_{it} + \varepsilon_{i,t+1}} = e^{\mu_{0,i} + \varepsilon_{i,t+1}} \left(\frac{E_{i,t+1}}{E_{i,t}} \right)^{\beta_i} = e^{\mu_{0,i} + \varepsilon_{i,t+1}} \left(\frac{e_{i,t+1}}{e_{i,t}} \right)^{\beta_i},$$

where $\left(\frac{E_{i,t+1}}{E_{i,t}} \right)^{\beta_i}$ represents the generalized η-term defined above, and $e_{i,t} \equiv E_{i,t}/\bar{E}_{i,1}$ are the current emissions' share with respect to $\bar{E}_{i,1}$: the national carbon budget for country i in the first period (from 2026 onward).

Given that our main goal is to study the fiscal sustainability of transition paths that lead to net zero emissions, it is appropriate to transform the previous equation as follows:

$$g_{i,t+1} = e^{\mu_{0,i} + \varepsilon_{i,t+1}} \left(\frac{c + e_{i,t+1}}{c + e_{i,t}} \right)^{\beta_i}, \tag{9}$$

where c is the percentage of emissions over the country's carbon budget $\bar{E}_{i,1}$, that will be possible to recapture by the CCS (carbon capture and storage) technologies which are currently developing, and therefore can still be sustained in a green economy with net zero emissions. This value is assumed to be 1% of the global carbon budget, as at the beginning of 2026. CCS technology capacity was, in 2021, around 41Mt CO2 per year, or 205Mt CO2 over 5 years (our assumed period length). They are projected to develop to a capacity of 230Mt CO2 per year by 2030, which amounts over 5 years, to around 0.1% of our benchmark global carbon budget in 2021 (1116 Gt CO2, as illustrated below). The realized growth rate would be of 85%. At this rate, a capacity of 1% of the carbon budget would be achieved before 2050. In the following results, the transition is generally slow, and never achieved before 2050, therefore, we can safely assume a value of $c = 1\%$ with respect to the 2026 carbon budget $\bar{E}_{i,1}$. Globally, this would amount to around 1.8 Gt CO_2 per year, which is way below the assumptions of various IPCC scenarios (AR 6 Synthesis Report, 2023). Nonetheless, in Appendix

D.1, the results of a comparative analysis for lower values are reported.[11] The parameter is taken as constant for the sake of simplicity. In the following the subscript i will be dropped for the sake of simplicity.

3 Fiscal limits, emissions' abatement and the carbon budget

In this section, I incorporate the two elements introduced in the previous section: the fiscal limit measure and the growth rate as a function of green growth and emissions' increase. I introduce, then, the concepts of carbon budget, "green" fiscal limit, debt sustainability and the maximization problems which underline the results of this paper.

3.1 The maximum sustainable debt under the carbon budget

In light of the growth functional form in terms of emissions (9), assumed in the previous section, the MSB (8) can be rewritten as:[12]

$$b_t^M = \frac{\gamma e^{\mu_0}}{R}(\alpha + b_{t+1}^M)\frac{\eta(E_{t+1})}{\eta(E_t)}, \text{ where } \eta(E_t) = (c\bar{E} + E_t)^\beta = [(c+e_t)\bar{E}]^\beta \tag{10}$$

This specification of the function η also finds additional interpretations in light of the research by Dietz and Venmans (2019). Their model represents a further justification of the reduced-form growth function in terms of emission employed in this paper.[13] The MSB equation can be represented in two other additional ways. First, by iterating we get:

$$b_t^M = \alpha \sum_{j=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R}\right)^j \prod_{l=0}^{j-1} \frac{\eta(E_{t+l+1})}{\eta(E_{t+l})} \tag{11}$$

$$= \underbrace{\alpha \frac{\gamma e^{\mu_0}}{R} \frac{\eta(E_{t+1})}{\eta(E_t)}}_{b_S^M(\{E_t\})} \underbrace{\sum_{j=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R}\right)^{j-1} \prod_{l=1}^{j-1} \frac{\eta(E_{t+l+1})}{\eta(E_{t+l})}}_{\Gamma_{\{E_t\}}} \tag{12}$$

[11] Notice that the role of c becomes most relevant in the last period of the transition, where, since $e_{i,j} = 0$, $\forall j > T$, we have:

$$g_{i,T+1} = e^{\mu_{0,i} + \varepsilon_{i,t+1}} \left(\frac{1}{1 + ae_{i,T}}\right)^{\beta_i}, \text{ where } a = 1/c.$$

A higher value of a, or lower value of c, means that the last transition period's growth rate is more depressed, for the same value of $e_{i,T}$. It follows that governments will tend to postpone more and to reduce the level of emissions in this final period. This results in smoothness of the optimal transition in our framework, when governments aim to maximize current sustainability, as introduced in Section 3.

[12] We drop also the explicit dependence of MSB from the path of emissions $\{E_t\}$.

[13] See Appendix A for a discussion.

where the latter formulation is aligned with the standard model by Collard, Habib, and Rochet (2015). The first component $b_S^M(\{E_t\})$ is the maximum static borrowing[14], and $\Gamma_{\{E_t\}}$ is the borrowing multiplier, which measures the "increase in present borrowing made possible by infinitely repeated reliance on future borrowing", given a feasible path $\{E_t\}$. The product $\gamma e^{\mu_0} \frac{\eta(E_{t+1})}{\eta(E_t)}$ represents the total borrowing factor, including the growth rate contribution.
Alternatively, the product in (11) can be simplified into $\prod_{l=0}^{j-1} \frac{\eta(E_{t+l+1})}{\eta(E_{t+l})} = \frac{\eta(E_{t+j})}{\eta(E_t)}$, in order to get:

$$b_t^M = \frac{\alpha}{\eta(E_t)} \sum_{j=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R} \right)^j \eta(E_{t+j}) \tag{13}$$

This second form is the most convenient, since it highlights the dependence of current MSB both on E_t, which is assumed to be predetermined (as explained before), and on the sequence of future carbon emissions.

Given (7), the maximum sustainable debt (MSD) corresponds to :

$$d_t^M = x_M(\alpha + b_{t+1}^M) e^{\mu_0} \frac{\eta(E_{t+1})}{\eta(E_t)} \equiv \frac{R}{1 - F(x_M)} b_t^M. \tag{14}$$

This represents the fiscal limit in period t: the maximum face value the debt issued at period t can have in order to be sustainable. This is the value to be compared with current debt-to-GDP levels, in order to determine their sustainability.

The carbon budget. Let now take into consideration that each economy has a total available carbon budget, that the government has to respect:

$$\sum_{j=0}^{+\infty} E_j \leq \bar{E}_0 \tag{15}$$

The imposition of this constraint captures the political commitments signed under the Paris Agreement and implicitly the willingness to avoid adverse effects of climate change, generated by the use of carbon emissions. The negative economic impact of climate change, which is the focus of the last decades' advances in environmental economics, through the modelling of the so-called "damages function", is ignored, for now. Potentially, an increasing consumption of emissions could guarantee an higher economic growth in the short term, but decrease it in the long run, because of depletion of the country's natural resources and as a result of a higher frequency of climate disasters. The willingness and necessity of avoiding these adverse effects is simply captured, for now, only by imposing the carbon budget, as in Gollier (2022). In Section 6, I will then explicitly introduce economic climate damages, by employing the damage function by Dietz and Venmans (2019), both in the green transi-

[14]the MSB if borrowing were not possible in the subsequent period.

tion scenario, and in alternative business-as-usual scenarios.

At time t, the environmental constraint rewrites as:

$$\sum_{j=t}^{+\infty} E_j \leq \bar{E}_t \tag{16}$$

where $\bar{E}_t = \bar{E}_0 - \sum_{j=0}^{t-1} E_j$ is the remaining carbon budget.

Debt sustainability. Here, as in the standard model by Collard, Habib, and Rochet (2015), debt is defined as sustainable if the sequence b_t^M is bounded: "$b > 0$ is sustainable if and only if there exists a bounded sequence of borrowings $(b_t^M)_t$ such that $b_0^M = b$ and $b_t^M \leq \tau(b_{t+1}^M)\ \forall t$". Furthermore, Collard, Habib, and Rochet (2015) looks for a debt level that is stable, and thus, "more properly described as sustainable". Here, the same approach is adopted. Therefore, in order to have convergence of the MSB to a finite value, the following condition has to be satisfied:

$$\exists j > t \text{ s.t. } \gamma e^{\mu_0} \frac{\eta(E_l)}{\eta(E_{l-1})} < R, \forall l \geq j.$$

There must exist a time j after the period t, when we are measuring the MSB b_t^M, such that the total borrowing factor (the product of the borrowing factor net of growth γ and the gross growth rate $g_l = e^{\mu_0} \frac{\eta(E_l)}{\eta(E_{l-1})}$) is lower than the gross risk-free interest rate R. This is ensured when imposing a carbon budget (given that this constraint will lead to zero emissions in the medium/long term) if $\gamma e^{\mu_0} < R$, which is always quantitatively verified in the data range of this paper.

3.2 The MSD of the green economy

The framework introduced here can provide an answer to a first important question: What is the MSD of an economy after having succeeded in the transition? This is the limit towards which the economy will tend in the long-run, given the necessity of respecting the carbon budget: in the long-run all countries will have to transition to a green economy. This will be characterized by the feasible path $\{E_t\}$: $E_j = 0$, $\forall j \geq t$, and by the green growth rate $g_{t+1}^G = e^{\mu_0 + \varepsilon_{j+1}}$, $\forall j \geq t$. In this case, the MSB becomes:

$$b_t^{M,G} = \frac{\gamma e^{\mu_0}}{R}(\alpha + b_{t+1}^M) \equiv \tau(b_{t+1}^M).$$

Equivalently to the standard analysis by Collard, Habib, and Rochet (2015), by iterating, we get:

$$b_t^{M,G} = \frac{\alpha \gamma e^{\mu_0}}{R} \Gamma_G, \text{ where } \Gamma_G = \left[1 + \frac{\gamma e^{\mu_0}}{R} + \left(\frac{\gamma e^{\mu_0}}{R}\right)^2 + \ldots\right].$$

The product γe^{μ_0} represents the "green" borrowing factor. When $\gamma e^{\mu_0} < R$, the series converges to a finite borrowing multiplier $\Gamma_G = \frac{1}{1-\frac{\gamma e^{\mu_0}}{R}}$. When $\gamma e^{\mu_0} \geq R$, the series diverges, and Γ_G is infinite. In the green economy where $E_j = 0$, $\forall j \geq t$, and it is assumed $\gamma e^{\mu_0} < R$, the MSB capacity is finite and equal to:

$$b_{M,G} \equiv \frac{\alpha \gamma e^{\mu_0}}{R - \gamma e^{\mu_0}} \tag{17}$$

and the function τ is a contraction that has the unique fixed point $b_{M,G}$.[15] This represents the "green MSB", that is the borrowing capacity of the green economy, towards which the government will converge at the end of the transition. Using (7), the green MSD is described by:

$$d_{M,G} \equiv (\alpha + b_{M,G}) x_M e^{\mu_0}. \tag{18}$$

The probability of default for a given level of debt d, in the green economy, can be found through:

$$\mathsf{PD}_{M,G}(d) \equiv F\left(\frac{d}{[\alpha + b_{M,G}]e^{\mu_0}}\right) = \Phi\left(\frac{\ln(d) - \ln(\alpha + b_{M,G}) - \mu_0}{\sigma}\right) \tag{19}$$

where Φ represents the standard normal c.d.f.

3.3 The government's maximization problems

This section introduces the two maximization problems of current debt sustainability and "welfare" (or the present value of future GDPs), which underline the results presented in Section 5. Further details are available in Appendix B.

3.3.1 Maximizing the current MSB under the carbon budget.

A government might want to maximize the MSB of the current period, in order to ensure repayment of the current outstanding debt-to-GDP d_{t-1}. For current political choices, especially for countries with fiscal problems, this value is also fundamental: it represents the maximum possibly obtainable fiscal limit of the country, under the constraint of the impending transition. For simplicity, let call the initial period $t = 0$, which will represent the current NDC cycle 2021-25. The MSB for an initial period $t = 0$ is:

$$\begin{aligned} b_0^M &= \frac{\gamma e^{\mu_0}}{R}(\alpha + b_1^M)\frac{\eta(E_1)}{\eta(E_0)} \\ &= \frac{\alpha}{\eta(E_0)} \sum_{t=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R}\right)^t \eta(E_t) \end{aligned} \tag{20}$$

[15]If it was $\gamma e^{\mu_0} > R$, any borrowing $b > 0$ would be sustainable.

The maximization problem at $t = 0$, given an initial level of emissions E_0 is:

$$\max_{\{E_t\}} \; b_0^M = \frac{\alpha}{\eta(E_0)} \sum_{t=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R} \right)^t \eta(E_t) \tag{21}$$

$$\text{s.t.} \; \sum_{t=1}^{+\infty} E_t \leq \bar{E}_1 \tag{22}$$

$$E_t \geq 0 \tag{23}$$

3.3.2 Maximizing "welfare" under the carbon budget

The previous maximization problem will be compared to the transition path a benevolent social planner would undertake, by ignoring the constrain of an outstanding debt to be repaid. The benevolent social planner would maximize the summation of the present value of expected future GDP (which is taken as a representation of social welfare), where the latter, in period t, is:

$$\begin{aligned}
\mathbb{E}_0[Y_t] &= Y_0 e^{t\mu_0} \mathbb{E}_0 \left[\prod_{j=1}^{t} e^{\varepsilon_j} \right] \frac{\eta(E_t)}{\eta(E_0)} \\
&= Y_0 e^{t\mu_0} \prod_{j=1}^{t} \mathbb{E}_0 \left[e^{\varepsilon_j} \right] \frac{\eta(E_t)}{\eta(E_0)} = \frac{Y_0}{\eta(E_0)} e^{t(\mu_0 + 1/2\sigma_0^2)} \eta(E_t) \\
\text{where} \;\; & \eta(E_t) = (c\bar{E}_1 + E_t)^{\beta}
\end{aligned}$$

given that $\exp(\varepsilon_j)$ is assumed to be i.i.d. and lognormal. The welfare maximization writes as

$$\max_{\{E_t\}} \; \sum_{t=0}^{+\infty} \frac{\mathbb{E}_0[Y_t]}{R^t} = \frac{Y_0}{\eta(E_0)} \sum_{t=0}^{+\infty} \left(\frac{\bar{g}}{R} \right)^t \eta(E_t) \tag{24}$$

$$\text{s.t.} \; \sum_{t=1}^{+\infty} E_t \leq \bar{E}_1 \tag{25}$$

$$E_t \geq 0 \tag{26}$$

by defining $\bar{g} = e^{\mu_0 + 1/2\sigma_0^2}$, as the expected green growth rate.

4 Data and Calibration

This section presents the data used for the calibration of the model for 31 advanced economies. The analysis is restricted to these OECD countries for data limitation on two aspects: countries who signed the Paris Agreement, and submitted their plans for climate action, known as "Nationally Determined Contributions" (NDCs); countries studied in the OECD paper which constructs the Environmentally Adjusted Multifactor Productivity indicator, from which we take estimates of countries' β.

Period length and risk-free interest rate The period length for repaying outstanding government debt is selected to be 5 years. This matches average maturity of outstanding US government debt in 2020.[16] The risk-free interest rate $r \equiv R - 1$ is chosen to match the real yield on the United States five-year Treasury Bond over the period 2003–2020.[17] Its maximum value of the annual return over this period is 2.44% (2006). Its average over 2003-2013 and 2003-2020 are respectively 0.76%, and 0.5%. CBO (Congressional Budget Office)'s projections, predicts an increase in real interest rates over the next decades, with respect to the current unprecedent low rates scenario. Given our long-run horizon, we take, therefore, as benchmark the maximum value over the available data period of 2.44%. We also perform a comparative analysis for higher and lower values: 3%, to represent the possible increase of interest rates to the high values of the past; and 1.88% to align with the CBO's projections for mid-century.[18]

The maximum primary surplus is taken to match the national historical maximum primary surplus: $\alpha = \max_t \frac{s_t}{Y_t}$, where s_t is the annual primary surplus, (source IMF). A lower value indicates a worse situation for the fiscal sustainability of the country, called "fiscal fatigue". From the IMF datasets, I also take value of debt-to-GDP in 2020, for the 31 countries in our analysis. The data are reported on the last two columns of Table 1.

GDP growth rate and the abatement cost β. I take the mean and volatility of the growth rate from OECD data. The first four columns in Table 1 represent respectively the average GDP growth rate μ and its volatility , and the average green GDP growth rate (adjusted for pollution abatement) μ_0 and volatility σ_0, in the OECD results from the construction of the Environmentally Adjusted Multifactor Productivity indicator (Rodríguez et al., 2018, reference period 1990-2013). Notice that for most countries, μ_0 is higher than μ (and σ_0 is lower than σ). The reason is that those countries actually reduced their emissions over the reference period. I will assume, in a first analysis, that this corresponds to the green growth rate in the long term, that is when pollution abatements will be terminated. Nonetheless, it is possible for this assumption to be too optimistic. Therefore, I will also analyse scenarios where μ_0 and σ_0 are equal to μ and σ ("parallel hypothesis") or μ_0 lower than μ. The fifth column represents the β coefficient, equivalent to the elasticity of output with respect to emissions, and the abatement short-term cost parameter. This represents an average of the elasticities that result significant in the OECD study, for CO2 and CH4 (methane), weighted for the average relevance of the type of emission over the national total emissions in the period 1990-2013, to be

[16]Since we are considering zero-coupon bonds, using duration would be more appropriate. This would be around 5.5 years, as shown by Andreolli 2021 (JMP). We take 5 years for the sake of simplicity.

[17]The selected period is due to data limitation: data.nasdaq.com.

[18]I assume a term premium between 5-year yield and 10-year yields of 45 percentage points, its average over the past two decades for inflation-linked treasury bonds.

Country	μ	σ	μ_0	σ_0	β	Debt/GDP$_{2020}$	MPS(α)
Australia	3.29	1.08	3.14	1.01	0.067	57.8	4.15
Austria	1.90	1.77	2.20	1.75	0.022	83.2	3.32
Belgium	1.80	1.53	2.07	1.48	0.074	112.8	6.84
Canada	2.36	1.88	2.38	1.78	0.036	117.8	10.05
Czech Republic	2.55	2.96	3.24	2.57	0.165	37.7	2.14
Denmark	1.48	2.16	1.62	2.09	0.040	42.1	11.62
Estonia	4.51	6.40	4.59	5.98	0.077	19.0	3.36
Finland	1.77	3.76	2.00	3.72	0.033	69.0	9.63
France	1.57	1.48	1.88	1.43	0.064	115.2	3.65
Germany	1.44	2.18	2.13	2.19	0.092	68.7	4.34
Greece	1.01	4.24	1.09	3.83	0.079	211.9	4.37
Hungary	1.76	2.86	2.26	2.55	0.116	80.0	7.84
Iceland	2.68	3.57	2.71	3.49	0.032	77.4	8.48
Ireland	4.62	4.40	4.66	4.22	0.063	58.4	6.72
Italy	0.73	1.94	1.03	1.68	0.101	155.3	6.55
Japan	0.93	2.06	1.34	1.90	0.080	259.0	5.53
Latvia	4.59	6.55	4.82	6.44	0.086	43.3	1.70
Lithuania	4.58	5.89	4.81	5.12	0.131	46.6	1.63
Luxembourg	3.68	3.42	3.97	3.24	0.133	24.8	4.44
Netherlands	1.99	2.12	2.39	2.14	0.077	52.8	5.62
New Zealand	2.63	2.23	2.54	2.18	0.034	43.1	7.53
Norway	2.48	1.69	2.67	1.54	0.027	46.8	20.57
Poland	3.69	2.98	3.76	3. 00	0.015	57.4	3.62
Portugal	1.53	2.29	1.61	2.02	0.055	135.2	3.47
Slovak Republic	4.15	3.42	4.39	3.30	0.102	59.7	0.23
Slovenia	2.53	3.50	2.67	3.13	0.108	79.8	2.54
Spain	2.04	2.36	1.96	2.13	0.072	120.0	4.01
Sweden	2.04	2.70	2.32	2.62	0.066	39.6	7.05
Switzerland	1.55	1.64	1.78	1.65	0.089	42.4	3.44
Turkey	4.01	4.90	2.90	4.03	0.110	39.5	7.03
United Kingdom	2.02	1.51	2.09	1.50	0.042	102.6	6.65
United States	1.77	1.37	1.84	1.24	0.043	134.2	3.93

Table 1: Summary of Economic Indicators. Columns 1-4, based on Rodríguez et al., 2018 (reference period 1990-2013): average GDP growth rate (μ), its volatility (σ), average "green" GDP growth rate adjusted for pollution increase/reduction (μ_0), and its volatility (σ_0). Columns 5-6, IMF data: historical maximum primary surplus ($\alpha = \max_t \frac{s_t}{Y_t}$) and debt-to-GDP in 2020.

consistent with the OECD reference period.[19] The average weights are presented in Table 2, together with their standard deviation. Appendix C present robustness results for β, when taking as weights the share of the type of emission over the national total emissions in 2013 and 2020. Except for some few countries, these percentages have been generally stable over time. For Netherlands, for which data are missing, I take the average across countries to calibrate β. Notice that, by following the OECD results, $\beta_i \leq 0.165$ for every country. This is in line with the existing empirical literature, e.g. Känzig (2023), Känzig and Konradt (2023), Metcalf and Stock (2020 and forthcoming). They estimate that the negative effect of climate policies, on the level and growth of economic output is generally quite modest and/or not significant. The adverse impact is more substantial for emission trading systems

[19]Data source: ourworldindata.org/greenhouse-gas-emissions, based on emissions data from Jones et al., *National contributions to climate change due to historical emissions of carbon dioxide, methane and nitrous oxide [Data set]*, In Scientific Data (2023.1), Zenodo

Country	CO2 mean	CO2 std	CH4 mean	CH4 std
Australia	59.87%	8.88%	25.18%	7.85%
Austria	68.14%	15.60%	25.85%	12.69%
Belgium	79.57%	4.36%	16.64%	4.45%
Canada	83.55%	2.70%	12.38%	1.90%
Czech Republic	69.06%	13.32%	27.27%	13.98%
Denmark	70.12%	5.51%	21.48%	5.42%
Estonia	76.54%	7.24%	16.62%	4.66%
Finland	71.29%	10.34%	21.46%	7.66%
France	58.69%	11.22%	35.19%	14.02%
Germany	77.22%	9.72%	18.76%	9.33%
Greece	61.81%	10.99%	30.46%	9.86%
Hungary	59.69%	11.84%	33.90%	15.30%
Iceland	68.37%	16.92%	18.71%	7.80%
Ireland	24.89%	23.13%	65.81%	28.95%
Italy	62.11%	15.91%	31.10%	15.00%
Japan	78.32%	13.53%	16.71%	10.86%
Latvia	64.50%	9.80%	27.08%	7.67%
Lithuania	66.29%	6.18%	24.05%	6.00%
Luxembourg	42.86%	44.60%	43.21%	33.55%
Netherlands	–	–	–	–
New Zealand	56.91%	20.49%	30.62%	15.45%
Norway	67.11%	6.53%	24.00%	4.71%
Poland	63.83%	10.29%	31.07%	9.56%
Portugal	52.21%	14.35%	40.49%	14.04%
Slovak Republic	75.78%	8.54%	20.13%	8.85%
Slovenia	62.98%	9.38%	31.89%	9.24%
Spain	64.14%	10.83%	27.93%	10.55%
Sweden	77.37%	4.88%	16.05%	2.97%
Switzerland	67.96%	12.22%	25.90%	11.25%
Turkey	45.85%	22.35%	42.48%	21.58%
United Kingdom	76.50%	3.61%	20.07%	4.29%
United States	84.37%	3.54%	11.85%	2.75%
average	65.74%	11.90%	26.91%	10.72%

Table 2: Mean and standard deviation of carbon dioxide (CO_2) and methane (CH_4) percentages over total emissions.

than carbon taxes, and instead become lower or might even turn positive in the presence of revenue recycling. In general, these results are in line with the estimates recovered from computable general equilibrium models. For example, in the E3 model by Goulder and Hafstead (2018), a $40 per ton carbon tax for the United States starting in 2020, and rising at 5 percent real annually, would reduce annual emissions by 40% and GDP by only 1.5% in 2035. Notice, however that the focus of this paper is on the relationship between the growth rates of emissions and GDP, and that we want to keep generality in terms of the carbon policies enacted by the individual countries. This is the reason why I do not resort directly to the results of this empirical and theoretical literature on carbon pricing to calibrate the function $\eta(\cdot)$, but employ instead the results of OECD (2018), which directly estimates the impact of emissions' reduction on GDP growth. Furthermore, it is worth mentioning that this low impact of emissions' reduction is not consensual in the entire literature. For example, as we have discussed in Section A, some analytical models implement MAC (marginal abatement cost) functional forms that would be in the same space of a $\beta > 1$. In the following, I will show the implications of a higher β for debt sustainability. It is also worth recalling that β only captures the short-term cost of the transition. A value for μ_0 lower than μ would potentially capture instead its long-term cost, due, for example, to a lower rate of development of a green economy with respect to a dirty economy, as represented in the pessimistic scenario (3) introduced in Section 5.

Global carbon budget, National Determined Contributions and national carbon budgets. By referring to the IPCC (2023) estimates[20] for the remaining global carbon budget in 2020, I pick the 2°C over pre-industrial levels scenario with 67% probability, which corresponds to a carbon budget of 1150 Gt of CO_2 emissions. In order to estimate the national carbon budgets, I follow Gollier (2022), by dividing this global budget per-capita for the advanced countries in our data. Given a global population in 2021 of 7794 million people, and global level of carbon emissions of around 34 Gt CO_2e in 2020, the remaining budget for 2021 onwards of 1116 Gt CO_2e is divided as depicted in Table 1. Beyond the ethical reason for adopting this rule, this is also due to a lack of clear national benchmarks in the political spectrum. Indeed, how much each country should contribute to the fight against climate change by reducing emissions is notably object of long-standing debate. Nonetheless, the engagement of keeping global average temperatures beyond 2°C over pre-industrial levels (and pursuing efforts to limit the increase even further to 1.5°C) has become legally binding with the Paris Agreement, adopted in 2015 and entered into force in 2016. Therefore this carbon budget is seriously taken and rightly assumed as a hard constraint in this paper.

Also notice that in equation (5) emissions are denoted as $E_{t|t-1}$ in order to capture that the amount of emissions in period t are decided at $t-1$, capturing the so called "carbon lock-in" phenomenon. The latter is mainly due to technological and political constraints. Regarding the political constraints it is important to notice that, under the Paris Agreement, countries have to submit their nationally determined contributions (NDCs)– according to a 5-year cycle.[21] The first cycle corresponds to the period 2021-2025. According to the current NDCs submitted by "the Parties",[22] I extrapolated the following emissions' targets for this period, which will be taken as given in the following simulations of the model, as the initial 5-year emissions value E_0 (second column to the right in Tables 4 and 5). Recalling that the national carbon budget writes as:

$$\sum_{j=0}^{+\infty} E_j \leq \bar{E}_0 \quad \text{or} \quad \sum_{j=1}^{+\infty} E_j \leq \bar{E}_1, \quad \text{where} \quad \bar{E}_1 = \bar{E}_0 - E_0,$$

we would have that the carbon budget in 2021 $\bar{E}_0$ and in 2026 $\bar{E}_1$ respectively correspond to the last columns of Tables 3 and 4/5.

Reference emissions are taken from the OECD dataset as GHG emissions including LULUCF (Land

[20] P. 29, Table SPM.2 in IPCC (2023).
[21] https://unfccc.int/NDCREG
[22] As of beginning of 2023.

	Total population 2021 (Mln)	**Percentage over global population**	**National Carbon budget ($GtCO_2e$)**
Australia	25.8	0.33%	3.694
Austria	9.0	0.12%	1.289
Belgium	11.6	0.15%	1.661
Canada	38.1	0.49%	5.455
Czech Republic	10.7	0.14%	1.532
Denmark	5.8	0.07%	0.830
Estonia	1.3	0.02%	0.186
Finland	5.5	0.07%	0.788
France	65.4	0.84%	9.364
Germany	83.9	1.08%	12.013
Greece	10.4	0.13%	1.489
Hungary	9.6	0.12%	1.375
Iceland	0.3	0.00%	0.043
Ireland	5.0	0.06%	0.716
Italy	60.4	0.77%	8.648
Japan	126.0	1.62%	18.042
Latvia	1.9	0.02%	0.272
Lithuania	2.7	0.03%	0.387
Luxembourg	0.04	0.001%	0.006
Netherlands	17.2	0.22%	2.463
New Zealand	4.9	0.06%	0.702
Norway	5.5	0.07%	0.788
Poland	37.8	0.48%	5.412
Portugal	10.2	0.13%	1.461
Slovak Republic	5.5	0.07%	0.788
Slovenia	2.1	0.03%	0.301
Spain	46.7	0.60%	6.687
Sweden	10.2	0.13%	1.461
Switzerland	8.7	0.11%	1.246
United Kingdom	68.2	0.88%	9.765
United States	332.9	4.27%	47.667

Table 3: National carbon budgets from 2021 onward (3rd column), based on a per capita criterion (1st and 2nd columns).

Country	**Ref. year**	**Ref. Em**	**Em 2020**	**2030 target**	**Em 2021–30**	**2025 target**	**Em 2021–25: estimated target**	**CB 2026**
Australia	2005	0.621	0.498	0.354	4.381		2.191	1.504
Canada	2005	0.739	0.672	0.425		0.549	2.967	2.488
Iceland	1990	0.013	0.013	0.006		0.010	0.056	-0.013
Japan	2013	1.408	1.096	0.760		0.928	4.977	13.065
New Zealand	2005	0.086	0.055	0.043	0.571		0.286	0.416
Norway	1990	0.052	0.029	0.025		0.027	0.138	0.649
Switzerland	1990	0.054	0.042	0.027		0.035	0.189	1.057
United Kingdom	1990	0.810	0.409	0.259		0.334	1.822	7.943
United States	2005	6.635	5.222	3.251		4.844	24.98	22.69
EU	1990	4.632	3.081	2.085		2.583	13.911	50.093

Table 4: NDCs 2021-2025 (E_0) and national carbon budgets from 2026 onward ($\bar{E}_1$).
According to NDCs (submitted by 2023): reference year (col.1), emissions in the reference year (col.2, GHG emissions including LULUCF, OECD data), emissions in 2020 (col.3, GHG emissions including LULUCF, OECD data), emissions target for 2030 (col.4), cumulative emissions target for 2030 if available (col.5), emissions target for 2025 (col.6, in green if stated, in black if extrapolated through linear reduction from 2020), estimated target as cumulative emissions E_0 over the NDC cycle 2021-2025 (col.7), and consequent carbon budgets from 2026 onwards (col.8, based on Table 3). "Em" stands for "Emissions", reported in Gt of CO_2.

Use, Land Use Changes and Forests).[23] Countries which stated explicitly a 2025 target in their NDCs

[23]Except for the NDCs of Canada, Norway and Switzerland, for which emissions in the reference year are clearly defined as excluding LULUCF.

EU Country	Em 2021–2025: estimated target	CB 2026 onwards
Austria	0.280	1.009
Belgium	0.361	1.300
Czech Republic	0.333	1.199
Denmark	0.181	0.650
Estonia	0.040	0.146
Finland	0.171	0.616
France	2.035	7.329
Germany	2.611	9.402
Greece	0.324	1.165
Hungary	0.299	1.076
Ireland	0.156	0.560
Italy	1.880	6.769
Latvia	0.059	0.213
Lithuania	0.084	0.303
Luxembourg	0.001	0.004
Netherlands	0.535	1.928
Poland	1.176	4.236
Portugal	0.317	1.143
Slovak Republic	0.171	0.616
Slovenia	0.065	0.235
Spain	1.453	5.233
Sweden	0.317	1.143

Table 5: EU – NDCs 2021-2025 (E_0) and national carbon budgets from 2026 onward ($\bar{E}_1$). Based on last row of Table 4 on a per-capita basis. "Em" stands for "Emissions", reported in Gt of CO_2.

are highlighted in green. Countries with an explicit cumulative objective (for 2030) are highlighted in red. All other countries defined the NDC objective in terms of annual emissions in 2030. I estimated their 2025 objective, and thus the cumulative emissions during the first cycle (2021-2025) by assuming a linear reduction rule from 2020 emissions' levels. For the EU countries in Table 5, the NDC is unique: their 2021-2025 objectives and the remaining carbon budgets are defined on a per-capita basis according to the EU NDC, whose information are reported at the bottom of Table 4. The MSB and the present value of future GDPs, will be maximized under the constraint represented by the carbon budgets $\bar{E}_1$ for 2026 onward reported in Tables 4 and 5.[24]

5 Results

This section shows the results for the maximization problem of the current fiscal limit introduced in section 3.3.1. I will analyse first the impact of the transition under three long-term scenarios:

(1) optimistic: $\mu_0 \neq \mu$, $\sigma_0 \neq \sigma$, as reported in Table 1;[25]

[24]Iceland will be ignored in the following analysis, because of the negativity of its budget constraint. The reason is geological, and coming from the methane release of the melting permafrost in its lands.

[25]The characterization of this scenario as "optimistic" reflects the observation that for the majority of countries $\mu_0 > \mu$, in the OECD study. This is also implied by the fact that these countries decreased their emissions over the observation

(2) parallel hypothesis (PL): $\mu_0 = \mu$, $\sigma_0 = \sigma$;

(3) pessimistic:[26] $\mu_0 = \mu[1 - m(E_t)]$, where $m(E_t)$ is decreasing in E_t, and when $E_t = 0$, μ_0 is reduced by a certain fraction (which will be calibrated to 11%, as explained below) with respect to μ. It is convenient to define the function m in terms of relative emissions:

$$m(e_t) = \sqrt{\theta \sum_1^t e_t} \tag{27}$$

α and β are taken for each country as in Table 1.

The parallel hypothesis scenario (2) is consistent with the evidence provided by Metcalf and Stock (2020 and forthcoming), and with the classical assumptions of computable general equilibrium (CGE) models (such as Goulder et al., 2019), where the long-run stationary growth rate is only determined by fundamentals, that would remain unaffected by temporary transition policies. The pessimistic scenario (3) can be interpreted by making use of the theoretical framework by Acemoglu et al. (2012) of directed endogenous technological progress: a lower long-run growth rate of the green economy would be the result of a lower success rate in innovation in the green sector with respect to the dirty sector. The latter's higher growth rate would be achieved instead in a laissez-faire scenario, leading to an environmental disaster (notice that climate economic damages are ignored in this framework). On the contrary, the optimistic scenario (1) is in line with the opposite situation where innovation is more frequently successful in the green sectors. Evidence in this regard is mixed. The knowledge spillovers, measured through the number of patent citations, appears to be higher for clean technologies, as shown by Dechezleprêtre and al. (2021, and 2017) and Perrons et al. (2021). Nonetheless the latter paper also shows that the pass-through delay of green innovations from scientific discovery to actual implementation is higher that for dirty innovations. I take the ratio of success rate of clean over dirty technology as the product of the ratio of their number of citations per patent (1.43 according to Dechezleprêtre and al. (2017)) and the inverse of the ratio of the pass-through years (5/8 according to Perrons et al. (2021). This results in a success rate of green technology equal to 89% the success rate of dirty technology. We assume then $\mu_0 = \mu(1 - 11\%)$, at the end of the transition. On the basis of these considerations, equation (27) is calibrated for scenario (3), with a value for $\theta = 0.0121$.

period 1990-2013. The denomination as "optimistic" also intend to reflects the dimensionality curse of estimations of short-term and long-term abatement costs based on historical data (the abatement effort of the past is only a small part of what would be needed to respect the Paris Agreement). For few countries, such as Australia, New Zealand and Spain, μ_0 is lower than μ. This second small group of advanced economies didn't reduce their emissions over the observation period.

[26]When maximizing the MSB under this long-term scenario, we add a monotonicity constraint in emissions over time and we assume the carbon budget to hold in equality. These assumptions are needed to avoid unrealistic paths in emissions' reductions. Notice that when these constraints are imposed for the other two scenarios, our results remain unchanged.

5.1 Abatement costs and fiscal limits

Table 6, and tables A2 and A3 in Appendix, show, respectively, the results for a 2.44%, 3% and 1.88% risk-free real interest rate. Each table lists values for the optimistic (1), the parallel-hypothesis (2), and the pessimistic (3) scenarios, of the stationary values at the end of the transition and the current MSB and MSD under the carbon budget constraint.[27] As expected, initial values are lower than the respective stationary values.[28] Countries that have general debt sustainability problems, such as Greece, Japan and Portugal, are highlighted in red. Countries for which this issue appears only in the context of performing the transition are highlighted in brown, such as Italy with a 2.44% risk-free rate, and France with 3%.

Country	Green b_M			Max b_0^M st cb			Green d_M			d_0^{M*} st cb			D/Y
	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	
Australia	788.12	906.13	499.36	598.63	685.19	520.54	896.47	1031.30	568.35	680.93	779.85	592.45	57.80
Austria	142.12	122.82	112.87	137.23	118.87	113.69	162.83	140.75	129.35	157.22	136.22	130.29	83.20
Belgium	312.88	268.56	246.81	277.51	240.13	228.22	357.46	306.99	282.14	317.05	274.50	260.88	112.80
Canada	464.49	436.02	387.14	416.43	391.53	366.77	532.28	500.19	444.12	477.20	449.15	420.75	117.80
Czech Republic	105.62	67.03	60.80	80.49	53.96	51.12	122.04	77.80	70.56	93.01	62.62	59.32	37.70
Denmark	343.90	321.94	305.62	326.14	305.91	296.70	395.43	370.42	351.65	375.01	351.98	341.39	42.10
Estonia	96.49	85.78	74.38	87.42	78.25	72.32	116.28	103.97	90.16	105.36	94.84	87.66	19.00
Finland	209.65	195.63	186.41	202.21	189.01	183.96	245.50	229.20	218.40	236.79	221.45	215.53	69.00
France	155.79	132.38	123.55	140.75	120.51	115.60	177.91	151.26	141.17	160.73	137.69	132.08	115.20
Germany	150.52	118.03	112.27	132.01	105.26	102.25	173.22	135.84	129.21	151.92	121.14	117.68	68.70
Greece	74.60	68.48	66.94	69.51	64.14	63.36	87.47	80.71	78.89	81.50	75.59	74.68	211.90
Hungary	251.81	194.65	184.00	215.23	170.13	164.85	290.92	225.66	213.32	248.67	197.24	191.11	80.00
Ireland	343.16	312.12	246.77	307.96	281.28	247.90	404.25	368.51	291.36	362.78	332.11	292.69	58.40
Italy	181.85	153.35	149.95	160.27	136.92	135.18	208.18	176.05	172.14	183.48	157.18	155.19	155.30
Japan	158.54	132.72	128.91	139.54	118.07	116.09	181.92	152.56	148.18	160.11	135.72	133.45	259.00
Latvia	47.49	43.08	37.30	42.60	38.88	35.91	57.60	52.33	45.31	51.66	47.24	43.62	43.30
Lithuania	64.34	47.60	40.52	52.84	40.25	36.82	76.66	57.30	48.78	62.95	48.45	44.33	46.60
Luxembourg	238.31	189.23	158.42	190.61	154.69	139.57	277.52	220.78	184.83	221.97	180.48	162.84	24.80
Netherlands	221.35	188.70	173.96	197.02	169.46	161.49	254.62	217.01	200.05	226.62	194.88	185.71	52.80
New Zealand	312.92	319.08	280.35	287.89	293.41	273.93	360.12	367.39	322.79	331.31	337.83	315.40	43.10
Norway	1290.37	1050.44	910.25	1234.88	1009.14	947.84	1475.18	1202.54	1042.05	1411.74	1155.26	1085.08	46.80
Poland	193.97	188.81	152.94	189.04	184.06	173.13	225.23	219.19	177.54	219.51	213.67	200.98	57.40
Portugal	104.74	94.15	89.29	97.31	87.85	85.16	120.33	108.47	102.87	111.80	101.22	98.12	135.20
Slovak Republic	15.77	12.56	9.85	13.00	10.52	9.14	18.37	14.66	11.50	15.15	12.28	10.67	59.70
Slovenia	78.94	67.95	62.39	68.32	59.52	56.72	91.80	79.36	72.87	79.45	69.52	66.25	79.80
Spain	132.82	126.41	116.79	120.23	114.73	109.51	152.77	145.75	134.66	138.30	132.28	126.26	120.00
Sweden	226.70	200.15	186.13	207.06	183.92	176.29	262.10	231.60	215.38	239.39	212.83	204.00	39.60
Switzerland	127.76	116.64	109.36	116.41	106.91	102.88	146.22	133.48	125.14	133.24	122.35	117.73	42.40
Turkey	184.91	215.05	185.59	159.01	182.66	168.09	217.35	255.52	220.51	186.90	217.03	199.72	39.50
United Kingdom	301.47	246.01	224.31	271.39	223.85	211.87	345.71	282.47	257.56	311.22	257.03	243.27	102.60
United States	212.25	191.23	166.66	178.57	161.54	149.12	242.97	219.13	190.97	204.42	185.10	170.88	134.20

Table 6: Debt sustainability in advanced countries in the transition. $r = 2.44\%$. b_M and d_M: respectively, the maximum sustainable borrowing and the debt limit in the green economy. Max b_0^M st cb and d_0^{M*} st cb: respectively, the maximum current "maximum sustainable borrowing" consistent with the carbon budget and the corresponding debt limit. In red, countries that have general debt sustainability problems (D/Y$>d_M$); in brown, countries for which this issue appears only in the context of performing the transition (D/Y$>d_0^{M*}$ st cb).

From these tables, we can notice that the US debt ceiling ($31.4 trillion, suspended in June 2023 until January 2025), corresponding over GDP in 2021 to 134%, is always under the MSD in these results.

[27]Under the scenario (2), the stationary MSB and MSD are equivalent to the classical measure in Collard, Habib, and Rochet (2015)

[28]Except for very few cases in scenario (3): this anomaly comes from computational limitations, that prevent studying a true infinite horizon, and from the fact that for Australia, Austria, Ireland, Norway and Poland, the maximization of current sustainability leads to dividing the carbon budget in very small pieces among all the available years. This result appears only for Norway and Poland when $r = 3\%$.

It can be argued that the ceiling would be more appropriately defined as a percentage of GDP, than in US dollars. Given that GDP historically increases over time, the resulting debt ceiling-to-GDP would decrease over time, if the ceiling was not systematically politically re-discussed and increased every few years. Notice, that the current study remains agnostic regarding the evolution of actual debt-to-GDP ratios. Nonetheless it is reasonable to expect that they will be needed to raise to cope with the green transition and/or climate damages. The risk is that, not only they would overcome the official debt ceilings, but also MSD levels that would entail a true higher probability of default. Notice that the following figures will show the probability of default of *current* debt-to-GDP. This measure would naturally increase for higher levels of actual debt-to-GDP.

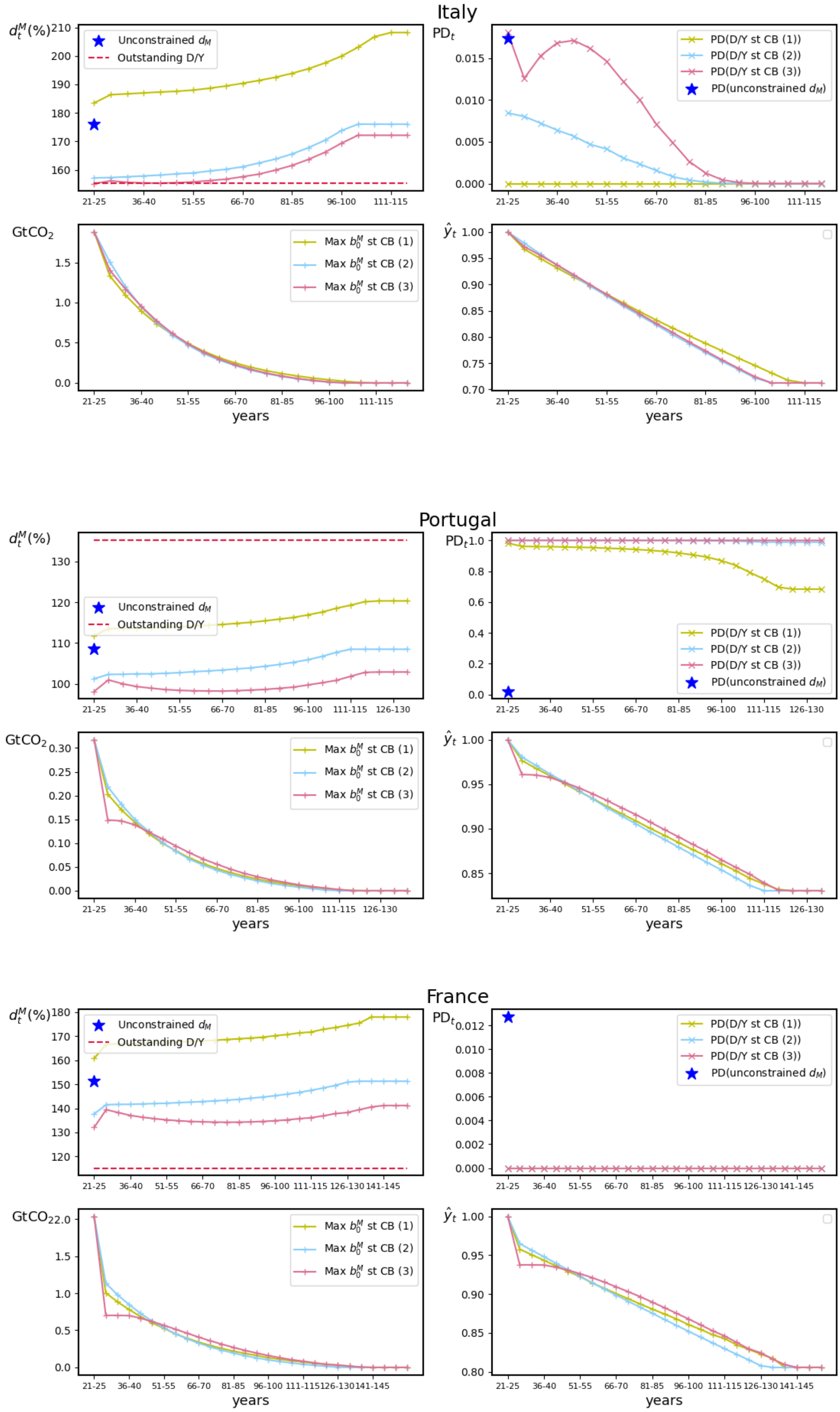

Figure 1: Comparison of the impact on debt sustainability of different scenarios for the long-term green growth rate. $r = 2.44\%$. Plots for the fiscal limit d_t^M, the annual probability of default PD_t (1.0 is 100%) of 2020 debt-to-GDP ratios (outstanding D/Y, assumed to remain constant), the emissions' path expressed in $GtCO_2$ maximizing the current maximum sustainable borrowing b_0^M under the carbon budget, and the normalized detrended GDP $\hat{y}_t$. Scenarios: (1) optimistic, (2) parallel hypothesis, (3) pessimistic.

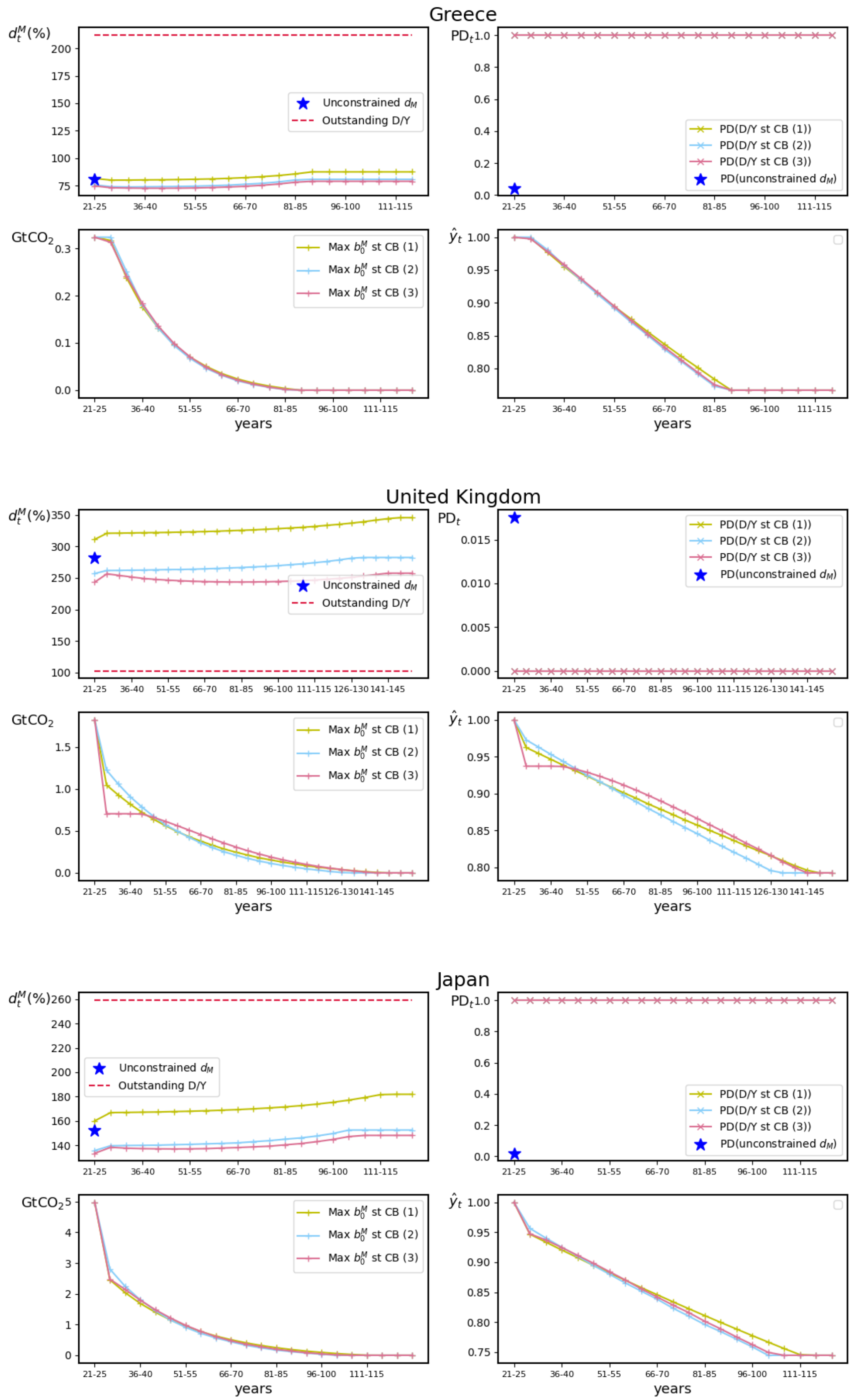

Figure 1: Comparison of the impact on debt sustainability of different scenarios for the long-term green growth rate. $r = 2.44\%$. (cont.)

Figure 1 plots the dynamic results for some of the countries in our dataset,[29] for $r = 2.44\%$ and the alternative three long-term scenarios. The four pictures for each country shows: the optimizing transition path in $GtCO_2$ (bottom left), the corresponding normalized detrended GDP (bottom right), the MSD (top left) and the probability of default of outstanding debt-to-GDP (top right). Each point shows their respective values for a 5-year period, starting from the current NDC cycle 2021-2025, for which we take the level of emissions as given in Tables 4 and 5. The depicted path is the one which guarantees current maximum debt sustainability. This is the main question asked in this paper: what is the emission path compatible with a national fair carbon budget which maximizes the current MSB? The objective is to compare the resulting MSD with the outstanding debt-to-gdp, in order to understand how sustainable advanced economies are in the face of the urgently needed transition. Notice that for some countries, such as France, United Kingdom, Portugal, the drop in optimal emissions in period 1 (2026-2030), with respect to period 0 (2021-2025) is substantial. This drop is higher for countries with a lower abatement cost β, and, thus, a lower dependence on emissions. In this regard, Figure 2 shows a sensitivity analysis for higher values of β (in scenario (2)) for Italy, France and the United States, for which higher abatement costs imply a shift from a sustainable to an unsustainable current debt-to-GDP. A higher β implies a faster transition, with an initial overshooting if $\beta = 0.6$. The consequent convergence to the stationary MSD is faster, but passing through very adverse levels of fiscal limit. This evidence points towards the importance of a careful quantification of future abatement costs, and the importance of managing them efficiently in order to avoid impairing debt sustainability in the short-term, and making financing the transition an unfeasible task.

In these and the following graphs, the blue star represents the traditional country-specific fiscal limit measure of Collard, Habib, and Rochet (2015), which does not take into account the role of transition and/or climate costs (introduced only in Section 6). Also, in Appendix D.1, Figure A1 shows sensitivity analysis for lower values of the recapture parameter c.

[29]For space reasons, the dynamic results for only few notable countries are shown. Results for all countries and scenarios are available upon request to the author.

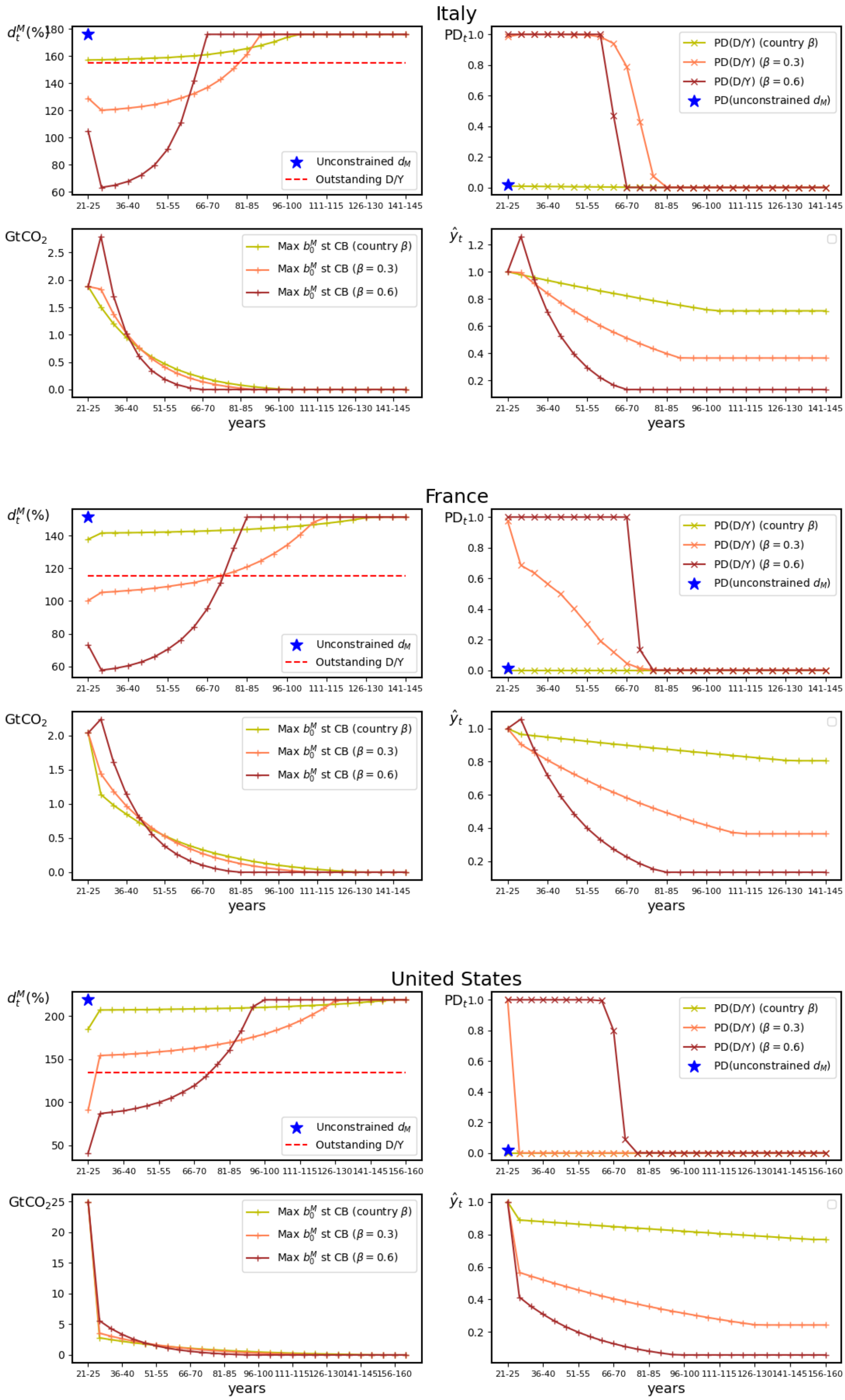

Figure 2: Sensitivity analysis for higher values of β. $r = 2.44\%$, parallel hypothesis scenario. Plots for the fiscal limit d_t^M, the annual probability of default PD$_t$ (1.0 is 100%) of 2020 debt-to-GDP ratios (outstanding D/Y, assumed to remain constant), the emissions' path expressed in $GtCO_2$ maximizing the current maximum sustainable borrowing b_0^M under the carbon budget, and the normalized detrended GDP $\hat{y}_t$. Country βs are reported in Table 1.

5.2 Current debt sustainability versus "welfare" maximization

The objective function analysed so far aims to find the current fiscal limit: the current maximum sustainable debt level achievable when facing the transition constraints and their costs in terms of growth, in order to evaluate the minimum probability of default of current debt-to-GDP levels. Here, the results of performing the maximization problem of Section 3.3.1 are compared with results of maximizing the present value of expected future GDP, as outlined in Section 3.3.2. This second objective, which can be taken as an approximation of welfare maximization by a benevolent social planner, leads to a very fast initial decarbonization in the first period, leaving a great fraction of the remaining carbon budget available for the future. A benevolent social planner maximizing welfare would prefer to perform most of the green transition of the economy as fast as possible, and leave the possibility of emitting for future generations. The resulting initial debt limit is lower, but its subsequent levels are higher, the convergence to the stationary fiscal limit slower. Probability of defaults remain generally unaffected.

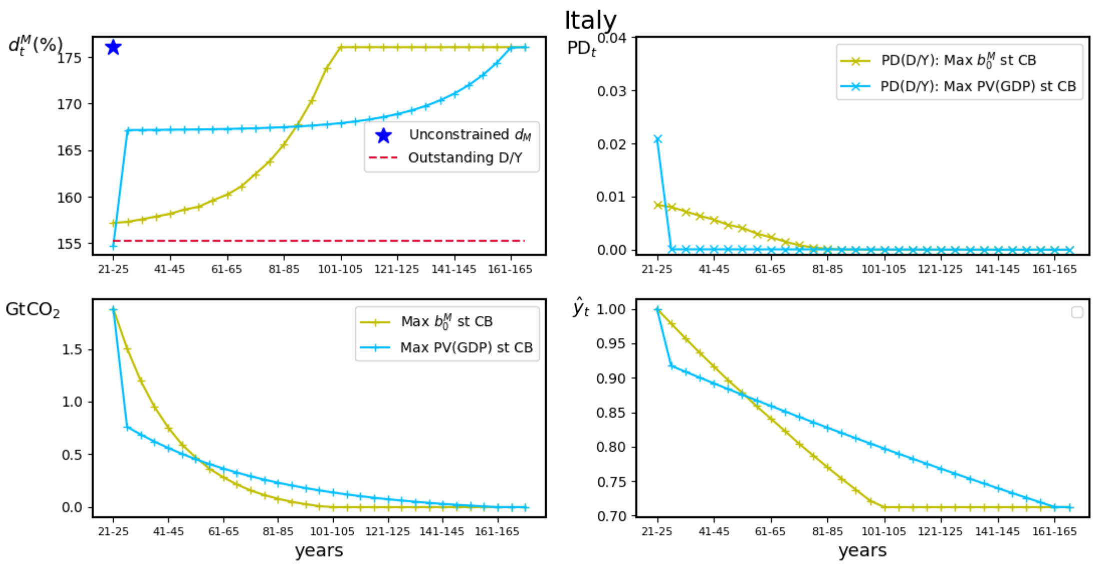

Figure 3: Maximizing under the carbon budget the current maximum sustainable borrowing versus welfare maximization. $r = 2.44\%$, parallel hypothesis scenario. Plots for the fiscal limit d_t^M, the annual probability of default PD_t (1.0 is 100%) of 2020 debt-to-GDP ratios (outstanding D/Y, assumed to remain constant), the emissions' path expressed in $GtCO_2$ maximizing the current maximum sustainable borrowing b_0^M or welfare (the present value of future GDPs, PV(GDP)) under the carbon budget, and the normalized detrended GDP $\hat{y}_t$.

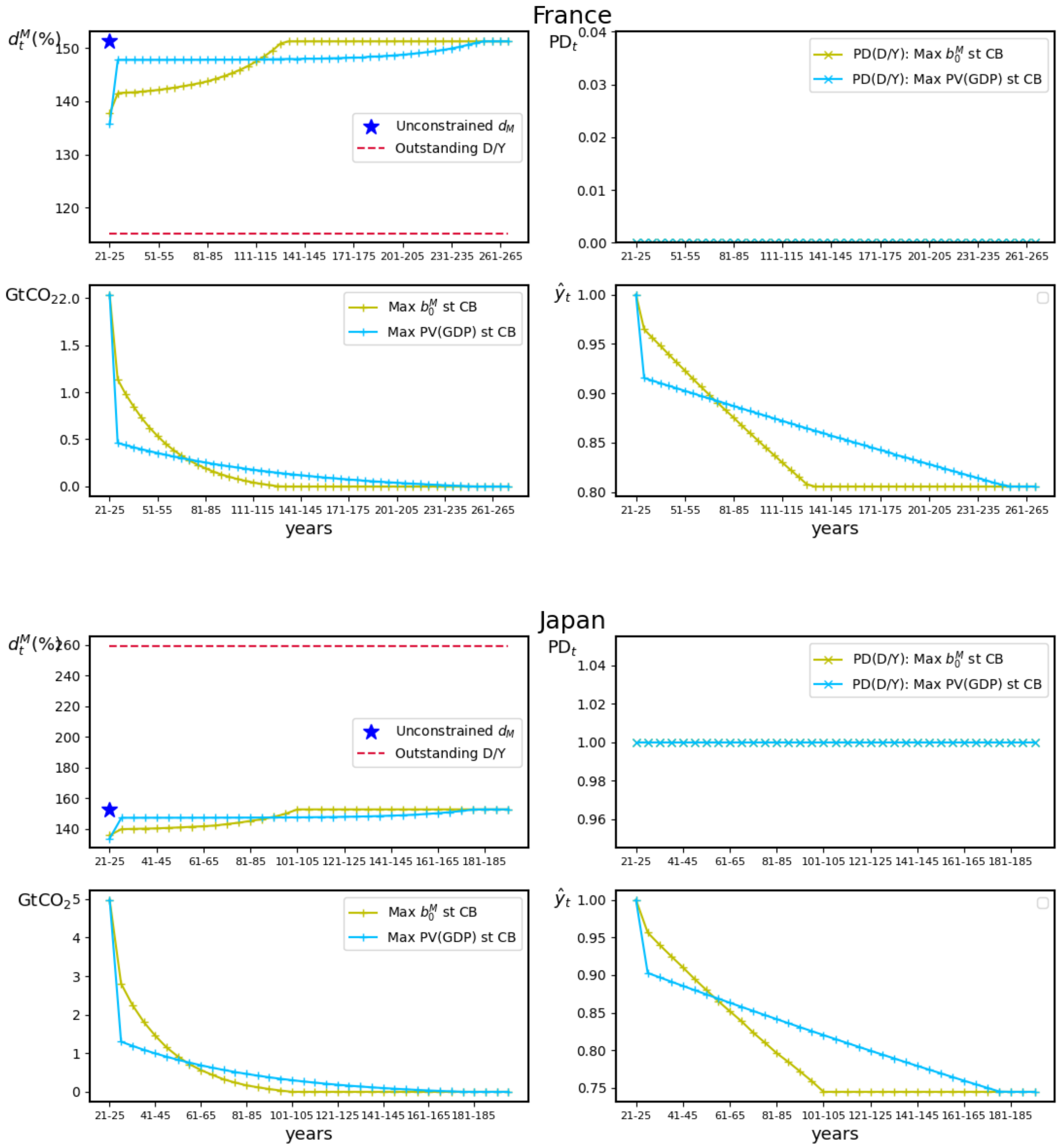

Figure 3: Maximizing under the carbon budget the current maximum sustainable borrowing versus welfare maximization. $r = 2.44\%$, parallel hypothesis scenario.

6 Introducing climate damages and the need for global coordination

Until now, we overlooked a fundamental aspect of the transition, that is its fundamental motivation of reducing climate damages. Their economic impact is captured in the economic-climate literature through the so called "damage function". I follow Dietz, Venmans (2019) (DV in the following) and I assume it to be exponential in temperature, which is linear in cumulative emissions:

$$D(T_t) = \exp\left(-\frac{\rho}{2}T_t^2\right), \text{ where } T_t = \zeta C_t. \tag{28}$$

T_t represents the global average temperature increase, with respect to pre-industrial levels, and C_t global cumulative emissions since 1850. The parameters ρ and ζ are calibrated by taking their central value in DV: $\rho = 0.01$, and $\zeta = 0.000480$.[30] The time delay from cumulative emissions to temperature is assumed to be negligible, following the evidence provided by DV.
By adding the damage function, the GDP growth rate becomes:

$$g_{i,t+1} = e^{\mu_{0,i}+\varepsilon_{i,t+1}-\frac{\rho}{2}[(\zeta C_{t+1})^2-(\zeta C_t)^2]} \left(\frac{c+e_{i,t+1}}{c+e_{i,t}}\right)^{\beta_i} \tag{29}$$

Notice that the damage function is global, and thus, C_t represent global cumulative emissions. In the coordinate transition scenario, it is assumed that each country, when selecting its own transition path, will automatically set the path for the global economy, through, for example, political bargaining efforts, or carbon border adjustment mechanisms. Therefore, for each country, we define $C_t = C_{2020} + \frac{\bar{E}_0^G}{\bar{E}_{i,0}} C_{i,t}$, where $\bar{E}_0^G$ and $\bar{E}_{i,0}$ are respectively the global and the national carbon budget from 2021 onward. Cumulative emissions until 2020, C_{2020}, are estimated to have been 3336 Gt CO2. National cumulative emissions at time t are defined as: $C_{i,t} = \sum_0^t E_{i,t}$, where period 0 represents the period 2021-2025.

We are able now to properly compare the coordinated transition paths with a business-as-usual scenario, where national and global emissions continue to develop overtime as historically. In this case, the growth rate and volatility of countries are set respectively to their historical averages μ_i and σ_i,[31] whereas the annual growth rate of global emissions is set to its average from 1990 to 2020, as 1.126%.

When we add the damage function to our growth function, we can indeed observe the trade-off that countries are facing with respect to climate issues. Figure 4 shows the dynamic results for some countries, France, Italy and the US. In the long term, the business-as-usual scenario (black lines) always reaches a 5-year probability of default of 100% (for the US in periods subsequent to the horizon reported in the graph). For most countries, the trade-off between mitigation and unmanaged climate costs for the level of fiscal limit switches in favor of a coordinated global transition (light green lines) very rapidly, in the first 5/10 years of our analysis, and for all countries before 2080. The precise period of the shift is reported in Table A4 for each country. In general, countries with higher β tend to change the sign of the trade-off later.

[30] In this analysis I employed a global damage function. In future extensions of this work, the parameter ρ could potentially be adapted locally. Nonetheless, further research in the literature needs to be conducted in order to provide this granularity to the exponential (and other) damage function(s).

[31] Notice that it is assumed $e^{\mu_i} \equiv e^{\mu_{0,i}+\varepsilon_{i,t+1}} \left(\frac{c+e_{i,t+1}^{\text{business-as-usual}}}{c+e_{i,t}^{\text{business-as-usual}}}\right)^{\beta_i}$.

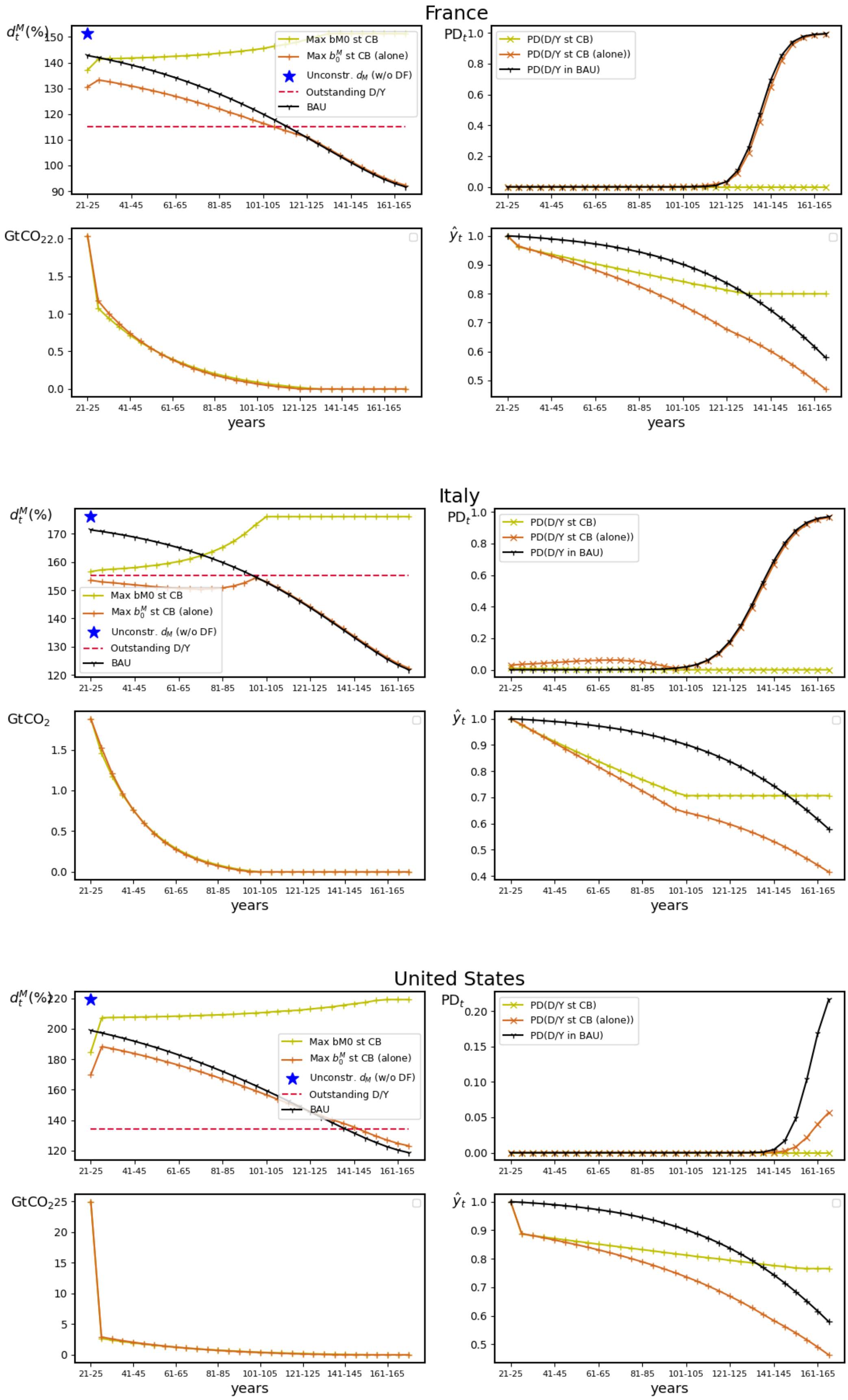

Figure 4: Debt sustainability in a coordinated transition (light green), a "solitary" transition (orange) and a global business-as-usual scenario (black). $r = 2.44\%$, parallel hypothesis scenario. Plots for the fiscal limit d_t^M, the annual probability of default PD_t (1.0 is 100%) of 2020 debt-to-GDP ratios (outstanding D/Y, assumed to remain constant), the emissions' path expressed in $GtCO_2$ maximizing the current maximum sustainable borrowing b_0^M under the carbon budget, and the normalized detrended GDP $\hat{y}_t$.

The orange lines depict the fiscal limit when the considered country is the unique one respecting its fair carbon budget ("solitary" transition), and all the rest of the world continues on the business-as-usual path. As we could expect, this converges to the black line, except for countries such as the US, whose population and emissions are a great share of global population and emissions. Notice that the probability of default rises only when the current debt-to-GDP overcomes the fiscal limit, except for countries, such as Japan or Greece, with particular adverse situations, this happens in the business-as-usual scenario toward the end of this century or in the next for most countries. Nonetheless the depicted probabilities of default refers to current debt-to-GDP ratios. Given the great costs that unmanaged climate damages would impose on public finances, we might expect that debt-to-GDP ratios will tend to increase over the next decades in the business-as-usual scenario. This might be true or not also for the scenario of a globally successful transition, depending on the policy mix between carbon taxes and public investment that will be implemented by each country. This question pertains to the research agenda following this paper.

7 Conclusions

This paper proposes an adjustment in the quantification of traditional measures of fiscal limit (such as Collard, Habib, and Rochet (2015)), by taking into account the need of respecting the Paris Agreement and performing the transition to a green economy, and the important costs attached to it. Despite the choice of a quite conservative baseline growth function in terms of emission (by following the OECD paper by Rodríguez et al. (2018)), fiscal limits are importantly and negatively impacted by the carbon budget constraint. The reduction of fiscal limits can also imply an increase in the probability of default of current debt-to-GDP ratios, in scenarios with more adverse short-term or long-term transition costs (as shown in Section 5). Importantly, the decrease in fiscal limit in a coordinated transition scenario is always lower in the medium-long term than in the scenario of a failed transition and unmanaged climate damages (Section 6). In the long-term, ignoring the need of reducing carbon emissions globally imply plunging fiscal limits, and serious sustainability problems.

Another interesting question would be: is a transition which respects the fair carbon budget economically and fiscally feasible? In the current reduced-form modeling approach, the economic details of the alternative policy mixes for implementing the transition are not studied. Indeed there are different available instruments through which the transition could be implemented: such as carbon taxes, cap and trade systems, subsidies, direct public investment. The latter twos could be financed either through revenue recycling of the carbon tax, or by increasing other taxes, or through issuing additional debt. These issues could be analysed by complementing this (or a) model of debt sustainability and fiscal limits, with a general equilibrium model which studies the optimal transition policy mix and its financing. The result of this framework would be a comparison of the actual levels of debt-to-GDP needed to finance the transition, and the corresponding debt limit. In general, we can expect that the transition would tend to increase current debt levels, and, as demonstrated in this paper, reduce the debt limit, reducing, as a result, countries' available fiscal space (the debt limit minus the actual debt-to-GDP). This analysis belongs to the future research agenda.

Nonetheless, the evidence provided in this paper is strong enough to argue for a swift and globally coordinated transition to mitigate climate damages on growth and fiscal limits, in order to avoid plunging fiscal limits. For EU and advanced countries, it is not sufficient to implement national or communitarian mitigation policies. They also need to ensure that the green transition is performed globally, in order to avoid witnessing a degradation in their own public debt sustainability. This is fundamental to guarantee the viability of financing the green transformation, for which all governments will need energetic policy measures and ample fiscal spaces.

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. "The Environment and Directed Technical Change". *American Economic Review* 102, no. 1 (Feb. 2012): 131–66.

Aguiar, Mark, Satyajit Chatterjee, Harold Cole, and Zachary Stangebye. "Self-Fulfilling Debt Crises, Revisited". *Journal of Political Economy* 130, no. 5 (May 2022): 1147–1183.

Bacchetta, Philippe, Elena Perazzi, and Eric van Wincoop. "Self-fulfilling debt crises: What can monetary policy do?" *Journal of International Economics* 110 (2018): 119–134.

Barrage, Lint. "The Fiscal Costs of Climate Change". *AEA Papers and Proceedings* 110 (2020): 107–112.

— . *The Fiscal Costs of Climate Change in the United States*. Tech. rep. WP 23/380. CER-ETH, 2023.

Batini, Nicoletta, Giovanni Melina, and Stefania Villa. *Fiscal Buffers, Private Debt, and Stagnation: The Good, the Bad and the Ugly*. IMF Working Papers 2016/104. International Monetary Fund, May 2016.

Beirne, John, Nuobu Renzhi, and Ulrich Volz. "Feeling the heat: Climate risks and the cost of sovereign borrowing". *International Review of Economics Finance* 76 (2021): 920–936.

Bi, Huixin. "Sovereign default risk premia, fiscal limits, and fiscal policy". *European Economic Review* 56, no. 3 (2012): 389–410.

Bi, Huixin, and Eric M. Leeper. *Analyzing Fiscal Sustainability*. Staff Working Papers 13-27. Bank of Canada, 2013.

Boehm, H. "Physical climate change and the sovereign risk of emerging economies". *Economic Structures* 11 (2022): 31.

Bolton, Patrick, Mitu Gulati, and Ugo Panizza. *Sovereign Debt Puzzles*. IHEID Working Papers 27-2022. Economics Section, The Graduate Institute of International Studies, Dec. 2022.

Chatterjee, Satyajit, and Burcu Eyigungor. "Maturity, Indebtedness, and Default Risk". *American Economic Review* 102, no. 6 (May 2012): 2674–99.

Collard, Fabrice, Michel Habib, Ugo Panizza, and Jean-Charles Rochet. *Debt Sustainability with Involuntary Default*. CEPR Discussion Papers 17357. C.E.P.R. Discussion Papers, 2022.

Collard, Fabrice, Michel Habib, and Jean-Charles Rochet. "Sovereign Debt Sustainability In Advanced Economies". *Journal of the European Economic Association* 13, no. 3 (2015): 381–420.

Collender, Sierra, Baoqing Gan, Christina S. Nikitopoulos, Kylie-Anne Richards, and Laura Ryan. "Climate transition risk in sovereign bond markets". *Global Finance Journal* 57 (2023): 100868.

Corsetti, Giancarlo, Keith Kuester, André Meier, and Gernot J. Müller. "Sovereign Risk, Fiscal Policy, and Macroeconomic Stability". *The Economic Journal* 123, no. 566 (2013): F99–F132.

Darracq Pariès, Matthieu, Pascal Jacquinot, and Niki Papadopoulou. *Parsing financial fragmentation in the euro area: a multi-country DSGE perspective*. Working Paper Series 1891. European Central Bank, Apr. 2016.

Dechezlepretre, Antoine, Ralf Martin, and Myra Mohnen. *Knowledge Spillovers from clean and dirty technologies*. GRI Working Papers 135. Grantham Research Institute on Climate Change and the Environment, Oct. 2017.

Dechezleprêtre, Antoine, Cal B. Muckley, and Parvati Neelakantan. "Is firm-level clean or dirty innovation valued more?" *The European Journal of Finance* 27 (2021): 31–61.

Dées, Stéphane. "Assessing the Role of Institutions in Limiting the Environmental Externalities of Economic Growth". *Environmental and Resource Economics* 76 (2020): 429–445.

Dietz, Simon, and Frank Venmans. "Cumulative carbon emissions and economic policy: In search of general principles". *Journal of Environmental Economics and Management* 96 (2019): 108–129.

Eaton, Jonathan, and Mark Gersovitz. "Debt with Potential Repudiation: Theoretical and Empirical Analysis". *The Review of Economic Studies* 48, no. 2 (1981): 289–309.

Ghosh, Atish R., Jun I. Kim, Enrique G. Mendoza, Jonathan D. Ostry, and Mahvash S. Qureshi. "Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies". *The Economic Journal* 123, no. 566 (2013): F4–F30.

Gollier, Christian. *The cost-efficiency carbon pricing puzzle*. TSE Working Papers 18-952. Toulouse School of Economics (TSE), 2022.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. "Optimal Taxes on Fossil Fuel in General Equilibrium". *Econometrica* 82, no. 1 (Jan. 2014): 41–88.

Goulder, Lawrence H., and Marc A. C. Hafstead. *Confronting the Climate Challenge: U.S. Policy Options*. Columbia University Press, 2018.

Goulder, Lawrence H., Marc A.C. Hafstead, GyuRim Kim, and Xianling Long. "Impacts of a carbon tax across US household income groups: What are the equity-efficiency trade-offs?" *Journal of Public Economics* 175 (2019): 44–64.

Grossman, Herschel I., and John B. Van Huyck. "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation". *The American Economic Review* 78, no. 5 (1988): 1088–1097.

Hatchondo, Juan Carlos, and Leonardo Martinez. "Long-duration bonds and sovereign defaults". *Journal of International Economics* 79, no. 1 (2009): 117–125.

Hatchondo, Juan Carlos, Leonardo Martinez, and Francisco Roch. “Fiscal Rules and the Sovereign Default Premium”. *American Economic Journal: Macroeconomics* 14, no. 4 (Oct. 2022): 244–73.

International Monetary Fund. Fiscal Affairs Dept. *Fiscal Monitor, October 2023: Climate Crossroads: Fiscal Policies in a Warming World*. Vol. 2023. 002. International Monetary Fund, 2023.

IPCC. “Climate Change 2023: Synthesis Report”. In *Contribution of Working Groups I, II and III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Core Writing Team, H. Lee and J. Romero (eds.)]*. 35–115. IPCC, Geneva, Switzerland, 2023.

— . “Summary for Policymakers”. In *Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, 3–32. Cambridge University Press, 2023.

Känzig, Diego R. *The Unequal Economic Consequences of Carbon Pricing*. Working Paper, Working Paper Series 31221. National Bureau of Economic Research, May 2023.

Känzig, Diego R, and Maximilian Konradt. *Climate Policy and the Economy: Evidence from Europe's Carbon Pricing Initiatives*. Working Paper, Working Paper Series 31260. National Bureau of Economic Research, May 2023.

Lorenzoni, Guido, and Iván Werning. “Slow Moving Debt Crises”. *American Economic Review* 109, no. 9 (Sept. 2019): 3229–63.

Metcalf, Gilbert E., and James H. Stock. “The Macroeconomic Impact of Europe's Carbon Taxes”. *American Economic Journal: Macroeconomics* (forthcoming).

Nordhaus, William. *A question of balance: Weighing the options on global warming policies*. Yale University Press, 2014.

Perrons, Robert K., Adam B. Jaffe, and Trinh Le. “Linking scientific research and energy innovation: A comparison of clean and dirty technologies”. *Energy Research Social Science* 78 (2021): 102122.

Renne, Jean-Paul, and Kevin Pallara. “Fiscal Fatigue, Fiscal Limits and Sovereign Credit Spreads”. *Proceedings of Paris December 2019 Finance Meeting EUROFIDAI - ESSEC* (July 2023).

— . “Fiscal Limits and the pricing of Eurobonds”. *Management Science* (forthcoming).

Rodríguez, Miguel, Ivan Haščič, and Martin Souchier. “Environmentally Adjusted Multifactor Productivity: Methodology and Empirical Results for OECD and G20 Countries”. *Ecological Economics* 153 (Nov. 2018): 147–160.

Seghini, Caterina, and Stéphane Dees. *The Green Transition and Public Finances*. SSRN Working Paper, Jan. 2024.

Stock, James H., and Gilbert E. Metcalf. “Measuring the Macroeconomic Impact of Carbon Taxes”. *American Economic Review* 110, no. May (2020): 101–06.

Tanner, Evan C. *Fiscal Sustainability: A 21st Century Guide for the Perplexed*. IMF Working Papers 2013/089. International Monetary Fund, Apr. 2013.

Weitzman, Martin L. “GHG Targets as Insurance Against Catastrophic Climate Damages”. *Journal of Public Economic Theory* 14, no. 2 (2012): 221–244.

Yeyati, Eduardo Levy, and Ugo Panizza. “The elusive costs of sovereign defaults”. *Journal of Development Economics* 94, no. 1 (2011): 95–105.

Appendix

A Comparison with other models

It can be shown that the i.i.d. growth function $g_{t+1} \equiv e^{\mu+\varepsilon_{t+1}}$ in the Collard, Habib, and Rochet (2015) framework is compatible with a stochastic Solow model, with a Cobb-Douglas production function with labor L and capital K as inputs, and technological progress Z_t (labor-intensive).[32] Emissions can then be introduced as an additional input to this model, as in Dietz and Venmans (2019). Total output becomes:

$$Y_t = e^{\zeta_t} K_t^{\kappa} (Z_t L_t)^{1-\kappa} \eta(E_t) D(T_t) = e^{\zeta_t} \hat{k}_t^{\kappa} Z_t L_t \eta(E_t) D(T_t)$$

where ζ_t is a technological shock, κ the share of capital in production and $\hat{k}_t \equiv \frac{K_t}{Z_t L_t}$ capital per effective labor . Dietz and Venmans (2019) also impose:

$$\eta_{DV}(E_t) = \exp\left(\phi E_t - \frac{\varphi}{2} E_t^2\right)$$
$$D(T_t) = \exp\left(-\frac{\rho}{2} T_t^2\right), \text{ where } T_t = \zeta C_t$$

where C_t are global cumulative emissions. Notice that this function η is concave in emissions, as assumed in this paper. By assuming that $\hat{k}_t$ is already close to the steady state, which will be achieved at the end of the transition when emissions are zero, their growth rate writes approximately as:

$$g_{i,t+1} \approx \exp[\mu_{0,i} + \varepsilon_{i,t+1} - \frac{\rho}{2}[(\zeta C_{t+1})^2 - (\zeta C_t)^2] + \phi(E_{t+1} - E_t) - \frac{\varphi}{2}(E_{t+1}^2 - E_t^2)]$$

Notice that following DV, we can interpret $\frac{\eta'(E_t)}{\eta(E_t)}$ as the MAC (marginal abatement cost) function. Whereas DV have: $\frac{\eta'(E_t)}{\eta(E_t)} = \phi - \varphi E_t$, which is decreasing in E_t, in the current model, the MAC function becomes:

$$\frac{\eta'(E_t)}{\eta(E_t)} = \frac{\beta}{c\bar{E}_0 + E_t},$$

also decreasing in emissions. Therefore, following DV, we can give an exact interpretation to η, and in principle open the path to implementing different MAC functions, and study their impact on fiscal limits. This is left for future research.

[32] These results are available upon request.

B The government's maximization problems

B.1 Maximizing the current MSB under the carbon budget.

The maximization problem at $t = 0$, given an initial level of emissions E_0 is:

$$\max_{\{E_t\}} \; b_0^M = \frac{\alpha}{\eta(E_0)} \sum_{t=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R}\right)^t \eta(E_t) \tag{30}$$

$$\text{s.t.} \; \sum_{t=1}^{+\infty} E_t \leq \bar{E}_1 \tag{31}$$

$$E_t \geq 0 \tag{32}$$

aining carbon budget at time t with respect to a total budget $\bar{E}_0$: $\bar{E}_t \equiv \bar{E}_0 - \sum_{j=0}^{t-1} E_j$.

The associated Lagrangian is:

$$\mathscr{L} = \frac{\alpha}{\eta(E_0)} \sum_{t=1}^{+\infty} \left(\frac{\gamma e^{\mu_0}}{R}\right)^t \eta(E_t) - \lambda\left(\sum_{t=1}^{+\infty} E_t - \bar{E}_1\right) + \sum_{t=1}^{+\infty} \psi_t E_t$$

The FOC with respect to E_t is:

$$\frac{\delta \mathscr{L}}{\delta E_t} = \frac{\alpha}{\eta(E_0)} \left(\frac{\gamma e^{\mu_0}}{R}\right)^t \eta'(E_t) - \lambda + \psi_t = 0, \forall t > 0$$

$$\Rightarrow \eta'(E_t) = \frac{(\lambda - \psi_t)\eta(E_0)}{\alpha} \left(\frac{R}{\gamma e^{\mu_0}}\right)^t, \forall t > 0$$

For a higher period t, $\left(\frac{R}{\gamma e^{\mu_0}}\right)^t$ is higher, given that $\gamma e^{\mu_0} < R$. Therefore $\eta'(E_t)$ increases with time and, when $\beta < 1$ as in our standard scenario, E_t decreases:

$$\eta(E_t) = [c\bar{E}_1 + E_t]^\beta \implies \eta'(E_t) = \beta(c\bar{E}_1 + E_t)^{\beta-1}$$

Therefore:

$$\beta(c\bar{E}_1 + E_t)^{\beta-1} = \frac{(\lambda - \psi_t)\eta(E_0)}{\alpha} \left(\frac{R}{\gamma e^{\mu_0}}\right)^t$$

$$\implies E_t = \left[\frac{\alpha\beta}{(\lambda - \psi_t)[c\bar{E}_1 + E_0]^\beta} \left(\frac{\gamma e^{\mu_0}}{R}\right)^t\right]^{1/(1-\beta)} - c\bar{E}_1 \tag{33}$$

When $\beta < 1$ (in line with the OECD estimation results for ε_{YE}), since the condition $\gamma e^{\mu_0} < R$ holds, E_t decreases over time, as mentioned above. This guarantees a smooth transition path.[33][34]

[33] If, instead $\beta > 1$, E_t has to increase over time under the maximization of current debt sustainability. This goes against the usual and reasonable planning of a green transition.

[34] The calculations to find a pseudo-analytical solution are available upon request to the author.

B.2 Maximizing "welfare" under the carbon budget

The welfare maximization writes as

$$\max_{\{E_t\}} \sum_{t=0}^{+\infty} \frac{\mathbb{E}_0[Y_t]}{R^t} = \frac{Y_0}{\eta(E_0)} \sum_{t=0}^{+\infty} \left(\frac{\bar{g}}{R}\right)^t \eta(E_t) \tag{34}$$

$$\text{s.t.} \sum_{t=1}^{+\infty} E_t \leq \bar{E}_1 \tag{35}$$

$$E_t \geq 0 \tag{36}$$

by defining $\bar{g} = e^{\mu_0 + 1/2\sigma_0^2}$, as the expected fundamental growth rate. The associated Lagrangian is:

$$\mathscr{L} = \frac{Y_0}{\eta(E_0)} \sum_{t=0}^{+\infty} \left(\frac{\bar{g}}{R}\right)^t \eta(E_t) - \varphi(\sum_{t=1}^{+\infty} E_t - \bar{E}_1) + \sum_{t=1}^{+\infty} \theta_t E_t$$

The FOC with respect to E_t is:

$$\frac{\delta \mathscr{L}}{\delta E_t} = \frac{Y_0}{\eta(E_0)} \left(\frac{\bar{g}}{R}\right)^t \eta'(E_t) - \varphi + \theta_t = 0, \forall t > 0$$

$$\Rightarrow \eta'(E_t) = \frac{(\varphi - \theta_t)\eta(E_0)}{Y_0} \left(\frac{R}{\bar{g}}\right)^t, \forall t > 0$$

$$E_t = \left[\frac{\beta Y_0}{(\varphi - \theta_t)(c\bar{E}_1 + E_0)^\beta} \left(\frac{\bar{g}}{R}\right)^t\right]^{1/(1-\beta)} - c\bar{E}_1 \tag{37}$$

Let recall that, as demonstrated by Collard, Habib, and Rochet (2015), the total borrowing factor is lower than the average growth rate: $\gamma e^{\mu_0} < \bar{g}$. Therefore, we can expect emissions maximizing welfare to decrease slower than the emissions maximizing the current MSB, from an initial level:

$$E_1 = \left[\frac{\beta Y_0}{\varphi(c\bar{E}_1 + E_0)^\beta} \left(\frac{\bar{g}}{R}\right)\right]^{1/(1-\beta)} - c\bar{E}_0$$

by setting $\theta_1 = 0$ in the case initial emissions $E_1 > 0$.[35]

C Robustness analysis for the shares of CO2 and CH4 over total emissions, and the β.

The β coefficient is calibrated by taking the elasticities of output with respect to CO2 and CH4 estimated in Rodríguez et al. (2018). In the main results, the β is defined as a weighted average of these elasticities, where the weights are the average relevance of the type of emission over the

[35]The calculations to find a pseudo-analytical solution of this second optimization problem are available upon request to the author.

national total emissions in the period 1990-2013 (the first and fourth columns in Table A1).[36] The table also shows the share of CO2 and CH4 over the national total emissions in 2013 and 2020, and the consequent βs alongside the main calibration used in the paper (seventh column). Except for some few countries, such as Luxembourg, New Zealand and Turkey, these percentages have been generally stable over time. For Netherlands, for which data are missing we take the average across countries to calibrate β.

country	CO2 av_{90-13}	$CO2_{13}$	$CO2_{20}$	CH4 av_{90-13}	$CH4_{13}$	$CH4_{20}$	β	β_{13}	β_{20}
Australia	0.5987	0.696	0.6689	0.2518	0.2113	0.2119	0.0666	0.0763	0.0734
Austria	0.6814	0.7977	0.7993	0.2585	0.1532	0.1477	0.022	0.013	0.0126
Belgium	0.7957	0.7785	0.7766	0.1664	0.1512	0.1543	0.0738	0.0712	0.0714
Canada	0.8355	0.8033	0.8136	0.1238	0.1463	0.1276	0.0355	0.0339	0.0345
Czech Republic	0.6906	0.8183	0.8119	0.2727	0.1266	0.1284	0.1649	0.1547	0.154
Denmark	0.7012	0.7475	0.6712	0.2148	0.1673	0.2115	0.0402	0.0381	0.0389
Estonia	0.7654	0.7888	0.6709	0.1662	0.1567	0.2346	0.0767	0.0787	0.0694
Finland	0.7129	0.6533	0.6499	0.2146	0.2778	0.2662	0.033	0.0336	0.033
France	0.5869	0.7348	0.6975	0.3519	0.1793	0.2046	0.0643	0.0607	0.0602
Germany	0.7722	0.8755	0.8625	0.1876	0.0828	0.091	0.0925	0.0916	0.0912
Greece	0.6181	0.7949	0.7188	0.3046	0.1537	0.212	0.079	0.0959	0.0885
Hungary	0.5969	0.6895	0.7193	0.339	0.1837	0.1487	0.116	0.1163	0.1177
Iceland	0.6837	0.7975	0.7823	0.1871	0.1279	0.1382	0.0316	0.0292	0.0297
Ireland	0.2489	0.5019	0.4661	0.6581	0.3439	0.3728	0.0635	0.0532	0.0535
Italy	0.6211	0.8314	0.8141	0.311	0.1275	0.14	0.101	0.0904	0.0909
Japan	0.7832	0.9591	0.9545	0.1671	0.0254	0.0281	0.0804	0.066	0.0662
Latvia	0.645	0.6374	0.682	0.2708	0.2203	0.1851	0.0863	0.0867	0.0942
Lithuania	0.6629	0.6322	0.6112	0.2405	0.1734	0.1649	0.1314	0.1142	0.1098
Luxembourg	0.4286	0.9153	0.9007	0.4321	0.0549	0.0656	0.133	0.0281	0.0311
Netherlands							0.0768	0.0707	0.0707
New Zealand	0.5691	0.1679	0.2747	0.3062	0.6125	0.5229	0.034	0.068	0.058
Norway	0.6711	0.6221	0.6018	0.24	0.3249	0.3462	0.0274	0.0306	0.0312
Poland	0.6383	0.7348	0.7458	0.3107	0.1978	0.1825	0.0147	0.0169	0.0172
Portugal	0.5221	0.7616	0.7307	0.4049	0.1832	0.21	0.0553	0.0534	0.0535
Slovak Republic	0.7578	0.7818	0.751	0.2013	0.1482	0.1762	0.1019	0.0985	0.0984
Slovenia	0.6298	0.7948	0.7717	0.3189	0.1692	0.1828	0.1085	0.111	0.1098
Spain	0.6414	0.7801	0.7393	0.2793	0.1532	0.1822	0.0715	0.0581	0.0608
Sweden	0.7737	0.7199	0.7314	0.1605	0.1734	0.1789	0.0661	0.0627	0.0638
Switzerland	0.6796	0.8374	0.8067	0.259	0.1168	0.1397	0.0888	0.0799	0.0809
Turkey	0.4585	0.7132	0.7172	0.4248	0.1884	0.1849	0.1096	0.1705	0.1714
United Kingdom	0.765	0.8531	0.8072	0.2007	0.096	0.1229	0.0733	0.0765	0.0736
United States	0.8437	0.8461	0.835	0.1185	0.1109	0.1215	0.0558	0.0558	0.0553

Table A1: Share of CO2 and CH4 over total national emissions.

D Additional results

As shown in Table A2, with an higher interest rate of 3%, additional countries' debt levels, in particular of France, Latvia, Lithuania and Spain, become unsustainable when taking into account transition's costs.

[36]Data source: ourworldindata.com

Country	Green b_M			Max b_0^M st cb			Green d_M			d_0^{M*} st cb			D/Y
	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	
Australia	379.41	405.54	295.02	296.14	315.68	272.84	443.5	474.32	345.05	346.17	369.21	319.11	57.8
Austria	112.42	99.69	92.86	108.95	96.8	93.15	132.36	117.39	109.35	128.27	114	109.7	83.2
Belgium	244.35	215.77	201.14	219.6	195.23	187.29	286.88	253.47	236.28	257.82	229.34	220.01	112.8
Canada	362.02	344.02	312.08	326.62	310.8	294.27	426.31	405.56	367.9	384.63	366.4	346.91	117.8
Czech Republic	81.22	55.83	51.32	63.86	45.97	43.93	96.44	66.58	61.21	75.84	54.82	52.39	37.7
Denmark	288.7	272.65	260.55	275.22	260.37	253.62	341.13	322.39	308.08	325.21	307.86	299.89	42.1
Estonia	81.33	73.4	64.68	74.42	67.56	63.06	100.73	91.42	80.56	92.17	84.15	78.55	19
Finland	182.63	171.62	164.3	176.75	166.35	162.39	219.77	206.63	197.81	212.7	200.28	195.52	69
France	123.3	107.79	101.71	112.61	99.08	95.74	144.69	126.57	119.43	132.16	116.34	112.41	115.2
Germany	123.45	100.2	95.91	109.75	90.38	88.17	145.99	118.51	113.43	129.79	106.89	104.28	68.7
Greece	66.49	61.46	60.18	62.38	57.93	57.3	80.11	74.44	72.89	75.16	70.15	69.4	211.9
Hungary	208.94	167.1	158.99	181.5	148.02	144.04	248.07	199.08	189.41	215.49	176.34	171.6	80
Ireland	261.98	243.02	200.53	237.99	221.52	199.32	317.15	294.86	243.31	288.1	268.78	241.84	58.4
Italy	153.97	132.54	129.92	137.42	119.66	118.34	181.14	156.36	153.27	161.67	141.16	139.61	155.3
Japan	133.67	114.39	111.47	118.86	102.68	101.19	157.61	135.12	131.67	140.15	121.29	119.53	259
Latvia	40.18	36.89	32.47	36.43	33.64	31.38	50.08	46.06	40.53	45.4	41.99	39.17	43.3
Lithuania	51.64	40.03	34.78	43.32	34.43	31.91	63.23	49.52	43.03	53.05	42.6	39.48	46.6
Luxembourg	179.96	149.8	129.34	147.82	125.27	115.28	215.35	179.61	155.07	176.9	150.2	138.21	24.8
Netherlands	177.75	155.54	145.12	160.17	141.23	135.65	210.11	183.81	171.5	189.33	166.9	160.31	52.8
New Zealand	248.88	252.87	227.3	230.27	233.86	220.59	294.34	299.2	268.94	272.32	276.71	261.01	43.1
Norway	938.91	801.95	715.5	903.94	774.46	730.3	1103.04	943.43	841.73	1061.96	911.1	859.14	46.8
Poland	146.53	143.49	121.27	143.22	140.28	133.38	174.84	171.17	144.67	170.89	167.34	159.11	57.4
Portugal	87.69	79.95	76.32	82.07	75.11	73.13	103.53	94.66	90.37	96.89	88.93	86.58	135.2
Slovak Republic	11.21	9.45	7.79	9.47	8.08	7.23	13.42	11.33	9.35	11.33	9.69	8.67	59.7
Slovenia	65.82	57.8	53.63	57.81	51.31	49.23	78.66	69.38	64.37	69.08	61.58	59.08	79.8
Spain	109.71	105.18	98.26	100.32	96.4	92.68	129.67	124.62	116.42	118.58	114.21	109.81	120
Sweden	188.08	168.97	158.6	173.36	156.57	151	223.46	200.93	188.6	205.97	186.18	179.56	39.6
Switzerland	103.63	96	90.88	95.72	89.12	86.33	121.88	112.89	106.87	112.58	104.81	101.52	42.4
Turkey	157.64	179.7	158.14	137.35	154.9	144.32	190.41	219.42	193.1	165.91	189.13	176.22	39.5
United Kingdom	235.86	199.67	184.76	215.1	183.74	175.58	277.95	235.6	218.01	253.48	216.8	207.17	102.6
United States	160.05	147.49	132.06	136.16	125.89	118.03	188.28	173.68	155.51	160.18	148.24	138.99	134.2

Table A2: Debt sustainability in advanced countries: in the green economy, and under the carbon budget at the beginning of the transition. $r = 3\%$. b_M and d_M: respectively, the maximum sustainable borrowing and the debt limit in the green economy. Max b_0^M st cb and d_0^{M*} st cb: respectively, the maximum current "maximum sustainable borrowing" consistent with the carbon budget and the corresponding debt limit. In red, countries that have general debt sustainability problems (D/Y$>d_M$); in brown, countries for which this issue appears only in the context of performing the transition (D/Y$>d_0^{M*}$ st cb).

Country	Green b_M			Max b_0^M st cb			Green d_M			d_0^{M*} st cb			D/Y
	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	(1)	(2)-*chr*	(3)	(1)	(2)	(3)	
Australia	∞	∞	1549.25	∞	∞	2322.6	∞	∞	1715.6	∞	∞	2571.98	57.8
Austria	191.67	158.91	143.03	184.23	153.17	145.94	213.66	177.18	159.48	205.36	170.78	162.72	83.2
Belgium	431.19	353.06	317.3	376.23	311.19	291.53	479.3	392.68	352.91	418.21	346.11	324.24	112.8
Canada	642.4	590.51	506.29	571.39	526.3	488.83	716.25	659.11	565.1	637.08	587.44	545.61	117.8
Czech Republic	149.58	83.42	74.21	109.56	65.38	61.16	168.17	94.19	83.79	123.17	73.83	69.06	37.7
Denmark	423.04	391.1	367.89	398.75	369.49	356.29	473.27	437.84	411.85	446.1	413.65	398.86	42.1
Estonia	118	102.74	87.17	105.71	92.76	84.62	138.36	121.16	102.8	123.95	109.39	99.79	19
Finland	245.13	226.66	214.69	235.52	218.19	211.56	279.3	258.37	244.73	268.34	248.72	241.16	69
France	209.9	170.41	156.41	187.11	153.32	145.49	233.23	189.44	173.88	207.9	170.45	161.74	115.2
Germany	191.64	142.91	134.76	165.38	125.78	121.48	214.58	160.03	150.91	185.17	140.85	136.03	68.7
Greece	84.71	77.1	75.2	78.34	71.71	70.73	96.63	88.41	86.23	89.37	82.22	81.11	211.9
Hungary	315.04	232.08	217.48	264.2	199.77	192.45	354.13	261.79	245.31	296.98	225.34	217.08	80
Ireland	492.45	432.4	318.66	435.15	384.17	333.89	564.43	496.73	366.07	498.75	441.32	383.56	58.4
Italy	221	181.19	176.58	191.95	159.74	157.35	246.17	202.39	197.23	213.81	178.42	175.76	155.3
Japan	193.83	157.4	152.21	168.58	138.6	135.87	216.39	176.04	170.23	188.2	155.01	151.96	259
Latvia	57.78	51.54	43.66	51.18	45.98	41.9	68.19	60.91	51.6	60.4	54.35	49.53	43.3
Lithuania	84.7	58.39	48.32	67.79	48.41	43.5	98.2	68.4	56.6	78.59	56.7	50.95	46.6
Luxembourg	349.05	254.85	203.09	269.78	202.58	177.21	395.49	289.3	230.54	305.67	229.96	201.17	24.8
Netherlands	291.26	238.44	215.94	255.38	211.35	199.1	325.97	266.79	241.61	285.81	236.48	222.77	52.8
New Zealand	418.2	428.96	363.31	382.1	391.7	363.41	468.26	480.55	407.01	427.84	438.81	407.11	43.1
Norway	2036.03	1507.44	1240.65	1933	1438.56	1334.99	2264.71	1679.05	1381.89	2150.1	1602.33	1486.97	46.8
Poland	283.91	273.27	205.41	275.67	265.44	243.58	320.75	308.66	232.01	311.44	299.81	275.12	57.4
Portugal	129.35	113.94	107.08	119.15	105.5	101.65	144.59	127.73	120.04	133.19	118.27	113.95	135.2
Slovak Republic	26.19	18.54	13.29	20.9	15.12	12.58	29.69	21.05	15.09	23.69	17.17	14.28	59.7
Slovenia	98.08	82.03	74.27	83.42	70.77	66.82	110.97	93.21	84.39	94.39	80.42	75.93	79.8
Spain	167.31	157.52	143.22	149.65	141.33	133.5	187.24	176.71	160.66	167.47	158.54	149.76	120
Sweden	283.7	244.25	224.19	256.33	222.28	211.27	319.13	274.99	252.41	288.35	250.25	237.86	39.6
Switzerland	165.46	147.72	136.51	148.3	133.35	127.09	184.25	164.47	151.99	165.14	148.47	141.51	42.4
Turkey	222.59	266.3	223.54	188.53	222.32	201.06	254.56	307.86	258.42	215.61	257.01	232.44	39.5
United Kingdom	414.21	318.28	283.73	366.91	285.72	266.58	462.15	355.58	316.98	409.38	319.2	297.82	102.6
United States	311.7	269.39	224.1	258.53	224.62	202.92	347.17	300.34	249.85	287.95	250.43	226.23	134.2

Table A3: Debt sustainability in advanced countries: in the green economy, and under the carbon budget at the beginning of the transition. $r = 1.88\%$. b_M and d_M: respectively, the maximum sustainable borrowing and the debt limit in the green economy. Max b_0^M st cb and d_0^{M*} st cb: respectively, the maximum current "maximum sustainable borrowing" consistent with the carbon budget and the corresponding debt limit. In red, countries that have general debt sustainability problems (D/Y$>d_M$); in brown, countries for which this issue appears only in the context of performing the transition (D/Y$>d_0^{M*}$ st cb).

D.1 Sensitivity analysis for the CCS parameter c

Figure A1 shows some sensitivity analysis for the parameter c in the growth rate function, which represents the percentage of emissions that could be recapture in each 5-year period in the green economy, after the end of the transition. The higher value $c = 0.01$ is the one adopted in the rest of the paper. Lower values of c imply: an almost identical but slightly slower transition path, lower detrended GDP levels, a slower convergence to the stationary MSD, and a generally unchanged probability of default.

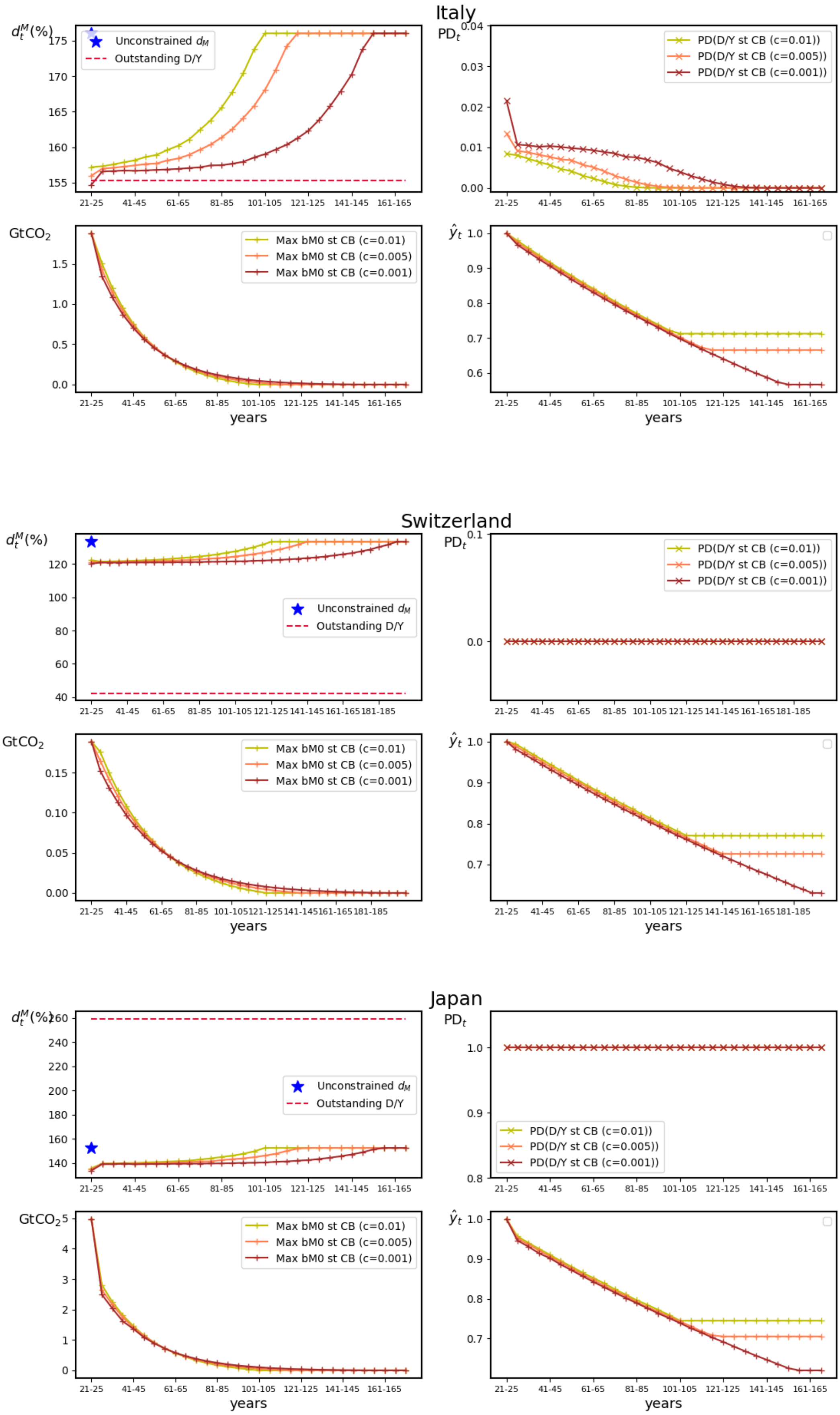

Figure A1: Sensitivity analysis for the CCS parameter c. $r = 2.44\%$. Plots for the fiscal limit d_t^M, the annual probability of default PD_t (1.0 is 100%) of 2020 debt-to-GDP ratios (outstanding D/Y, assumed to remain constant), the emissions' path expressed in $GtCO_2$ maximizing the current maximum sustainable borrowing b_0^M under the carbon budget, and the normalized detrended GDP $\hat{y}_t$.

D.2 Trade-off shift in debt sustainability with climate damages

Table A4 reports, for each country, the period when the national fiscal limit under a successful global transition overcomes the national fiscal limit under a business-as-usual scenario with global emissions growing at 1.126%.

	Shift period
Australia	2021-25
Austria	2021-25
Belgium	2026-30
Canada	2026-30
Czech Republic	2076-80
Denmark	2026-30
Estonia	2061-65
Finland	2041-45
France	2026-30
Germany	2066-70
Greece	2071-75
Hungary	2076-80
Ireland	2026-30
Italy	2071-75
Japan	2061-65
Latvia	2066-70
Lithuania	2071-75
Luxembourg	2051-55
Netherlands	2041-45
New Zealand	2026-30
Norway	2021-25
Poland	2021-25
Portugal	2041-45
Slovak Republic	2026-30
Slovenia	2071-75
Spain	2046-50
Sweden	2051-55
Switzerland	2046-50
United Kingdom	2031-35
United States	2026-30

Table A4: Period when the national fiscal limit under a successful global transition overcomes the national fiscal limit under a business-as-usual scenario with global emissions growing at 1.126%. $r = 2.44\%$, PL.