Foreign Exchange Intervention with UIP and CIP Deviations: The Case of Small Safe Haven Economies [1]

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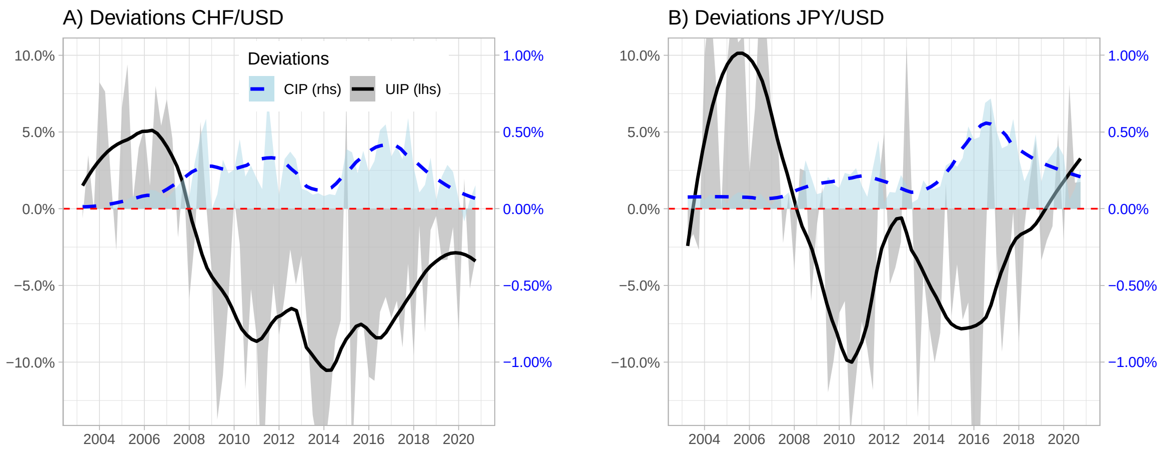
Abstract

We examine the opportunity cost of foreign exchange (FX) intervention when both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through constrained international financial intermediaries. Deviations from CIP come from limited arbitrage or through a convenience yield, while UIP deviations are also affected by risk. We show that the sign of CIP and UIP deviations may differ for safe haven countries. We find that there may be a benefit, rather than a cost, of FX reserves if international intermediaries value more the safe haven properties of a currency that domestic households. We show that this has been the case for the Swiss franc and the Japanese Yen. We examine the optimal policy of a constrained central bank planner in this context.

1 Introduction

A vast literature examines the optimal level of central bank international reserves in emerging markets (see Bianchi and Lorenzoni (2022) for a recent survey). A recurrent feature is that the accumulation of reserves bears an opportunity cost arising from a return differential between the liabilities and the assets of the central bank. In the recent literature on optimal Foreign Exchange (FX) interventions, some authors focus on Uncovered Interest Rate Parity (UIP) wedges (see Basu et al., 2020; Maggiori, 2021; Itskhoki and Mukhin, 2022). In contrast, other researcher argue that what matters are deviations from Covered Interest Rate Parity (CIP) (e.g., Amador et al., 2020; Fanelli and Straub, 2021). This distinction between CIP and UIP appears particularly relevant for safe haven countries, since CIP and UIP deviations may be of different signs. Figure 1 shows CIP and UIP deviations for Switzerland and Japan.[1] They are computed from the perspective of international investors and UIP deviations are estimated using survey expectation data. They show that since 2008, both countries have experienced positive CIP deviations and negative UIP deviations. The latter implies a negative excess return, which is typical of safe haven currencies.

Figure 1: UIP and CIP Deviations



Notes: This figure shows the UIP and CIP deviations in percentage points as defined in (5) and (4), taking the USD as the foreign currency and considering a 3-month horizon. Panel A) and B) consider the CHF and the JPY as the domestic currencies. UIP deviations are computed using monthly data from Datastream for the 3-month Libor rates and from Consensus Economics for the exchange rate forecasts and the spot exchange rates. The CIP deviations are monthly averages of daily observations and are computed using 3-month Libor rates, spot exchange rates and forward rates with a 3-month maturity from Datastream. All returns are annualised.

To clarify these issues, we develop a model where both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through international financial intermediaries as in Gabaix and Maggiori (2015). The

[1] See Appendix A for data description. Interestingly, Rime et al. (2022) show that CIP deviations for the CHF and the JPY with respect to the USD have been the most profitable for financial institutions.

structure of the model is similar to those in recent papers examining the role of international reserves (see Cavallino, 2019; Amador et al., 2020; Fanelli and Straub, 2021; Basu et al., 2020; Maggiori, 2021), but financial intermediaries are risk averse. Fang and Liu (2021) propose a related framework in a two-country model, but they focus on the US dollar and do not analyze FX interventions. The international financial intermediaries are the marginal investors and determine both UIP and CIP deviations through their unhedged and hedged portfolio choices. These deviations typically do not coincide and may even be of different sign.

In this environment, we examine the opportunity costs of FX intervention in terms of welfare. We identify the conditions under which CIP or UIP deviations matter for this cost. We find that there may be no opportunity cost, and that there may even be a benefit, of FX intervention in a safe haven country, even if that country faces a positive CIP deviation. We examine the implications for optimal FX intervention in these cases.

The presence of systematic deviations from CIP in the wake of the Global Financial Crisis is a major development in international finance (see Du and Schreger (2022) or Cerutti et al. (2021) for recent surveys). The theoretical literature has provided explanations for CIP deviations, but has devoted limited attention to the link between CIP and UIP deviation. Several papers analyzing interest rate differentials assume complete markets so that either there is no UIP deviations or CIP deviations are equal to UIP deviations. This is not consistent with the data.

The recent literature has followed two main approaches to explain interest rate differentials. First, there may be financial frictions that limit arbitrage, e.g., by assuming constrained financial intermediaries. The other approach is to assume differences in convenience yields. The two approaches are present in our model and determine deviations from CIP. But we do not assume complete markets, so that UIP deviations differ from CIP deviations. A basic result from this analysis is the following relationship between UIP and CIP deviations:

$$devUIP_t = devCIP_t - \frac{cov_t(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} \tag{1}$$

where m^*_{t+1} is the stochastic discount factor (SDF) of financial intermediaries and X^*_{t+1} is the foreign currency excess return from the international intermediary perspective. If the small open economy is a safe haven country, we have $cov_t(m^*_{t+1}, X^*_{t+1}) > 0$, i.e., the safe haven currency yields a higher return in bad times. Therefore, it is possible to have a positive CIP deviation with a negative UIP deviation.

We derive equation (1) in a simple two-period small economy model with two assumptions that differ from most of the literature. First, international financial intermediaries face exchange rate risk. This risk could be hedged on the forward market, but it is not optimal to fully hedge a safe haven currency. The other assumption is that the financial constraint applies to the whole foreign exchange investment of financial intermediaries, whether it is hedged or not.[2]

We analyze the welfare cost of accumulating international reserves in this framework. We show that if domestic households attribute less value to the safe haven properties of their currency than international financial intermediaries (i.e., the domestic SDF is less correlated to the excess return than for financial intermediaries), then FX reserves may have a benefit, not a cost. We examine this issue empirically by estimating the SDF of financial intermediaries following He et al. (2017). When considering the CHF and JPY with respect to the USD, we find that it is indeed the case that the SDF of financial intermediaries is more correlated with excess returns than the SDF of domestic households.

We examine the implications of this analysis for optimal FX intervention, by modeling the central bank as a constrained planner. We determine the various factors influencing optimal policy decisions, focusing on various types of FX interventions. We show that the central bank incentives are similar for sterilized interventions and unsterilized interventions at the Zero Lower Bound (ZLB).

For a more specific analysis, we consider a linearized version of the model where the distribution of shocks is such that the domestic currency is perceived as a safe haven by international investors. This allows us to derive precise expressions for the cost of FX intervention and CIP and UIP deviations and examine the impact of various parameters on these variables and on optimal FX interventions. For example, an increase in global risk leads to more beneficial FX interventions, larger positive CIP deviations and larger negative UIP deviations.

This paper complements the literature on the opportunity cost of FX reserves. There is a long tradition of estimating the cost and benefits of accumulating FX reserves (e.g., Jeanne and Rancière, 2011). Adler and Mano (2021) estimate the quasi-fiscal cost of interventions for 73 countries using UIP deviations. Using survey expectations or assuming a random walk for the nominal exchange rate, they find that the ex ante cost of intervention is negative for Japan and Switzerland in the period 2002-2013, while it is positive for most other countries.[3] In this paper, we examine the cost of intervention from the welfare point of view, and find that it is also negative for Japan and Switzerland, but that it is not equal to UIP deviations in general.

By focusing on countries like Switzerland or Japan, this paper provides a different perspective on safe haven economies. A growing literature has been analyzing the special role of the US dollar as a reserve currency. In particular, several papers have focused on the role of convenience yields in generating currency movements and expected excess

[2] In contrast, in Itskhoki and Mukhin (2021), intermediation frictions generate UIP deviation without CIP deviations. This is because the intermediation frictions originate in the intermediaries' risk aversion.

[3] In the case of developing or emerging economies, the opportunity cost may based on the country's borrowing cost, which implies a credit risk (e.g., Edwards, 1985). However, Yeyati and Gómez (2022) argue that when reserves are used for leaning-against-the-wind interventions, it is more appropriate to use UIP deviations.

returns (e.g., Jiang et al., 2021b,a; Valchev, 2020; Kekre and Lenel, 2021; Bianchi et al., 2022). We show that convenience yields are not the sole determinant for exchange rate movements and UIP deviations in safe haven economies.

The rest of the paper is organized as follows. Section 2 describes the model and the decentralized equilibrium. Section 3 analyzes the opportunity cost of reserves in this context. Section 4 discusses optimal FX intervention and Section 5 proposes linearized model of a safe haven country. Section 6 concludes.

2 The Model

This section presents a two-period model of a small open economy facing international financial intermediaries in the spirit of Gabaix and Maggiori (2015). These intermediaries buy domestic bonds and are the marginal investors both in the spot and the forward market.[4] They are risk averse so that there is a difference between their covered and uncovered positions. After presenting the financial intermediaries, we describe the households, the government and the central bank, as well as the equilibrium in the asset markets.

We call the foreign currency the dollar and assume that the foreign interest rate i_t^* is given. Purchasing power parity (PPP) is assumed to hold and the price of goods in dollars is normalized to one. S_t is the spot price of dollars in terms of domestic currency and F_t is the forward rate.

2.1 International Financial Intermediaries

International financial intermediaries value their expected profits with their stochastic discount factor m_{t+1}^*, which will be further described below. They typically invest in domestic bonds, but at the ZLB they may also hold domestic money.[5] Denote b_t^{H*} and h_t^{H*} their net positions in domestic bonds and money, expressed in dollars, and a_t^{H*} their total position: $a_t^{H*} = b_t^{H*} + h_t^{H*}$. Financial intermediaries have a zero net position and fund their investments in domestic assets in dollars. We also assume that they can use forward contracts in quantities f_t^* and that they are the only players in the forward market.[6]

Moreover, financial intermediaries may value the liquidity of dollar assets. We assume that investors have operating costs that are increasing in non-dollar assets holdings a_t^{H*} and that it is a linear function: $\chi \cdot a_t^{H*}$, with $\chi \geq 0$. Their objective function is in dollars (and equivalently, in goods terms since the dollar price is constant):

[4]Since the objective of the model is to highlight the consequences of the differences between CIP and UIP deviations, we abstract from various interesting factors affecting the dynamics of spot and forward rates that are considered in the recent literature.

[5]For notational convenience, we assume that financial intermediaries only potentially hold money at time t so that $h_{t-1}^{H*} = h_{t+1}^{H*} = 0$.

[6]These assumptions are similar to Fang and Liu (2021). They consider a two-country model with financial intermediaries in both countries, but only the Home country arbitrages CIP deviations.

$$V_t^* = E_t \left\{ m_{t+1}^* \left[a_t^{H*} \left((1+i_t)\frac{S_t}{S_{t+1}} - (1+i_t^*) \right) - f_t^* \left(\frac{1}{S_{t+1}} - \frac{1}{F_t} \right) \right] \right\} - \chi a_t^{H*}$$

a_t^{H*} represents the total funds invested in the country, covered or uncovered. $f_t^*/(1+i_t)S_t$ is the covered amount, and $a_t^{H*} - f_t^*/(1+i_t)S_t$ is the uncovered amount. Here we take into account the fact that when $i_t > 0$, $a_t^{H*} = b_t^{H*}$ as money is dominated by bonds.

To capture the role of financial intermediaries, we assume, as Gabaix and Maggiori (2015), that intermediaries can divert a fraction Γa_t^{H*} of the total invested funds, after the investment decisions are taken, but before shocks are realized. This yields a participation constraint for investors:

$$V_t^* \geq \Gamma (a_t^{H*})^2$$

Consider first the FOC w/f_t^*:

$$E_t \left\{ m_{t+1}^* \left(\frac{1}{S_{t+1}} - \frac{1}{F_t} \right) \right\} = 0 \tag{2}$$

The forward market is effectively frictionless, since it does not involve a transfer of funds ex ante. This implies a relationship between CIP and UIP deviations:

$$E_t(m_{t+1}^* Z_{t+1}^*) = E_t(m_{t+1}^* X_{t+1}^*) \tag{3}$$

where Z_{t+1}^* is the excess return hedged by a forward contract or the CIP deviation:

$$Z_{t+1}^* \equiv (1+i_t)\frac{S_t}{F_t} - (1+i_t^*) \tag{4}$$

and X_{t+1}^* is the domestic currency excess return, expressed in foreign currency:

$$X_{t+1}^* \equiv (1+i_t)\frac{S_t}{S_{t+1}} - (1+i_t^*) \tag{5}$$

Equation (3) can be rewritten as an equivalent of Equation (1):

$$E_t X_{t+1}^* = Z_{t+1}^* - \frac{cov_t(m_{t+1}^*, X_{t+1}^*)}{E_t m_{t+1}^*} \tag{6}$$

Covered and uncovered carry trades yield the same returns in expectation, up to a covariance term, because intermediaries are risk-averse.

Using Equation (2), the participation constraint can be simplified as follows:

$$E_t \left(m_{t+1}^* a_t^{H*} X_{t+1}^* \right) - \chi a_t^{H*} \geq \Gamma (a_t^{H*})^2 \tag{7}$$

If the participation constraint is binding, we have:

$$E_t \left(m_{t+1}^* X_{t+1}^* \right) = \Gamma a_t^{H*} + \chi \tag{8}$$

This, along with Equations (3) and (6), implies

$$Z^*_{t+1} = \frac{\Gamma a_t^{H*} + \chi}{E_t m^*_{t+1}} \tag{9}$$

and

$$E_t X^*_{t+1} = \frac{\Gamma a_t^{H*} + \chi}{E_t m^*_{t+1}} - \frac{cov_t(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} \tag{10}$$

The term $\Gamma a_t^{H*} + \chi$ in Equations (9) and (10) shows the impact of limited arbitrage and of the convenience yield on CIP and UIP deviations. Indeed, the intermediation frictions, which bear on both covered and uncovered intermediated funds, affect both CIP and UIP deviations.[7]

2.2 Domestic Households

Households' real consumption is c_t and they receive a real endowment y_t. They can hold money, H_t^H, domestic bonds B_t^H (both expressed in domestic currency), and foreign bonds b_t^F (expressed in foreign currency). Domestic bonds and money are perfect substitutes at the ZLB. We assume that households do not use the forward exchange market.

Since PPP holds, their budget constraint can be written as:

$$c_t = y_t - h_t^H - b_t^H - b_t^F + t_t \tag{11}$$

$$c_{t+1} = y_{t+1} + \frac{1}{1+\pi_t} h_t^H - h_{t+1}^H + \frac{1+i_t}{1+\pi_t} b_t^H + (1+i_t^*) b_t^F + t_{t+1} \tag{12}$$

where $b_t^H = B_t^H / P_t$ and $h_t^H = H_t^H / P_t$ are the real levels of domestic bonds and money holdings and $t_t = T_t / P_t$ and $t_{t+1} = T_{t+1} / P_{t+1}$ are real transfers.

Potentially, households face a cash-in-advance constraint in t and $t+1$:

$$h_t^H \geq y_t, \;\; h_{t+1}^H \geq y_{t+1} \tag{13}$$

They also face short-selling constraints :

$$b_t^H \geq 0, \qquad\qquad b_t^F \geq 0 \tag{14}$$

Their utility function is:

$$U(c_t) + \beta E_t U(c_{t+1}) \tag{15}$$

Domestic households choose bonds and money holdings to maximize (15) subject to constraints (11) to (14). Using the assumption of PPP ($P_t = S_t$), the first-order conditions associated with bond portfolio choices are:

$$U'(c_t) - E_t\left(\beta U'(c_{t+1})(1+i_t^*)\right) \qquad -\lambda^F = 0 \tag{16}$$

$$E_t\left(\beta U'(c_{t+1})\left[(1+i_t^*) - (1+i_t)\frac{S_t}{S_{t+1}}\right]\right) \qquad +\lambda^F - \lambda^H = 0 \tag{17}$$

[7] Fanelli and Straub (2021) discuss a similar setup with frictions in intermediation and frictionless forward markets in an extension of their model. They find that, in that case, intermediation frictions generate both UIP and CIP deviations.

where λ^H and λ^F are the multipliers associated with the short-selling constraints (14). Equation (16) shows that the borrowing constraints affect intertemporal allocations. Equation (17) shows that they prevent households from reaching their optimal portfolio allocation between domestic and foreign currency bonds.

2.3 The Government

At time t the nominal government issues debt B_t^G (expressed in domestic currency) and transfers the funds to households:

$$B_t^G = T_t \tag{18}$$

At $t+1$, the government receives the central bank profits, Π_{t+1}^{CB} and repays its debt :

$$T_{t+1} = -(1+i_t)B_t^G + \Pi_{t+1}^{CB} \tag{19}$$

We assume that the government is passive and that the level of real debt $b_t^G = B_t^G/P_t$ is exogenous.[8]

2.4 The Central Bank

In period t, the central bank issues money H_t, and buys domestic and foreign bonds B_t^{CB} and b_t^{CBF} (expressed respectively in domestic and foreign currency). In period $t+1$, the central bank issues new money $H_{t+1} - H_t$ and distributes its profits Π_{t+1}^{CB} to the government. The central bank's budget constraint write then as follows:

$$S_t b_t^{CBF} + B_t^{CB} = H_t \tag{20}$$

$$\Pi_{t+1}^{CB} = (1+i_t^*)S_{t+1}b_t^{CBF} + (1+i_t)B_t^{CB} + H_{t+1} - H_t \tag{21}$$

In period t, the central bank has as instruments the nominal interest rate i_t, the total money supply H_t and the choice of foreign reserves b_t^{CBF} and domestic bonds B_t^{CB}. However, the interest rate cannot be negative, so the central bank loses the interest rate instrument when it hits this zero lower bound (ZLB).

In period $t+1$, we assume that the supply of money H_{t+1} is exogenous: $H_{t+1} = \bar{H}e^h$ where h is an exogenous shock. It represents variations in the net money supply to households due for instance to liquidity trading, or money velocity shocks.

From the budget constraint (20), there are two ways to change the level of reserves b_t^{CBF}. First, through a *sterilized* intervention where an increase in b_t^{CBF} is compensated by a decline in B_t^{CB}. Second, through an *unsterilized* intervention where an increase in b_t^{CBF} is associated with an expansion in H_t. Another possibility would be to allow the central bank to transfer funds to households in both periods. In that case, an increase in b_t^{CBF} could be implemented by introducing transfers from the central bank in period t. We do not examine this fiscal foreign exchange intervention.

[8]Alternatively, we could assume that it is the nominal debt level B_t^G that is exogenous. However, there would be an incentive for the central bank to alter the real debt level by moving the exchange rate.

2.5 Gross and Net Foreign Liabilities

For the rest of our analysis, it is convenient to focus on the Home country's net and gross foreign liabilities. Moreover, since PPP holds, we replace P_t with S_t. Gross foreign liabilities are domestic bonds and money not held domestically. They are given by

$$gfl_t = \left(b_t^G - \frac{B_t^{CB}}{S_t} - b_t^H\right) + \left(\frac{H_t}{S_t} - h_t^H\right) = b_t^G + b_t^{CBF} - b_t^H - h_t^H \tag{22}$$

The first term of the first equality corresponds to the foreign holdings of domestic bonds. The second term corresponds to the foreign holdings of domestic money. The second equality is obtained by using the central bank's budget constraint. It shows that FX interventions directly affects the supply of domestic bonds to foreigners. Indeed, the central bank can increase its holding of foreign currency assets only through a balance-sheet expansion and hence through an increase in its domestic currency liabilities, whether it is bonds or money.

Net foreign liabilities are given by

$$nfl_t = gfl_t - (b_t^F + b_t^{CBF}) = b_t^G - b_t^H - b_t^F - h_t^H \tag{23}$$

where $b_t^F + b_t^{CBF}$ are the domestic holding of foreign assets. The second equality is obtained by replacing b_t^{CBF} with $(H_t - B_t^{CB})/S_t$, using the central bank budget constraint, and replacing gfl_t using (22).

It is useful to notice that FX intervention affects gfl_t, but not nfl_t: an increase in b_t^{CBF} will increase gfl one-for-one, through an increase in H_t (unsterilized intervention) or a decline in B^{CB} (sterilized intervention), while in nfl the changes in b_t^{CBF} are offset either by changes in B_t^{CB} or by changes in H.

2.6 Equilibrium in Asset Markets

The amount of domestic debt held by international intermediaries is equal to the net domestic supply: $b_t^{H*} = b_t^G - B_t^{CB}/S_t - b_t^H$. Similarly, foreign money holdings are equal to the net domestic supply: $h_t^{H*} = H_t/S_t - h_t^H$. In equilibrium, gross foreign liabilities are equal to the bonds and money held by foreigners: $gfl_t = a_t^{H*} = b_t^{H*} + h_t^{H*}$. From (10), this implies

$$\Gamma gfl_t + \chi = E_t(X_{t+1}^*) + \frac{cov_t(m_t^*, X_{t+1}^*)}{E_t m_{t+1}^*} \tag{24}$$

The net supply of domestic liabilities to foreigners, gfl_t, determines the equilibrium expected domestic currency excess return $E_t(X_{t+1}^*)$, which is defined in (5). A higher gfl_t can only be absorbed by the intermediaries if it offers a higher excess return. On the contrary, an increase in $cov_t(m_t^*, X_{t+1}^*)$ leads to a decline in the domestic currency excess return. Intuitively, the increase in covariance makes the domestic bonds more attractive to foreigners and generates an excess demand for domestic bonds. The decline in the domestic currency excess return clears this excess demand.

How does the domestic currency excess return adjust in practice? Consider for instance an increase in the excess return. Since the foreign interest rate i^*_t is exogenous, this implies a higher domestic real interest rate $(1+i_t)E_t(S_t/S_{t+1})$. At $t+1$, equilibrium on the money market yields $H^H_{t+1} = S_{t+1}y_{t+1} = He^h$, which determines S_{t+1}. Therefore, we can treat S_{t+1} as exogenous. As a result, a higher gfl_t increases $(1+i_t)S_t$. Whether it affects i_t or S_t depends on whether the economy is at the ZLB or not, as discussed later.

3 On the Cost of Foreign Reserves

In this section, we derive the utility cost of FX reserves and assess its relation to UIP and CIP deviations. We show that the key determinant of this cost is the covariance between excess returns and the SDF of domestic households on the one hand and the SDF of international financial intermediaries on the other. In the context of safe have currencies, it depends on whether international financial intermediaries value more the safe haven properties than domestic investors. We examine this issue empirically and show that it is the case for Switzerland and Japan.

3.1 Utility Cost of Reserves with UIP and CIP Deviations

After consolidating the household's budget constraints using the equilibrium in the domestic asset markets, and substituting transfers in the household's budget constraint, we obtain the period resource constraints:

$$\begin{aligned} c_t &= y_t + nfl_t \\ c_{t+1} &= y_{t+1} - (1+i^*_t)nfl_t - X^*_{t+1}gfl_t + i_t \tfrac{S_t}{S_{t+1}} \left(\tfrac{H_t}{S_t} - h^H_t \right) \end{aligned} \tag{25}$$

The last term, which represents the economy's seigniorage revenue from the foreign holding of domestic money, can be neglected since we will either have $\frac{H_t}{S_t} = h^H_t$ (if $i_t > 0$) or $i_t = 0$. The intertemporal resource constraint is:

$$(1+i^*_t)c_t + c_{t+1} = (1+i^*_t)y_t + y_{t+1} - X^*_{t+1}gfl_t. \tag{26}$$

Everything else equal, FX interventions affect the economy's intertemporal resources through changes in gfl_t.[9] By holding more foreign reserve b^{CBF}_t, the central bank increases the economy's gross foreign position by issuing more domestic bonds (decreasing B^{CB}_t) or more money if the economy is at the ZLB (increasing $\frac{H_t}{S_t} - h^H_t$).

The last term in (26) gives the monetary cost of holding reserves, since reserves affect gfl_t. It depends on the sign of X^*_{t+1}, which is the marginal cost of holding reserves, evaluated in units of goods. In a safe haven case with $E_tX^*_{t+1} < 0$, there is an expected

[9] If the central bank could make transfers in t, then it could also affect the consumption profile through the economy's net position nfl_t. If the household is constrained, then the central bank could increase the net borrowing of the economy and hence transfer consumption from $t+1$ to t.

gain of holding reserves, i.e., the central bank can exploit the UIP deviation. However, the increase in reserves also increases exchange rate risk, so the question is whether this could increase households' utility. We define the marginal *utility* cost of reserves as follows:

Definition 1 (The marginal utility cost of FX interventions) *The marginal utility cost of FX interventions is the expected product of the UIP deviation X^*_{t+1} and the SDF of domestic households m_{t+1}, divided by the expected discount factor:*

$$UCFX_t = \frac{E_t(m_{t+1}X^*_{t+1})}{E_t(m_{t+1})} \tag{27}$$

where $m_{t+1} = \beta U'(c_t)/U'(c_{t+1})$.

The excess return on domestic bonds X^*_{t+1} is valued using the utility-based stochastic discount factor. It is normalized by the expected discount factor so that it coincides with the monetary cost X^*_{t+1} in the absence of risk. In that sense, $UCFX_t$ can be seen as the certainty-equivalent cost of reserves in monetary terms.

The marginal utility cost of FX interventions can be rewritten as

$$UCFX_t = E_t X^*_{t+1} + \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} \tag{28}$$

The utility cost is composed of the excess return on foreign bonds, minus the risk premium associated with this excess return. Since $E_t X^*_{t+1} < 0$ for safe haven countries, there may be a utility gain.

Substituting $E_t X^*_{t+1}$ using Equation (10), we can rewrite the utility cost of foreign exchange interventions:

$$UCFX_t = \underbrace{\frac{\overbrace{\Gamma gfl_t + \chi}^{devCIP}}{E_t m^*_{t+1}} - \frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}}}_{devUIP} + \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} \tag{29}$$

Equation (29) shows how CIP and UIP deviations affect the utility cost. This can be summarized in the following proposition:

Proposition 1 *Consider the SDF of domestic households, m_{t+1}, and of international financial intermediaries m^*_{t+1} and the excess return in foreign currency, X^*_{t+1}. The utility cost (or benefit) of foreign exchange intervention depends on*

(i) CIP deviations when $cov(m_{t+1}, X^*_{t+1}) = cov(m^*_{t+1}, X^*_{t+1})$.

(ii) UIP deviations when $cov(m_{t+1}, X^*_{t+1}) = 0$.

In fact, the intermediation friction generates two wedges that are relevant for welfare. First, the CIP deviation, which is a riskless excess return. Second, the difference between the foreign and domestic risk premia. If the foreign and domestic agents have the same risk premium, only CIP deviations matter. This is the case in the absence of risk, as in Amador et al. (2020), or when financial intermediaries have the same discount factor as households. In contrast, in the limit case where the domestic agents have negligible risk aversion as compared to financial intermediaries, then the sum of the two wedges is equal to the UIP deviation, and the cost of reserves would be equal to UIP deviations.[10]

However, in general, the sum of the two wedges does not coincide with either the CIP or the UIP deviations. In particular, a safe haven currency may be more desirable for foreign investors as a diversification hedge than for the domestic investors so that $cov(m^*_{t+1}, X^*_{t+1}) > cov(m_{t+1}, X^*_{t+1})$. If the difference is large enough, there may be a utility gain from accumulating reserves, instead of a cost.

3.2 Estimating the Utility Cost for Switzerland and Japan

The theoretical analysis has shown that the utility cost FX of interventions depends crucially on the difference between $cov(m^*_{t+1}, X^*_{t+1})/E_t m^*_{t+1}$ and $cov(m_{t+1}, X^*_{t+1})/E m_{t+1}$ (Equation (29)). In this subsection, we provide estimates of these two terms for Switzerland and Japan. First, Appendix 2 confirms that both countries can be considered as safe haven, in the sense that the excess return on their currencies is positively related to global risk variables.

A key issue is the measurement of stochastic discount factors m_{t+1} and m^*_{t+1}. For domestic households, we simply assume that $m_{t+1} = \beta(c_{t+1}/c_t)^{-\gamma}$, where $1/\gamma$ is the rate of intertemporal substitution. For international financial intermediaries, we follow the literature on intermediary asset pricing (e.g., see He and Krishnamurthy (2011) or Brunnermeier and Sannikov (2014)), and assume that their SDF is proportional to their net worth NW_t:[11]

$$m^*_{t+1} = \beta \left(\frac{NW_{t+1}}{NW_t} \right)^{-\gamma} \tag{30}$$

As in He et al. (2017), we assume that the financial intermediaries' net worth is equal to the aggregate wealth in the economy (denoted by W_t) multiplied by the intermediaries' capital ratio (denoted by η_t). This specification implies that the financial intermediaries' marginal utility of wealth rises when either the aggregate wealth in the economy or the equity capital ratio is low. The first term captures the asset pricing effect of weaker

[10] This is what Itskhoki and Mukhin (2021) implicitly assume. In their linear approximation, they take the level of risk to zero but ensure that the risk premium of the financial intermediaries remains a first order object by rescaling their risk aversion, but not that of the households. This implies that the intermediaries' risk aversion is an order of magnitude higher than that of the households. As a result, it is optimal to eliminate UIP deviations.

[11] Here, we implicitly assume that, on top of the Gabaix-Maggiori constraint on their international arbitrage, financial intermediaries face a borrowing constraint on their overall balance sheet, such that their constraint depends on their net worth, e.g. as in Gertler and Kiyotaki (2010). This gives rise to intermediary asset pricing in the form of Equation (30).

fundamentals, while the second captures the idea that the intermediaries' risk bearing capacity is impaired when the capital ratio is low. As a result, risk aversion increases the marginal value of wealth. Using time-series and cross-sectional asset pricing tests, He et al. (2017) show that this specification captures well the marginal utility of wealth of financial intermediaries, and find supporting evidence that financial intermediaries are indeed marginal investors for a wide class of assets.

In our empirical exercise, we consider two measures of the capital ratio (η_t) and two measures of aggregate wealth (W_t), giving rise to four different possible specifications. For the first capital ratio measure, we consider the equity capital ratio of financial intermediaries (*Primary Dealer* counterparties of the New York Federal Reserve) from He et al. (2017), which we denote by η_{t+1}^{HKM}. The second measure is from Adrian et al. (2014), and is defined as the (inverse of) book leverage of security *Brokers & Dealers*.[12] We denote it as η_{t+1}^{AEM}. For total wealth, we consider a real measure using US GDP (W_t^{GDP}) and a financial measure using the US MSCI Equity Index (W_t^{MSCI}).[13]

As in He et al. (2017), our measure of net worth is obtained by interacting the capital ratio measure with the total wealth measure: $NW_t = \eta_t \times W_t$. To convert net worth into a growth rate (as suggested by (30)), we adopt an approach similar to He et al. (2017). For the capital ratio, we define the intermediary capital risk factor by dividing the residual from a regression of the capital ratio on its lag by the lagged capital ratio. For the financial measure of wealth (W_t^{MSCI}), we compute the excess returns on the equity index, using the 3M US Libor as the risk-free rate. For the real measure of wealth (W_t^{GDP}), we simply compute the growth rate. $\frac{NW_{t+1}}{NW_t}$ is then defined by the interaction of the intermediary capital risk factor and the growth rate measure of total wealth. Appendix B provides additional details about the sources of the data and the construction of the excess returns and the stochastic discount factors.

We consider excess returns using the CHF and the JPY as the domestic currency and the USD as the foreign one. Let us define the log excess returns of going long in the domestic currency from the international investors' perspective:

$$x_{t+1}^* = i_t - i_t^* + s_t - s_{t+1} \tag{31}$$

We use x_{t+1}^* as an approximation of X_{t+1}^*.

Table 1 displays an estimate of $cov(m_{t+1}^*, x_{t+1}^*)/E_t m_{t+1}^*$ and $cov(m_{t+1}, x_{t+1}^*)/Em_{t+1}$ using either the CHF or the JPY as the domestic currency, keeping the USD as the foreign one. We assume that $\beta = 0.99$ and $\gamma = 10$. For each currency, we consider two subsamples (2000M1-2009M12 and 2010M1-2020M2) to highlight potential time-variation in these measures, as suggested by Figure 1. Columns 2 to 5 display the covariance terms from the perspective of financial intermediaries using the capital ratio measure from He

[12] It is obtained using balance sheet data reported in the Flow of Funds from the Federal Reserve Board. It is computed as the ratio of total equity (total financial assets minus total financial liabilities) to total financial assets.

[13] As a robustness, we also consider a "world version" of these two variables.

Table 1: $\frac{Cov(x^*_{t+1}, m^*_{t+1})}{E_t(m^*_{t+1})}$ and $\frac{Cov(x^*_{t+1}, m_{t+1})}{E_t(m_{t+1})}$

A) CHF domestic currency, USD foreign currency					
	Fin. Intermediaries				HH
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{HKM}_{t+1} \times W^{GDP}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{GDP}_{t+1}$	C^{CH}_{t+1}
1999-2010	1.61	1.74	0.2	-1.17	0.25***
2010-2020	2.82**	1.32	5.1*	2.13**	0.01

B) JPY domestic currency, USD foreign currency					
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{MSCI}_{t+1}$	$\eta^{HKM}_{t+1} \times W^{GDP}_{t+1}$	$\eta^{AEM}_{t+1} \times W^{GDP}_{t+1}$	C^{JP}_{t+1}
1999-2010	1.85	-2.9	-3.57	-2.56**	0.7***
2010-2020	6.39***	3.31**	7.93***	2.63**	0.33

Note:
This table estimates $\frac{Cov(x^*_{t+1}, m^*_{t+1})}{E_t(m^*_{t+1})}$ and $\frac{Cov(x^*_{t+1}, m_{t+1})}{E_t(m_{t+1})}$ from equation (29) using different proxies of the SDF of (international) financial intermediaries and Swiss and Japanese households. Values are expressed in percentage points. Appendix B provides provides details on their construction and the source of the data. Statistical significance is assessed by regressing excess returns on the different measures of the SDF using Newey-West standard errors. $^{***}: p < 0.01$, $^{**}: p < 0.05$, $^{*}: p < 0.1$.

et al. (2017) and Adrian et al. (2014) and the two measures of total wealth to compute the SDF. The last column displays the covariance term for the Swiss and Japanese households, using real consumption growth to compute the SDF. Statistical significance is assessed by regressing the excess returns on the different measures of SDF and using Newey-West standard errors.

The results show that, since 2010, the covariance term for financial intermediaries is clearly positive and statistically significant for most of the specifications of the stochastic discount factor, and quantitatively in line with the UIP deviations depicted in Figure 1, reaching as high as 7.9% for Japan and 5.1% for Switzerland. Interestingly, the covariance term is generally an order of magnitude smaller (or negative) before 2010. In words, since 2010, being long in CHF or JPY tends to provide higher returns when the marginal utility of wealth of financial intermediaries is high, which supports that the CHF and the JPY behave as a hedge for international intermediaries. On the other hand, the covariance term between excess returns and SDF based on real consumption growth tend to be much smaller and statistically not significant since 2010. These observations can help rationalise the large UIP deviations (and the low expected excess returns) observed post 2010 in the data. For Switzerland and Japan, Proposition 1 implies that it is not CIP but UIP deviations that should matter for FX interventions, since $cov(m_{t+1}, x^*_{t+1})/E_t m_{t+1}$ is not significantly different from zero.

4 The Central Bank as a Constrained Planner

To determine how the cost of reserves influences the policy trade-offs of the central bank, we consider a central bank who maximizes households' welfare. The households' domestic participation constraint provides an incentive for the central bank to distort the domestic real interest rate. The cost of reserves may either conflict with this domestic objective, or facilitate it. We first reframe the central bank's problem as that of a constrained central planner. We then show how the resulting optimal allocation can be decentralized using foreign exchange interventions.

Before that, we relate the country's consolidated financial liabilities to the household participation constraints (14). Using the definition of gfl_t in (22), we can show that the households' constraint on domestic bond issuance translates into a constraint on gross foreign liabilities:

$$gfl_t \leq b_t^G + b_t^{CBF} - h_t^H \tag{32}$$

However, (32) is not an effective constraint since the central bank can change its holding of foreign bonds b_t^{CBF}.

Similarly, the foreign currency no-borrowing constraint implies:

$$nfl_t \leq gfl_t - b_t^{CBF} \tag{33}$$

This constraint cannot be relaxed by non-fiscal FX intervention since changes in gfl_t are offset by changes in b_t^{CBF}.[14] This constraint is effective except if we allowed the central bank to perform fiscal interventions, where changes in gfl_t need not be offset by changes in b_t^{CBF}. Equations (32) and (33) are equivalent to the no-borrowing constraints (14).

4.1 The Constrained Planner's Program

Based on the previous equations, we can examine the planner's optimal choices.

Definition 2 (Constrained planner equilibrium) *A constrained planner equilibrium is an equilibrium where a planner maximizes objective (15) subject to the economy's resource constraints (25); the asset pricing equation (8); the cash-in-advance constraints $h_t^H \geq y_t$ and $\bar{H}e^h = S_{t+1}y_{t+1}$; the non-negativity of foreign domestic money holdings $h_t^{H*} \geq 0$; the equilibrium on the market for money $H_t = S_t(h_t^H + h_t^{H*})$; the consolidated bond and money market equilibrium $a_t^{H*} = gfl_t$; the zero lower bound $i_t \geq 0$; and the foreign liability constraints (32) and (33). The planner's choice variables are $(i_t, S_t, S_{t+1}, gfl_t, nfl_t, b_t^{CBF}, H_t, h_t^H, h_t^*, a_t^*)$.*

[14]When capital controls are in place, however, Bacchetta et al. (2013) show that sterilized interventions can affect the country's intertemporal allocation.

The central bank's program is:

$$\begin{aligned}
&\max E\Bigg\{U(c_t)+\beta U(c_{t+1})\\
&+\eta_t\left(y_t-c_t+nfl_t\right)\\
&+\eta_{t+1}\left[y_{t+1}-c_{t+1}-(1+i_t^*)nfl_t+\left[(1+i_t^*)-(1+i_t)\tfrac{S_t}{S_{t+1}}\right]gfl_t+i_t\tfrac{S_t}{S_{t+1}}\left(\tfrac{H_t}{S_t}-h_t^H\right)\right]\\
&+\xi i_t\\
&+\Delta_t^H\left(h_t^H-y_t\right)\\
&+\Delta_t^F\left(\tfrac{H_t}{S_t}-h_t^H\right)\\
&+\Lambda\left(gfl_t-b_t^{CBF}-nfl_t\right)\\
&+\tilde{\Lambda}\left(b_t^G+b_t^{CBF}-h_t^H-gfl_t\right)\\
&+\alpha_0\left(E_t\left(m_{t+1}^*\left[(1+i_t^*)-(1+i_t)\tfrac{S_t}{S_{t+1}}\right]\right)+\Gamma gfl_t+\chi\right)\Bigg\}
\end{aligned}$$

and we treat S_{t+1} as an exogenous variable since $S_{t+1}=He^h/y_{t+1}$. Here, we substituted the foreign demand for domestic assets a_t^{H*} with gfl_t and h_t^{H*} with $H_t/S_t-h_t^H$.

Consider the first order conditions for assets:

$$/nfl_t: \quad \eta_t-E_t\left(\eta_{t+1}(1+i_t^*)\right) \quad -\Lambda \quad =0 \tag{34}$$

$$/gfl_t: \quad E_t\left(\eta_{t+1}\left[(1+i_t^*)-(1+i_t)\frac{S_t}{S_{t+1}}\right]\right) \quad +\Lambda-\tilde{\Lambda}+\alpha_0\Gamma \quad =0 \tag{35}$$

$$/H_t: \quad E_t\left(\eta_{t+1}\left[i_t\frac{S_t}{S_{t+1}}\right]\right) \quad +\Delta_t^F \quad =0 \tag{36}$$

$$/b_t^{CBF}: \quad -\Lambda+\tilde{\Lambda} \quad =0 \tag{37}$$

Equation (37) implies that $\tilde{\Lambda}-\Lambda=0$. This means that the central bank equalizes the marginal benefit of relaxing the foreign-currency and domestic-currency debt constraints by adjusting its assets and liabilities and going shorter in the asset whose shadow cost is higher and longer in the asset whose shadow cost is lower. Also note that $\eta_t=U'(c_t)$, $\eta_{t+1}=U'(c_{t+1})$, and that $m_{t+1}=\eta_{t+1}/\eta_t$ is the central bank's discount factor, which coincides with the household's (see Appendix C.1).

4.2 Optimal foreign exchange interventions

We can examine the impact of sterilized and unsterilized FX interventions by examining Equation (35) with $\Lambda-\tilde{\Lambda}=0$.

Sterilized interventions Equation (35), with $\Lambda-\tilde{\Lambda}=0$, can be rewritten as follows:

$$\underbrace{\overbrace{-E_tX_{t+1}^*-\frac{cov(m_{t+1},X_{t+1}^*)}{E_tm_{t+1}}}^{-UCFX_t}+\frac{\alpha_0}{\eta_tE_tm_{t+1}}}_{MBFX_t}\Gamma=0 \tag{38}$$

The left-hand side, $MBFX_t$, corresponds to the marginal benefit of sterilized foreign exchange interventions, that is, of expanding the central bank's balance sheet by going long in foreign bonds and short in domestic bonds. It is composed of the marginal utility benefit of FX interventions ($-UCFX_t$) and of the marginal benefit of the resulting price distortions. If, in the absence of interventions, $MBFX_t$ is positive, then it would be optimal for the central bank to accumulate FX reserves. These interventions can drive the marginal benefit to zero, achieving an optimal central bank balance-sheet, as we will see in more details later.

Finally, we examine the last term in equation $MBFX_t$, which arises from the price (interest rate and exchange rate) distortions implied by the central bank's interventions. The central bank has an incentive to not fully shut down its risk-adjusted foreign currency excess return in order to maximize its profit. Appendix C.2 shows that this term is equal to:

$$\frac{\alpha_0}{\eta_t E_t m_{t+1}}\Gamma = -\Gamma gfl_t \frac{E_t\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E_t m_{t+1} E_t\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)} \tag{39}$$

It is of the same sign as $-gfl_t$, home's gross external position in domestic currency. If the country is short in domestic currency ($gfl_t > 0$), then this term is negative. When accumulating foreign currency assets by issuing domestic currency liabilities, the planner reduces the foreign currency excess return (by increasing i_t or depreciating the domestic currency). The resulting opportunity cost is proportional to the economy's gross external position. This term also depends on Γ, which measures the impact of domestic currency bond supply on the excess return (see Equation (24)). The higher Γ, the more difficult it is for foreign intermediaries to absorb additional domestic currency assets, the higher the impact of domestic currency bond supply on the excess return. This term reflects the central bank's rent as a monopolistic issuer of domestic bonds.

To summarize, there could be a benefit of interventions for a safe haven currency if its hedging property is more valued by international investors. But the central bank has also to consider how these interventions affect its monopoly rent.

Unsterilized interventions In our framework, unsterilized interventions are ineffective outside the ZLB and are equivalent to sterilized intervention at the ZLB. Equation (36) implies that $\Delta^F > 0$ if $i_t > 0$, meaning that $H_t/S_t = h^H_t$ outside the ZLB. Therefore, issuing more money outside the ZLB would be purely inflationary since domestic households need a fixed real quantity of money and $H_t/S_t = h^H_t$. This would not increase the capacity of the central bank to buy foreign bonds. This arises from the absence of nominal friction in our model and the resulting money neutrality.[15]

At the ZLB, money and bonds are perfect substitutes, so that sterilized and unsterilized interventions become equivalent. Then, $MBFX_t$ is the marginal benefit of both

[15]Note that open market operations would be equally ineffective because they also rely on changing the money supply.

sterilized and unsterilized interventions, so that the above analysis applies. Whether foreign bonds are acquired by increasing H_t (unsterilized intervention) or by decreasing B_t^{CB} (sterilized intervention) does not matter.

Since unsterilized interventions have no specific impact, in what follows we fix arbitrarily the quantity of money H_t, and assume $H_t = 1$. In this case, the equilibrium is uniquely pinned down.

Lemma 1 (Equilibrium determinacy) *Conditional on H_t, the equilibrium is unique.*

To understand, suppose that H_t is not fixed. Then, outside the ZLB, S_t and i_t are undetermined. Indeed, according to the equilibrium in the domestic bond market (24), the excess return is affected by the central bank's optimal FX policy (through the supply of b^{CB} and hence through gfl_t, as can be seen in Equation (22)). For a given expectation of S_{t+1}, this equilibrium excess return determines the equilibrium value for $S_t(1 + i_t)$, which is compatible with different combinations of S_t and i_t. However, if H_t is fixed, then $H_t/S_t = y_t$, since foreigners hold no money ($h_t^{H*} = H_t/S_t - h_t^H = 0$) and households hold the minimum amount of money ($h_t^H = y_t$). This implies that the exchange rate S_t is determined, which means that the interest rate i_t is determined as well.

At the ZLB, $i_t = 0$, which implies that S_t is equal to the value that clears the domestic bond market (24). Note that it does not clear the money market as $H_t/S_t = y_t$ does not hold anymore, since now foreigners are willing to hold money. However, H_t and B_t^{CB} are not determined. Indeed, the optimal FX interventions can be obtained with an infinite number of combinations of sterilized and unsterilized interventions (H_t and B_t^{CB}). However, if H_t is fixed, then the amount of B_t^{CB} that obtains the optimal FX interventions is determined.

4.3 Implementation of the Optimum in a Decentralized Equilibrium

Here we discuss how the optimum is implemented in a decentralized equilibrium by analyzing households' optimal choices. Consider the central bank's foreign exchange interventions (sterilized or unsterilized). These interventions are relevant for the economy' gross foreign liabilities. Suppose that the optimal gross foreign liability position of the economy is $\widehat{gfl}_t$.

The households' optimal portfolio allocation, characterized by Equation (17), can be compared with Equation (35). For Equation (35) to be implemented in the decentralized equilibrium, we need that

$$\lambda^H - \lambda^F = -\alpha_0 \Gamma \tag{40}$$

where we used $\eta_t = U'(c_t)$, $\eta_{t+1} = U'(c_{t+1})$ and $\Lambda - \tilde{\Lambda} = 0$.

In safe haven countries, where typically $\widehat{gfl}_t > 0$, α_0 is more likely to be negative, so optimal foreign exchange interventions are only consistent with the households being financially constrained when issuing domestic-currency bonds ($\lambda^H > 0$), since $\lambda^F \geq 0$. The central bank, as we have seen, does not fully exhaust the –private– marginal benefit of going long in foreign currency and short in domestic currency. Households can be prevented from exploiting this residual marginal benefit only if their domestic-currency borrowing constraint is binding.

In this case, the central bank desires fewer domestic liabilities than households. The optimum is then easily implementable for the central bank by supplying just the right amount of domestic liabilities to complement the existing public domestic supply and reach $\widehat{gfl}_t$, through foreign exchange interventions. More precisely, the optimal foreign exchange intervention must satisfy

$$\widehat{b}_t^{CBF} = \widehat{gfl}_t - \left(b_t^G - h_t^H\right) \tag{41}$$

The domestic currency bonds issued by foreign exchange interventions $\widehat{b}_t^{CBF}$ must close the gap between the optimal gross foreign liabilities $\widehat{gfl}_t$ and the existing real supply of domestic bonds, which is equal to the amount of government bonds that are not held by the central bank to back households' asset holding $b_t^G - h_t^H$. For that level of domestic currency bonds, households would like to issue more domestic currency bonds ($b_t^H < 0$), but they are prevented from doing so by their no-borrowing constraints. That way, the optimum is implementable.[16]

4.4 Adding a Domestic Motive for FX Intervention

The focus of this paper is on the opportunity cost of FX intervention, so that we have abstracted from the benefits of interventions. The literature discusses numerous motives for intervention. To illustrate the cost-benefit analysis, we assume that the central bank has an additional incentive for intervention.[17] We now assume that households face short-selling constraints so that the real repayment on domestic debt $-b_t^H$ does not exceed some

[16] Note that if $\widehat{gfl}_t < 0$ ($\alpha_0 > 0$), then Equation (40) would imply that $\lambda^F > 0$ (since $\lambda^H \geq 0$), meaning that the household should be constrained in issuing foreign-currency bonds for the optimum to be implementable. Indeed, in that case, the central bank would typically save in domestic currency and borrow in foreign currency, but there would remain a private benefit of going short in foreign currency and long in domestic currency, which can only be consistent with a binding constraint on foreign liabilities in a decentralized economy. The optimum can be implemented by the central bank (or government) by supplying just the right amount of foreign-currency liabilities to reach $-\widehat{gfl}_t$.

[17] An alternative would be to assume that the central bank wants to stabilize the exchange rate, as Amador et al. (2020). One way to rationalize exchange rate stabilization would be to introduce nominal rigidities and imperfect substitution between domestic and foreign goods. In that case, variations in the exchange rate driven by financial shocks, such as shocks to Γ, shocks to σ_y, or noise trader shocks as in Itskhoki and Mukhin (2022), would introduce undesirable variations in the terms of trade that would warrant some exchange rate stabilization. Note that, in that case, open market operations would be effective outside the ZLB, but ineffective at the ZLB, whereas both sterilized and unsterilized would remain effective in the ZLB. This would also rationalize the fact that, in practice, FX interventions were heavily used only when the ZLB hit.

limit. The constraint in (14) is replaced by:

$$E_t \frac{(1+i_t)S_t}{S_{t+1}} b_t^H \geq \bar{b}^H \tag{42}$$

A non-zero level of $\bar{b}^H$ gives an additional motive for monetary policy. We assume that $\bar{b}^H$ can be either positive or negative. A positive sign for $\bar{b}^H$ implies an amount of forced savings.

This additional motive for intervention affects the last term in the first-order condition (38). Equation (39) becomes:

$$\frac{\alpha_0}{\eta_t E_t m_{t+1}} \Gamma = -\Gamma g f l_t \frac{E_t \left(m_{t+1} \frac{S_t}{S_{t+1}} \right)}{E_t m_{t+1} E_t \left(m^*_{t+1} \frac{S_t}{S_{t+1}} \right)} + \Gamma \frac{\Lambda}{\eta_t} \frac{\bar{b}^H}{E_t m_{t+1} E_t \left(\frac{(1+i_t)S_t}{S_{t+1}} \right) E_t \left(m^*_{t+1} \frac{(1+i_t)S_t}{S_{t+1}} \right)} \tag{43}$$

The second term on the right-hand side is active in the presence of a binding financial constraint ($\Lambda > 0$), and is of the same sign as $\bar{b}^H$, so it is positive if households are forced to save in domestic currency, i.e., if $\bar{b}^H > 0$. This generates an incentive for a higher interest rate (which improves the households intertemporal allocation by reducing the required savings) and therefore an accumulation of FX reserves. Note that if $\bar{b}^H < 0$ (if households face a borrowing limit), then the central bank would have a motive to depress the real domestic interest rate (which improves the households intertemporal allocation by increasing the allowed borrowing) and hence to reverse FX interventions.

5 A Linear-Quadratic Version of a Safe Haven Economy

In this section, we focus on the safe haven currency case. We assume that the SDF of financial intermediaries is inversely proportional to the growth of a global factor y^*_t. The safe haven feature is given by assuming that the domestic currency appreciates when the global factor declines.[18] We also assume that domestic output is only partly correlated with this factor. We consider an approximated version of the model and assume lognormal shocks. The objective is to derive $cov(m^*_{t+1}, X^*_{t+1})$ and $cov(m_{t+1}, X^*_{t+1})$ to evaluate the opportunity cost of FX interventions given in (29).

Assumption 1 (Specific case) *Our specific case is characterized by*

1. *The utility is logarithmic:* $U(c_t) = \log(c_t)$*;*

[18]There is a small literature trying to provide explanations for safe haven effects, but the focus is on the US and the mechanisms do not apply to a small countries. See Maggiori (2017) or Hassan et al. (2021). Papers that model time-varying safe haven effects include Gourinchas and Rey (2022), Devereux et al. (2022), and Kekre and Lenel (2021).

2. *The SDF of international financial intermediaries is driven by the global factor:* $m^*_{t+1} = \beta \frac{y^*_t}{y^*_{t+1}}$;

3. *The period-*$t+1$ *global factor is log-normally distributed:* $\tilde{y}^*_{t+1} \sim N(\sigma_y^2/2, \sigma_y^2)$*, with* $\sigma_y > 0$;

4. *Domestic output is correlated with the global factor:* $\log(y_{t+1}) = \alpha \log(y^*_{t+1}) + (1 - \alpha)\sigma_y^2/2$ *for some real* α;

5. *The exchange rate is correlated with the global factor* $S_{t+1} = He^{\rho \log(y^*_{t+1})}$ *for some real* ρ.

6. *In period* t *domestic output and the global factor are normalized to 1:* $y_t = y_t* = 1$;

Assumption 1.6 normalizes the expected foreign stochastic discount factor to β under log-utility: $E_t m^*_{t+1} = E_t(\beta/y^*_{t+1}) = \beta$. Assumption 1.4 ensures that $E(y_{t+1}) = E(y^*_{t+1})$.

We make the following assumption on the parameters:

Assumption 2 (Safe haven) $\rho > 0$ *and* $\alpha < 1$.

A positive ρ captures the hedging capacity of safe haven currencies: the exchange rate appreciates when the global factor is weak, so the domestic currency is a good hedge against fluctuations in the global factor. A low α reflects the small exposure of the domestic output to global risk. Domestic output comoves with the global factor, so the domestic currency is also a hedge for domestic households. However, since domestic output is less volatile, in equilibrium domestic households are willing to be short in domestic bonds.

5.1 The Utility Cost of Reserves

We first solve for the equilibrium for given nfl_t and gfl_t in order to evaluate the planner's optimality conditions as a function of nfl_t and gfl_t. We will consider in turn the case where the ZLB is not binding and the case where it is. We use second-order approximations. We denote from now on the variables in log with a tilde: $\tilde{y} = \log(y)$ and $\tilde{y}^* = \log(y^*)$. We also define $\tilde{i}^*_t = \log(1 + i^*_t)$ and $\tilde{i}_t = \log(1 + i_t)$.

The foreign and domestic interest rates must satisfy

$$E_t(e^{\tilde{m}^*_{t+1} + \tilde{i}^*_t}) = 1 \tag{44}$$

$$E_t(e^{\tilde{m}^*_{t+1} + \tilde{S}_t - \tilde{S}_{t+1} + \tilde{i}_t}) = 1 + \chi + \Gamma gfl_t \tag{45}$$

Besides, we have $\tilde{S}_{t+1} = \rho \tilde{y}^*_{t+1}$.

This yields:

$$1 + i^*_t = \frac{1}{\beta}$$

$$(1 + i_t)S_t = \frac{1}{\beta}(1 + \chi + \Gamma gfl_t)e^{-\frac{1}{2}(1+\rho)\rho\sigma_y^2} \tag{46}$$

See Appendix D.1 for a full derivation. These equations define respectively the foreign interest rate i_t^* as a function of the subjective discount factor and $(1 + i_t)S_t$ as a function of financial frictions (Γ and χ), of the hedging properties of the domestic exchange rate and of gross foreign liabilities. This determines i_t outside the ZLB (since $S_t = 1$) and S_t at the ZLB (since $i_t = 0$).

What are the implications for the cost of foreign exchange interventions? We can write the difference in risk premia, which is a component of the utility cost of reserves $UCFX_t$ (see Equation (29)), as follows

$$\frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} - \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} = \frac{1}{\beta}(1 + \chi + \Gamma gfl_t)\left(1 - e^{-\Delta cov}\right) \tag{47}$$

where $\Delta cov = cov(\tilde{m}_{t+1}, \tilde{S}_{t+1}) - cov(\tilde{m}^*_{t+1}, \tilde{S}_{t+1})$ is the difference between the covariance of the log-linearized domestic currency excess return with the intermediaries SDF and the covariance with the domestic SDF. See Appendix D.2 for a full derivation.

In Appendix D.2 we also show that

$$\Delta cov = \rho\sigma_y^2 \left[1 - \alpha(1 + nfl_t + gfl_t) - \rho gfl_t\right] \tag{48}$$

In a safe haven economy as defined in 2, if nfl_t and gfl_t are not too high, this covariance differential is positive. This implies that the difference in covariances introduces a gain from foreign exchange interventions. The domestic economy is less exposed to global risk, so the planner benefits from going short in domestic bonds and long in foreign bonds. Notice that this covariance differential is decreasing in gfl_t. By increasing its balance-sheet exposure to exchange rate risk, the planner would increase the risk exposure of the domestic household and the domestic covariance would catch up to the foreign one.[19]

5.2 Optimal Allocations with no Domestic Motive for FX Interventions

The marginal utility cost of FX interventions $UCFX_t$, and especially the covariance differential, is only one component of the planner's optimality condition (38). The following lemma lays down explicit expressions for the optimal FX interventions. For the moment, we abstract from the domestic motives of monetary policy by assuming that $\bar{b}^H = 0$.

Lemma 2 *Suppose that the economy is a safe haven as in Definition 2. Denote by $\widehat{gfl}_t$ and $\widehat{nfl}_t$ the optimal gross and net foreign liabilities. We focus on solutions where $\widehat{nfl}_t < -\log(\beta) + \alpha^2$. Then:*

(i) $\widehat{nfl}_t = min\{b_t^G - 1, \overline{nfl}(\widehat{gfl}_t)\}$, where $\overline{nfl}(gfl_t)$ is defined in Appendix D.3;

[19] Note that, as shown in Equation (47), the covariance differential is proportional to $1 + \chi + \Gamma gfl_t$, which corresponds to the risk-adjusted return on the domestic currency bond. Indeed, the higher the average return, the higher the covariances. As a result, stronger financial frictions also contribute to a higher covariance differential.

(ii) $\widehat{gfl}_t$ *is implicitly defined by:*

$$1-(1+\chi+2\Gamma gfl_t)e^{-\Delta cov}=0$$

where Δcov *is defined by Equation* (48)*.*

See the proof in Appendix D.3. Result (i) comes from the fact that the household may be financially constrained. In that case, $nfl_t = b_t^G - h_t^H$, where $h_t^H = 1$. Otherwise, the level of net foreign liabilities nfl equalizes the domestic and foreign discount factors in expectations. Note that in that case, the household can only issue foreign bonds, because, as we have seen, we must have $\lambda^H > 0$. Result (ii) derives from the optimality condition with respect to gfl_t, Equation (38). Interestingly, at the optimum, stronger financial frictions (a higher Γ or a higher χ) result in a higher covariance differential. Indeed, stronger financial frictions generate a more positive CIP, which limit the incentives to accumulate reserves (see the definition of the utility cost of reserves (29)). With less FX interventions, the households are less exposed to currency risk, and the covariance differential remains high. The extra term in Γ on the left-hand side reflects the fact that the planner does not fully shut down the rent arising from the domestic currency excess return. The planner intervenes enough to take advantage of the excess return, but takes into account the fact that interventions decrease the domestic excess return.

5.2.1 Comparative statics

Lemma 2 implies that, at the optimum,

$$\widehat{gfl}_t = \frac{\rho\sigma_y^2[1-\alpha(1+\widehat{nfl}_t)]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2} \tag{49}$$

This is shown formally in Appendix D.4. Since we analyze safe haven economies, we focus on the case where $\widehat{gfl}_t \geq 0$.

Note that $\widehat{gfl}_t$ depends negatively on $\widehat{nfl}_t$. Indeed, higher leverage makes the economy more vulnerable to global risk and hence reduces the incentives of the central bank to take more risk on its balance sheet. What parameters drive $\widehat{nfl}_t$? In the case where the households are unconstrained, we show in Appendix D.4 that, under some conditions, $\widehat{nfl}_t$ is positive if σ_y^2 is high and α is low, while χ and Γ are not too large, even though the path of domestic output is the same as the foreign one on average. This is due to the fact that domestic households are less exposed to global risk than the foreign investors, which lowers the domestic discount factor relative to the foreign one. This is akin to an "inverse precautionary saving motive". This low risk exposure generates a borrowing motive that can drive the domestic households to hit their no-borrowing constraint if the government debt is too low ($b_t^G - 1 < \overline{nfl}_t$).

In what follows, we suppose that the households are constrained so that $\widehat{nfl}_t = b_t^G - 1$ is given. In that case, the optimal level of intervention is given by

$$\widehat{b}_t^{CBF} = \frac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2} - (b_t^G - 1)$$

where we used equations (41) and (49).

The comparative statics for optimal FX intervention is given in the following proposition:

Proposition 2 *Consider a safe haven economy as defined in 2 and assume that* $\bar{b}^H = 0$. *Suppose that* $\widehat{gfl}_t \geq 0$ *and* $\widehat{nfl}_t = b^G - 1$. *Then optimal foreign exchange interventions,* $\widehat{b}_t^{CBF}$*:*

(i) are increasing in risk measures σ_y *and* ρ*;*

(ii) are decreasing in intermediaries financial frictions Γ *and* χ*;*

(iii) are decreasing in the domestic output exposure to global risk α*, as long as* $b_t^G > 0$*;*

(iv) are decreasing in the supply of government bonds b_t^G*.*

Points (i) to (iv) can be shown by taking the derivatives of $\widehat{b}_t^{CBF}$ with respect to σ_y, ρ,Γ, χ, α, and b_t^G. Risk tends to increase the covariance differential Δcov, which generates an excess benefit of foreign exchange interventions, while the intermediation frictions generate a cost. The exposure of domestic output to global risk decreases the covariance differential and generates a cost. Point (iv) arises from the substitutability between government bonds and the central bank's sterilization bonds (see Equation (41)). If the government issues more bonds, then this reduces the need for the central bank to issue domestic bonds through FX interventions.

Interestingly, an increase in risk, which increases the optimal gfl_t, typically generates both a more negative UIP deviation and a more positive CIP deviation, as the financial intermediaries have to absorb the excess domestic currency bonds. This is established formally in the following proposition:

Proposition 3 *Suppose that* $\widehat{gfl}_t \geq 0$ *and* $\widehat{nfl}_t = b^G - 1$. *Then:*

(i) Z_{t+1}^* *is increasing in* σ_y *(it becomes more positive);*

(ii) $E_t X_{t+1}^*$ *is decreasing in* σ_y *(it becomes more negative) if* Γ *is not too large.*

See the proof in Appendix D.5. The CIP deviation becomes more positive when risk increases, as financial intermediaries need to absorb more capital inflows (more gfl_t). As risk increases, the UIP deviation becomes more negative, because it affects positively the foreigners' risk premium. However, if the intermediation friction Γ is large, the increase in the CIP deviation can offset the increase in the risk premium, and the total impact on the UIP deviation becomes ambiguous.

5.2.2 Numerical Illustration

We examine in more details how the level of risk σ_y^2 affects the economy through a numerical example.

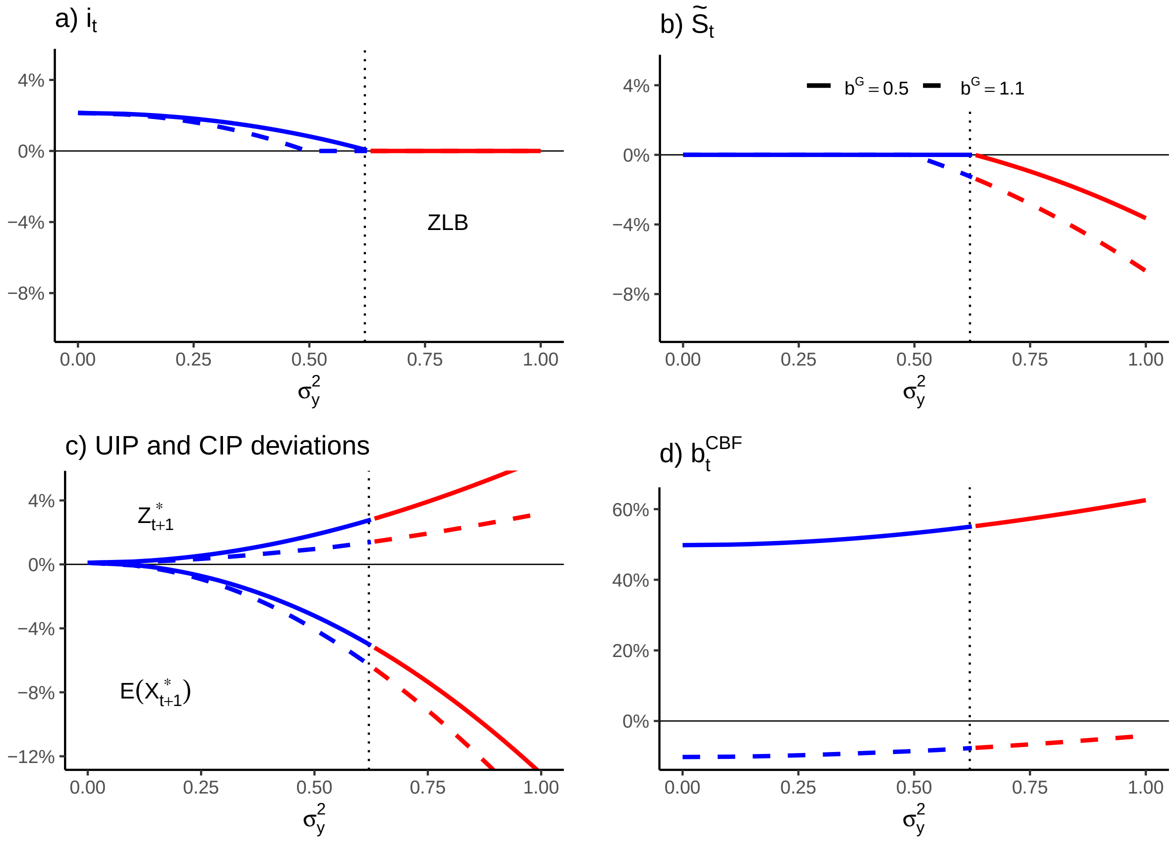
Figure 2 shows the comparative statics of σ_y^2 under a baseline specification of parameters. We also consider two levels of b_t^G: 0.5 and 1.1. Panel a) shows the negative relationship between the domestic interest rate and risk: the higher demand for domestic bonds is accommodated through a decline in the domestic nominal interest rate. The ZLB is attained at $\sigma_y^2 \geq 0.62$. Outside the ZLB, the interest rate adjusts adjusts to accommodate, through a decline, the higher demand for domestic bonds (see Panel b)), while at the ZLB, the exchange rate appreciates. Panel c) displays the deviations from UIP ($E_t X_{t+1}^*$) and CIP (Z_{t+1}^*). As we can see, an increase in risk leads to a more negative UIP deviation, and a more positive CIP deviation, as explained in Proposition 3.

For Panels a) to c), a higher public debt b_t^G reduces the domestic interest rate and the domestic currency excess return (generating a less positive CIP deviation and a more negative UIP deviation). Indeed, with a higher level of net foreign liabilities nfl_t, the central bank targets lower domestic gross liabilities gfl_t, as explained above (and as illustrated in Panels a) and b) of Figure E.1 in the Appendix). The lower equilibrium interest rate then results from the relative scarcity of domestic assets.

Panel d) shows that $\widehat{b}_t^{CBF}$ increases with risk because of the positive covariance differential (see equation (48)) resulting from the assumption of safe-haven ($\alpha < 1$ and $\rho > 0$). An increase in risk raises the benefit of FX interventions, which the central bank takes advantage of by buying FX reserves. However, the level of $\widehat{b}_t^{CBF}$ is only positive when $b_t^G = 0.5$. When b_t^G is large, the central bank is long in domestic bonds rather than foreign bonds, and short in foreign bonds rather than domestic bonds. In that case, an increase in risk pushes the central bank to sell domestic bonds and decrease its foreign currency leverage. However, this is possible only if the central bank is allowed to be short in foreign currency. Otherwise, the central bank cannot exploit its advantage. This perspective is consistent with the experience of Switzerland and Japan. Swiss public debt has been below 50% in the last 15 years, while it has been higher than 200% for Japan.

Note that households need not be constrained in their capacity to smooth consumption between periods for the central bank interventions to be effective. In Figure E.1 in the Appendix, we can see that in the case with a large public debt (dashed lines), the foreign-currency no-borrowing constraint is not binding for most values of σ_y^2 (see Panel c)), as government debt helps households achieve their desired level of nfl_t without having to borrow themselves. In fact, as risk increases, the discount factor of the domestic households decreases relative to the foreign one, leading to an increase in nfl_t (see Panel b)). This net foreign position is achieved by decreasing foreign currency bonds holdings b_t^F (see Panel e)), as long as the foreign-currency no-borrowing constraint is not binding. Despite that, the central bank can achieve its target gfl_t, because it can crowd out private domestic savings b_t^H by holding just the right amount of domestic bonds to achieve its

Figure 2: Comparative statics of σ_y^2



Notes: Baseline parameters : $\beta = 0.98, \chi = 0.002, \Gamma = 0.5, \alpha = 0.6, \rho = 0.2$. We assume that $\bar{b}^H = \bar{b}^F = 0$.

desired gfl_t. As discussed above, households want to issue *more* gross foreign liabilities than the central bank, but they are prevented from doing so by the domestic bond short-selling constraint. We can see that, indeed, households are still constrained in their capacity to issue domestic bonds ($\lambda^H/\eta_t > 0$, although it is very small, as shown in Panel d), and $(1+)b_t^H$ remains constant at $\bar{b}^H$, as shown in Panel f)).

5.3 Optimal Allocations with a Domestic Motive for FX Interventions

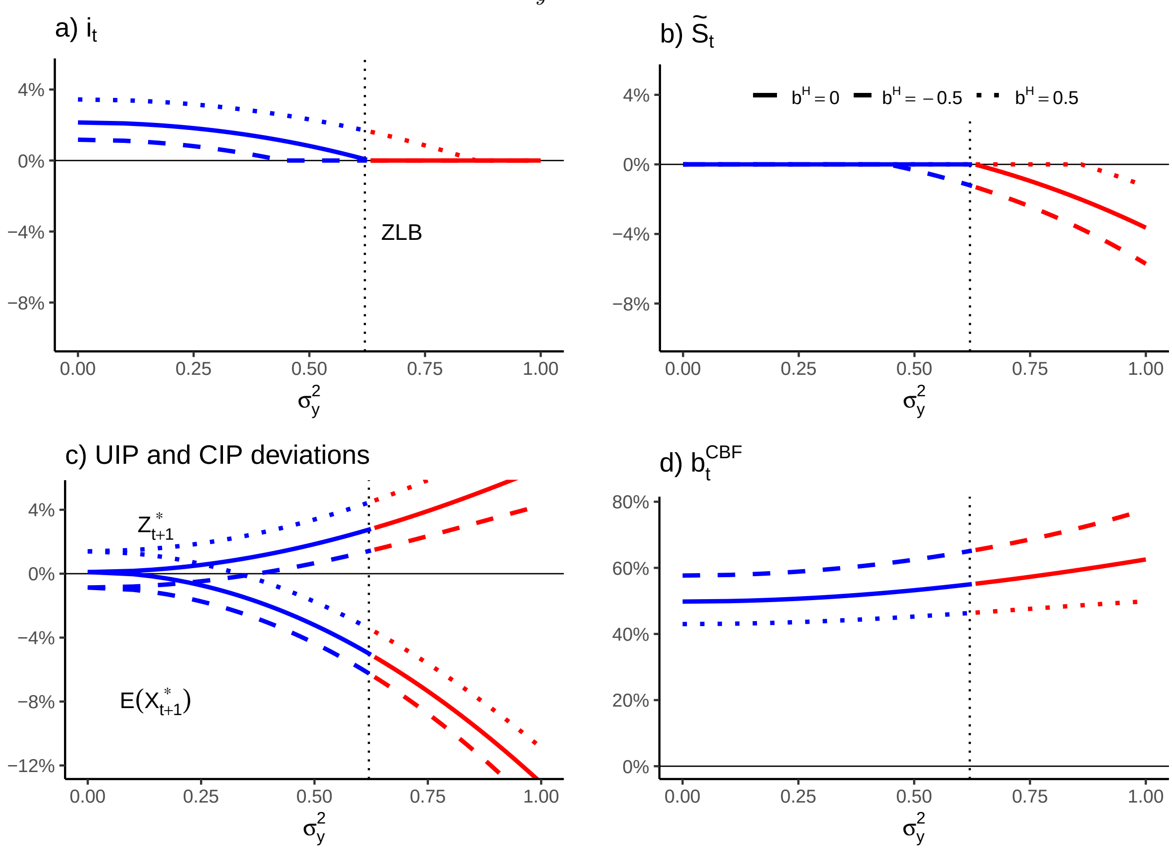
So far, we have examined the case where $\bar{b}^H$, the households' minimum domestic bond holdings, is equal to zero. This assumption suppresses any "domestic" motive for FX interventions, as distorting the domestic interest rate cannot improve the households' consumption smoothing. In that case, the only motive for FX interventions stems from the utility gains of holding FX reserves.

We now discuss the case where $\bar{b}^H \neq 0$. In that case, as we can infer from Equations (38) and (43), the central bank can have an additional motive of buying or selling FX reserves if $\Lambda > 0$. Note that, since $\lambda^F = \Lambda$, this means that this motive emerges when households are unable to achieve their desired nfl_t (both λ^H and λ^F are strictly positive).

If $\bar{b}^H$ is positive, households are forced to save. In that case, because the forced savings constraint bears on the total returns of savings, the central bank can reduce the amount of forced savings by maintaining a higher real interest rate. This is achieved through more FX interventions. If $\bar{b}^H$ is negative, households have a limited borrowing capacity. Here, on the opposite, the central bank would like to achieve a lower real interest rate to generate a higher borrowing capacity for the households, since the constraint bears on total debt repayments. This is achieved through less FX interventions.

Figure 3 illustrates this. Panel d) shows that a positive $\bar{b}^H$ shifts the central bank's FX reserve holdings upwards, while a negative $\bar{b}^H$ produces a downward shift. The price implications are shown in Panels a), b) and c). In the former case, the equilibrium nominal interest rate is higher (outside the ZLB) and the nominal exchange rate is more depreciated (in the ZLB), which generates a higher real interest rate, and hence a downward shift in both CIP and UIP deviations. In the latter case, the opposite happens.

Figure 3: Comparative statics of σ_y^2 with and without a "domestic motive"



Notes: Baseline parameters : $\beta = 0.98, \chi = 0.002\,\Gamma = 0.5, \alpha = 0.6, \rho = 0.2, b^G = 0.5.$

6 Conclusion

The GFC was followed by significant changes in the international monetary system. We have been observing systematic deviations from CIP, an increased demand for safe assets, a strengthening role of the USD as a reserve currency and strong increase in central bank balance sheets. In this context, there has been a stronger demand for safe haven currencies and more FX intervention by these countries' central banks. In the case of the Swiss National Bank, the spectacular increase in its balance sheet has occurred exclusively through the purchase of foreign assets.

The objective of this paper is to provide a simple framework to clarify some aspects of these developments. To explain UIP and CIP deviations in safe haven economies, we follow the recent literature that gives a key role to constrained international financial intermediaries. However, we assume that these intermediaries face exchange rate risk and value the hedging properties of safe haven currencies. The increased demand of these currencies may push the central bank to intervene and limit the extent of currency appreciation.

We examine the opportunity cost of FX intervention when CIP and UIP deviations are of different sign. We show that whether CIP or UIP matters depends on how domestic residents value the hedging property of their currency compared to international investors. If they give no value to its hedging property, UIP deviations should matter. This may imply a benefit, and thus a higher incentive, for FX accumulation. We show that the incentives to accumulate FX reserves in safe haven countries increase with the level of global risk or of effective risk aversion of international intermediaries. In contrast, the incentive decreases with the level of debt.

We also attempt to estimate the opportunity cost of intervention for Switzerland and Japan. We find that in both countries, domestic households value less the hedging properties of their currency than international investors. Overall, the incentives for intervention are stronger for Switzerland as the difference with international investors is larger and its public debt is much smaller than in Japan. While our analysis focuses on small safe haven countries, it also sheds some light on the difference between the properties of safe haven currencies and those of a reserve currency such the USD.

A CHF and JPY as Safe Haven Currencies

The safe haven properties of the Swiss franc and the Japanese yen have been documented by various authors, e.g., Stavrakeva and Tang (2021), Ranaldo and Söderlind (2010), Grisse and Nitschka (2015), or Fink et al. (2022). We confirm this by relating expected excess returns to various sources of risk.

We compute UIP deviations using short-term rates from Datastream and survey data from Consensus Economics.[20] Table A.1 shows the correlation between expected excess returns in CHF and JPY (Ex^*_{t+1}) and different measures of risk. Since 2010, this correlation is systematically positive, suggesting that agents tend to expect the CHF and JPY to yield excess returns at times of heightened uncertainty. When considering the entire sample (from 1999 to 2021), the correlation is systematically weaker or negative, which suggests that the CHF and JPY have reinforced their perceived safe-haven properties since 2010.

Table A.1: Correlation between UIP deviations and (global) risk variables

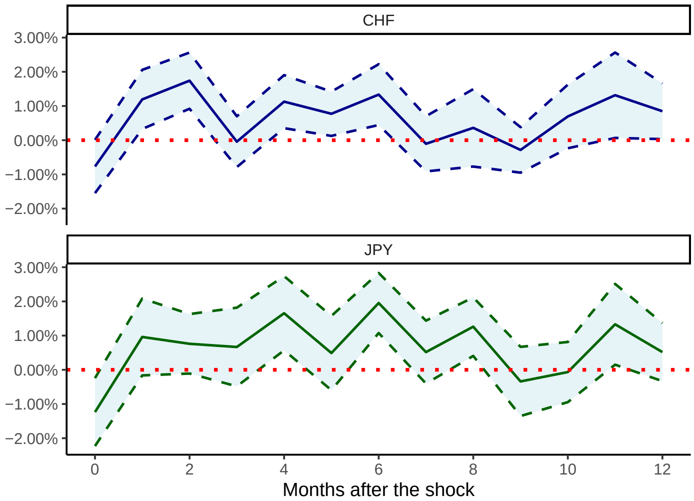
$Corr(RiskVariables, E(x^*_{t+1}))$						
	A) CHF/USD			B) JPY/USD		
Sample	USEPU	GEPU	WUI	USEPU	GEPU	WUI
1999-2021	-0.23	-0.29	-0.30	-0.11	-0.03	0.06
2010-2021	0.14	0.26	0.41	0.14	0.32	0.43

Notes: This table displays the correlation between Ex^*_{t+1} (at a 3-month horizon) and different risk variables for the whole sample and a subsample starting in 2010. Panel A) displays this correlation taking the CHF as the domestic currency and the USD as the foreign one. Similarly, Panel B) considers the JPY as the domestic currency. USEPU is the US Eonomic Policy Uncertainty index developed in Baker et al. (2016). GEPU is the Global EPU. WUI is the World Uncertainty Index developed in Ahir et al. (2022). Since WUI is only available at a quarterly frequency, we take the quarterly mean of UIP deviations when computing the correlation.

To examine the dynamic impact of uncertainty shocks, Figure A.1 runs a local-projection regression (Jordà (2005)) of a Global Economic Policy Uncertainty (EPU) shock on $E(x^*_{t+1})$ for the period 2010-2021. The results show that, following an unanticipated shock to the Global EPU, $E(x^*_{t+1})$ tends to increase both for the CHF and the JPY. In other words, the CHF and the JPY are generally expected to appreciate following an uncertainty shock.

[20] See Kalemli-Özcan and Varela (2021) for a recent analysis of UIP deviations using Consensus Economics survey.

Figure A.1: Local Projections to a Global EPU shock



Notes: This figure shows the results from the local projection of a Global EPU shock on the UIP deviations over the sample 2010-2021, using the CHF and the JPY as the domestic currency, respectively. Formally, we identify an uncertainty shock ($shock_t$) outside of the system by taking the residual of an AR(1) on our Global EPU variable in the spirit of Stock and Watson (2012) who uses the VIX. We then run $E(x^*_{t+h}) = \alpha^h + \beta_h shock_t + \phi^h x_t + u^h_{t+h}$ for $h = 0, ..., 12$ where x_t are control variables made of $p = 3$ lags of the dependent variable. We then report β^h at each horizon as well as the 90% confidence intervals using the Newey-West estimator.

B Computing excess returns and stochastic discount factors

In this section, we discuss the construction of $cov(x^*_{t+1}, m^*_{t+1})/E_t m^*_{t+1}$ and $cov(x^*_{t+1}, m_{t+1})/E_t m_{t+1}$ considering either the CHF and the JPY as the domestic currency, and keeping the USD as the foreign one.

B.1 Excess returns

First, we compute excess returns. For i_t, we rely on the domestic (CHF or JPY) 3-month risk-free rate, while i^*_t is the US 3-month risk-free rate. For s_t we rely on nominal spot exchange rate data expressed in amount of domestic currency per unit of USD. All data is from Datastream and retrieved at the daily frequency. The daily data is aggregated to the quarterly frequency by taking the mean within each quarter. To compute excess returns, we first compute quarterly excess returns according to (31). We assume that what matters for the financial intermediaries is the moving excess returns of this carry-trade over the past year by taking a moving sum of excess returns over that of the current and last three quarters. This allows to have a smoother version of excess returns.

B.2 Stochastic discount factors

International Financial Intermediaries We now discuss the construction of the SDF of financial intermediaries, which is defined as $m^*_{t+1} = \beta\left(NW_{t+1}/NW_t\right)^{-\gamma}$. Similar to He et al. (2017), we define $NW_{t+1} = \eta_{t+1} \times W_{t+1}$, where η_{t+1} is a measure of the capital ratio of financial intermediaries and W_t is a measure of total wealth. The SDF is obtained by interacting a measure of the growth rate of the capital ratio and total wealth. Below, we discuss the construction of these growth rates.

We consider two measures of the capital ratio. The first specification (HKM) relies on the capital ratio measure from He et al. (2017) which is retrieved from Zhiguo He's website at the daily frequency and aggregated at the quarterly frequency by taking the mean. The second specification (AEM) is based on Adrian et al. (2014) and is computed using quarterly balance sheet data from the Federal Reserve Flow Of Funds (Table L.130). To obtain an annual growth rate, we divide the residual of a regression of the capital ratio in t on its one-year lagged value by the one-year lagged value of the capital ratio. This gives rise to the intermediary capital risk factor. The two resulting measures are defined as $\Delta\eta^{HKM}_{t+1}$ and $\Delta\eta^{AEM}_{t+1}$, respectively.

For total wealth growth, we rely on a financial measure (MSCI US Equity Index) and a real measure (US GDP). For the financial measure, we consider moving annual excess returns. Every quarter, they are obtained by summing up daily excess returns over the past 4 quarters and subtracting the 3-month US risk-free rate. The resulting series is defined as ΔW^{MSCI}_{t+1}. For the real measure, we compute moving annual growth every quarter. The resulting series is defined as ΔW^{GDP}_{t+1}.

The SDF of financial intermediaries is then computed as $m^*_{t+1} = \beta(\Delta\eta^i_{t+1} \times \Delta W^j_{t+1})^{-\gamma}$ for $i \in \{AEM, HKM\}$ and $j \in \{MSCI, GDP\}$, with $\beta = 0.99$ and $\gamma = 10$. This gives rise to 4 potential specifications of the SDF of financial intermediaries.

Domestic Households For Households (HH), the SDF is defined as $m_{t+1} = \beta\left(C_{t+1}/C_t\right)^{-\gamma}$. Real consumption for Switzerland and Japan is retrieved from the FRED website at the quarterly frequency. As for the SDF of financial intermediaries, we compute a moving annual growth rate and assume $\gamma = 10$ and $\beta = 0.99$.

C Proofs - Constrained Planner Program

C.1 Other FOCs

We take the derivative with respect to h^H_t:

$$/h^H_t : -E_t\left(\eta_{t+1}\left[i_t \frac{S_t}{S_{t+1}}\right]\right) + \Delta^H_t - \Delta^F - \tilde{\Lambda} = 0 \tag{50}$$

Equations (50) and (36) then imply that $\Delta^H_t = \tilde{\Lambda} = \Lambda$. Therefore, when $\Lambda = \tilde{\Lambda} = 0$, $\Delta^H_t = 0$. This reflects the fact that, while households want to minimize their money holdings because they represent a cost (when $i_t > 0$), the amount of money held by the households is not relevant to the central bank when the economy is not constrained in its capacity to issue debt, since seigniorage is redistributed to households in period $t+1$. The cash-in advance constraint is relevant only to the extent that it also restrains the capacity of the economy to supply domestic assets to the rest of the world, just like the no-borrowing constraints.

We now take the derivatives with respect to prices:

$$
\begin{aligned}
/i_t: \quad & -E\left[\eta_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\left(gfl_t + \frac{H_t}{S_t} - h_t^H\right)\right] + (1+i_t)\xi \\
& -\alpha_0 E\left(m^*_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right) + \tilde{\Lambda}\frac{\bar{b}^H}{E_t\frac{(1+i_t)S_t}{S_{t+1}}} = 0 \qquad (51) \\
/S_t: \quad & -E\left(\eta_{t+1}\left[(1+i_t)\frac{S_t}{S_{t+1}}gfl_t - i_t\frac{S_t}{S_{t+1}}h_t^H\right]\right) - \Delta^F\frac{H_t}{S_t} \\
& -\alpha_0\left[E\left(m^*_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right)\right] + \tilde{\Lambda}\frac{\bar{b}^H}{E_t\frac{(1+i_t)S_t}{S_{t+1}}} = 0 \qquad (52)
\end{aligned}
$$

Finally, we derive with respect to consumption:

$$
\begin{aligned}
/C_t: \quad & U'(C_t) - \eta_t = 0 \qquad (53) \\
/C_{t+1}: \quad & E\left(\beta U'(C_{t+1}) - \eta_{t+1}\right) = 0 \qquad (54)
\end{aligned}
$$

These equations imply that $m^{CB}_{t+1} = \eta_{t+1}/\eta_t = \beta U'(C_{t+1})/U'(C_t) = m_{t+1}$.

C.2 Monopolistic term and interest and exchange rate determinacy

Here we have to distinguish two cases. Either $i_t > 0$, and in that case $\xi = 0$, $H_t/S_t = h_t^H$ and $\Delta^F > 0$. Or $i_t = 0$, and in that case $\xi > 0$ and $\bar{\Delta}^F = 0$.

In the former case (if $i_t > 0$), Equation (51) yields

$$
\frac{\alpha_0}{\eta_t} = -gfl_t\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)} + \frac{\tilde{\Lambda}}{\eta_t}\frac{\bar{b}^H}{E_t\left(\frac{(1+i_t)S_t}{S_{t+1}}\right)E_t\left(m^*_{t+1}\frac{(1+i_t)S_t}{S_{t+1}}\right)} \qquad (55)
$$

where we have used $E(\eta_{t+1}/\eta_t) = m_{t+1}$. If $\bar{\Lambda} = 0$, α_0 is of the same sign as $-gfl$, home's gross external position in domestic currency. In that case, if the country is short in domestic currency, then α_0 is negative.

Using Equation (36), Equation (52) yields the same equation, so it is redundant. This means that there is some nominal indeterminacy. This nominal indeterminacy does not come from the future exchange rate, which is exogenously fixed, but from the amount of excess return adjustment that comes from i_t and S_t. In other terms, the optimal nominal money supply H_t is undetermined. For instance, if the supply of money H_t is higher, then the exchange rate S_t will be higher (more depreciated), so that the optimal interest rate i_t will be have to be lower to generate a given excess return.

In the latter case (if $i_t = 0$), Equation (55) remains true. Note that in that case, the exchange rate is not undetermined, because $i_t = 0$.

D Proofs - Linear-Quadratic Case

D.1 Equations (46)

Equation (44) yields

$$\begin{aligned} & & E_t(e^{\tilde{m}^*_{t+1}+\tilde{i}^*_t}) &= 1 \\ \Leftrightarrow & & e^{E(\tilde{m}^*_{t+1})+\frac{1}{2}V(\tilde{m}^*_{t+1})+\tilde{i}^*_t} &= 1 \\ \Leftrightarrow & & e^{\log(\beta)+E(\tilde{-}y^*_{t+1})+\frac{1}{2}V(\tilde{y}^*_{t+1})+\tilde{i}^*_t} &= 1 \\ \Leftrightarrow & & e^{\log(\beta)+\tilde{i}^*_t} &= 1 \end{aligned}$$

Similarly, Equation (44) yields

$$\begin{aligned} & & E_t(e^{\tilde{m}^*_{t+1}-\tilde{S}_{t+1}+\tilde{i}_t+\tilde{S}_t}) &= 1+\chi+\Gamma gfl_t \\ \Leftrightarrow & & e^{E(\tilde{m}^*_{t+1}-\tilde{S}_{t+1})+\frac{1}{2}V(\tilde{m}^*_{t+1}-\tilde{S}_{t+1})+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t \\ \Leftrightarrow & & e^{\log(\beta)-E((1+\rho)\tilde{y}^*_{t+1})+\frac{1}{2}V((1+\rho)\tilde{y}^*_{t+1})+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t \\ \Leftrightarrow & & e^{\log(\beta)+\frac{(1+\rho)\rho}{2}\sigma_y^2+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t \end{aligned}$$

This yields (46).

D.2 Covariance differential

The difference in risk premia can be written as follows

$$\frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} - \frac{cov(m_{t+1}, X^*_{t+1})}{E(m_{t+1})} = \frac{1}{\beta}(1+\chi+\Gamma gfl_t)\left(1-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\right)$$

We used

$$\begin{aligned} cov(m^*_{t+1}, X^*_{t+1}) &= cov\left(m^*_{t+1},(1+i_t)\tfrac{S_t}{S_{t+1}}\right) - \underbrace{cov(m^*_{t+1},(1+i^*_t))}_{=0} \\ &= E\left(m^*_{t+1}(1+i_t)\tfrac{S_t}{S_{t+1}}\right) - E\left(m^*_{t+1}\right)E\left((1+i_t)\tfrac{S_t}{S_{t+1}}\right) \\ &= E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right) - E\left(e^{\tilde{m}^*_{t+1}}\right)E\left(e^{\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right) \\ &= \underbrace{E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}_{1+\chi+\Gamma gfl_t}\left[1-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right] \end{aligned}$$

where we used (45), and

$$E(m^*_{t+1}) = \beta$$

which yields

$$\frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} = \frac{1}{\beta}(1+\chi+\Gamma gfl_t)\left[1 - e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right] \tag{56}$$

Similarly:

$$\frac{cov(m_{t+1},X^*_{t+1})}{E(m_{t+1})} = \frac{cov\left(m_{t+1},(1+i_t)\frac{S_t}{S_{t+1}}\right) - \underbrace{cov(m_{t+1},(1+i^*_t))}_{=0}}{E(m_{t+1})}$$
$$= \frac{E\left(m_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})} - E\left((1+i_t)\frac{S_t}{S_{t+1}}\right)$$
$$= \frac{E\left(e^{\tilde{m}_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}{E\left(e^{\tilde{m}_{t+1}}\right)} - E\left(e^{\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)$$
$$= e^{-\log(\beta)+\tilde{i}_t+\tilde{S}_t-E(\tilde{S}_{t+1})+\frac{V(\tilde{S}_{t+1})}{2}-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\left[1 - e^{cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\right]$$
$$= \frac{1}{\beta}e^{\tilde{i}_t+\tilde{S}_t-E(\tilde{S}_{t+1})+\frac{V(\tilde{S}_{t+1})}{2}+E(\tilde{m}^*_{t+1})+\frac{V(\tilde{m}^*_{t+1})}{2}-cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\left[e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})} - e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right]$$
$$= \frac{1}{\beta}\underbrace{E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}_{1+\chi+\Gamma gfl_t}\left[e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})} - e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right]$$

where we used $-\log(\beta) = E(\tilde{m}^*_{t+1}) + \frac{V(\tilde{m}^*_{t+1})}{2}$. This yields Equation (47).

Now note that

$$\tilde{m}^*_{t+1} = \log(\beta) - \tilde{Y}^*_{t+1}, \tag{57}$$

.

To obtain $\tilde{m}_{t+1}$, we can rewrite the resource constraints (25) as

$$C_t = Y_t\left(1 + \frac{nfl_t}{Y_t}\right)$$
$$C_{t+1} = Y_{t+1}\left(1 - \frac{nfl_t}{Y_t}\frac{1+i^*_t}{1+g_{t+1}} - \frac{gfl_t}{Y_t}\frac{X^*_{t+1}}{1+g_{t+1}}\right)$$

with $1+g_{t+1} = Y_{t+1}/Y_t$. We used the fact that, in equilibrium, $(H_t/S_t - h^H_t)i_t S_t/S_{t+1}$ is equal to zero (either $H_t/S_t - h^H_t = 0$ or $i_t = 0$). Taking logs and using a second-order approximation (assuming $\tilde{Y}_{t+1}$, nfl_t/Y_t, gfl_t/Y_t, X^*_{t+1} and g_{t+1} are small), we obtain

$$\tilde{C}_t = \tilde{Y}_t + \frac{nfl_t}{Y_t} - \frac{1}{2}\left(\frac{nfl_t}{Y_t}\right)^2$$
$$\tilde{C}_{t+1} = \tilde{Y}_{t+1} - \frac{nfl_t}{Y_t}(1+i^*_t-g_{t+1}) + \frac{1}{2}\left(\frac{nfl_t}{Y_t}\right)^2(1+i^*_t) - \frac{gfl_t}{Y_t}(X^*_{t+1}-g_{t+1})$$

Finally, we use the approximation $g_{t+1} = \tilde{Y}_{t+1} - \tilde{Y}_t$ along with the assumption that $Y_t = 1$ and hence $\tilde{Y}_t = 0$ to obtain the approximated household's budget constraints:

$$\tilde{c}_t = nfl_t - \frac{1}{2}nfl_t^2$$
$$\tilde{c}_{t+1} = \tilde{y}_{t+1}(1 + nfl_t + gfl_t) - \left(nfl_t - \frac{1}{2}nfl_t^2\right)(1+i^*_t) - gfl_t X^*_{t+1} \tag{58}$$

Using (58), we get

$$\begin{aligned}\tilde{m}_{t+1} &= \tilde{C}_t - \tilde{C}_{t+1} \\ &= \log(\beta) - \alpha \tilde{Y}^*_{t+1}\left(1 + nfl_t + gfl_t\right) + (nfl_t - nfl_t^2/2)(2 + \tilde{i}^*_t) + gfl_t(\tilde{i}_t - \tilde{i}^*_t + \tilde{S}_t - \rho \tilde{Y}^*_{t+1}),\end{aligned} \tag{59}$$

using $\tilde{Y}_{t+1} = \alpha \tilde{Y}^*_{t+1}$, $X^*_{t+1} = \tilde{i}_t - \tilde{i}^*_t + \tilde{S}_t - \tilde{S}_{t+1}$ and $\tilde{S}_{t+1} = \rho \tilde{Y}^*_{t+1}$. Therefore, using (57) and (**??**), we find Equation (48).

D.3 Proof of Lemma 2

Another way to write Equation (29) is:

$$\begin{aligned}
E\left(m_{t+1}(1+i^*_t)\right) - E\left(m_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right) + \frac{\alpha_0 \Gamma}{\eta_t} &= 0 \\
\underbrace{(1+i^*_t)}_{\frac{1}{E(m^*_{t+1})}} E\left(m_{t+1}\right) - \underbrace{(1+i_t)}_{\frac{1+\chi+\Gamma gfl_t}{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}} E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right) + \frac{\alpha_0 \Gamma}{\eta_t} &= 0 \\
\frac{1}{E(m^*_{t+1})} - \frac{1+\chi+\Gamma gfl_t}{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)} \frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})} + \frac{\alpha_0 \Gamma}{\eta_t E(m_{t+1})} &= 0 \\
1 - (1+\chi+\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} + \frac{\beta \alpha_0 \Gamma E(m^*_{t+1})}{\eta_t E(m_{t+1})} &= 0
\end{aligned}$$

Equation (39) yields

$$\begin{aligned}
1 - (1+\chi+\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} - \Gamma gfl_t \frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} &= 0 \\
1 - (1+\chi+2\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} &= 0
\end{aligned}$$

Besides,

$$\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} = e^{cov(\tilde{m}^*_{t+1}, \tilde{S}_{t+1}) - cov(\tilde{m}_{t+1}, \tilde{S}_{t+1})} = e^{-\Delta cov} \tag{60}$$

Hence result (ii) of Lemma 2.

Note that (34) implies that

$$\frac{\Lambda}{\eta_t} = 1 - E[m_{t+1}(1+i^*_t)]$$

$\Lambda = 0$ is equivalent to

$$\begin{aligned}
E[m_{t+1}(1+i^*_t)] &= 1 \\
E(m_{t+1}) &= \beta \\
e^{E(\tilde{m}_{t+1}) + \frac{1}{2}V(\tilde{m}_{t+1})} &= \beta
\end{aligned}$$

where we used (46) and where $\tilde{m}_{t+1}$ is given by (59).

We have

$$E(\tilde{m}_{t+1}) = \log(\beta) - (1 + nfl + gfl)\frac{\sigma_y^2}{2} + [1 - \log(\beta)]\left(nfl_t - \frac{1}{2}nfl_t^2\right) + \left(-\frac{\rho^2\sigma_y^2}{2} - \rho\sigma_y^2 + \chi + \Gamma gfl_t\right) gfl_t$$

where we used $\log(1 + \chi + \Gamma gfl_t) \simeq \chi + \Gamma gfl_t$, and

$$\frac{1}{2}V(\tilde{m}_{t+1}) = \alpha^2(1 + nfl + gfl)\frac{\sigma_y^2}{2} + \frac{\rho^2\sigma_y^2}{2}gfl_t$$

Therefore, $\Lambda = 0$ is equivalent to $\tilde{m}(nfl_t, gfl_t) = 0$ with

$$\tilde{m}(nfl_t, gfl_t) = -(1 - \alpha^2)(1 + nfl + gfl)\frac{\sigma_y^2}{2} + [1 - \log(\beta)]\left(nfl_t - \frac{1}{2}nfl_t^2\right) + \left(-\rho\sigma_y^2 + \chi + \Gamma gfl_t\right) gfl_t \quad (61)$$

$\tilde{m}(nfl_t, gfl_t)$ is increasing in nfl_t if $nfl_t < -\log(\beta) + \alpha^2)$. We consider only solutions that satisfy this condition. In that case, the solution is unique. Denote by $\overline{nfl}(gfl_t)$ this solution. If $\overline{nfl}(gfl_t) > b^G - h_t^H = b^G - 1$, then $nfl_t = b^G - 1$ and $\Lambda > 0$.

D.4 Solutions for $\widehat{gfl}_t$ and $\widehat{nfl}_t$

For a given nfl_t, gfl_t is implicitly defined by

$$1 - (1 + \chi + 2\Gamma gfl_t)e^{-\rho\sigma_y^2[1 - \alpha(1 + nfl_t + gfl_t) - \rho gfl_t]} = 0$$

Using $\log(1 + \chi + 2\Gamma gfl_t) \simeq \chi + 2\Gamma gfl_t$, this yields

$$\chi + 2\Gamma gfl_t - \rho\sigma_y^2\left[1 - \alpha(1 + nfl_t + gfl_t) - \rho gfl_t\right] = 0$$

After rearranging, we obtain (49).

If $\lambda = 0$, (49) and $\tilde{m}(nfl_t, gfl_t) = 0$ jointly define nfl_t and gfl_t. If $\Lambda > 0$, then gfl_t is defined by (49) with $nfl_t = b_t^G - 1$.

Consider the case where $\Lambda = 0$. As before, consider solutions where $nfl_t < -\log(\beta) + \alpha^2)$ and denote by $\overline{nfl}(gfl_t)$ the unique solution. Suppose additionally that $1 + \overline{nfl}(gfl_t) + gfl_t > 0$. If σ_y^2 is large, and α, χ and Γ are small, then $\tilde{m}(nfl_t, gfl_t) = 0$ implies that $nfl_t - nfl_t^2/2 > 0$. As long as $nfl_t < 2$, this implies that $nfl_t > 0$.

Special case with $\alpha = 0$ In the special case where $\alpha = 0$, we can compute implicit solutions for nfl_t and gfl_t when $\Lambda = 0$.

First, in that case, (49) implies

$$\widehat{gfl}_t = \frac{\rho\sigma_y^2 - \chi}{2\Gamma + \rho^2\sigma_y^2}$$

and $\widehat{nfl}_t$ is the solution to the second-order polynomial equation $\tilde{m}(nfl_t, gfl_t) = 0$ that is on the increasing segment of the polynomial $\tilde{m}(nfl_t, gfl_t)$:

$$\widehat{nfl}_t = \frac{1}{2}\left[1 - \frac{\sigma_y^2}{2[1 - \log(\beta)]} - \sqrt{\left[1 - \frac{\sigma_y^2}{2[1 - \log(\beta)]}\right]^2 - 4\frac{\frac{\sigma_y^2}{2}(1 + \widehat{gfl}_t) - (-\rho\sigma_y^2 + \chi + \Gamma\widehat{gfl}_t)\widehat{gfl}_t}{1 - \log(\beta)}}\right]$$

D.5 Proof of Proposition 3

Note that the CIP deviation, as defined in (9), is increasing in gfl_t (hence (i)), since $E(m^*_{t+1}) = \beta$ is fixed, and $a_t^{H*} = gfl_t$.

Finally, note that the UIP deviation can be written as (we use (10), (56) and $E(m^*_{t+1}) = \beta$ as well):

$$\begin{aligned} E_t X^*_{t+1} &= \tfrac{1}{\beta}\left[\chi + \Gamma gfl_t - (1+\chi+\Gamma gfl_t)(1-e^{-\rho\sigma_y^2})\right] \\ &= -\tfrac{1}{\beta}\left[1 - (1+\chi+\Gamma gfl_t)e^{-\rho\sigma_y^2}\right] \end{aligned}$$

where we used the results in D.2. Replacing gfl_t with $\widehat{gfl}_t$ and nfl_t with $b_t^G - 1$, we obtain

$$\begin{aligned} E_t X^*_{t+1} &= -\tfrac{1}{\beta}\left[1 - \left(1+\chi+\Gamma\tfrac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2}\right)e^{-\rho\sigma_y^2}\right] \\ &\simeq -\tfrac{1}{\beta}\left[1 - e^{\chi+\Gamma\frac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2}-\rho\sigma_y^2}\right] \end{aligned}$$

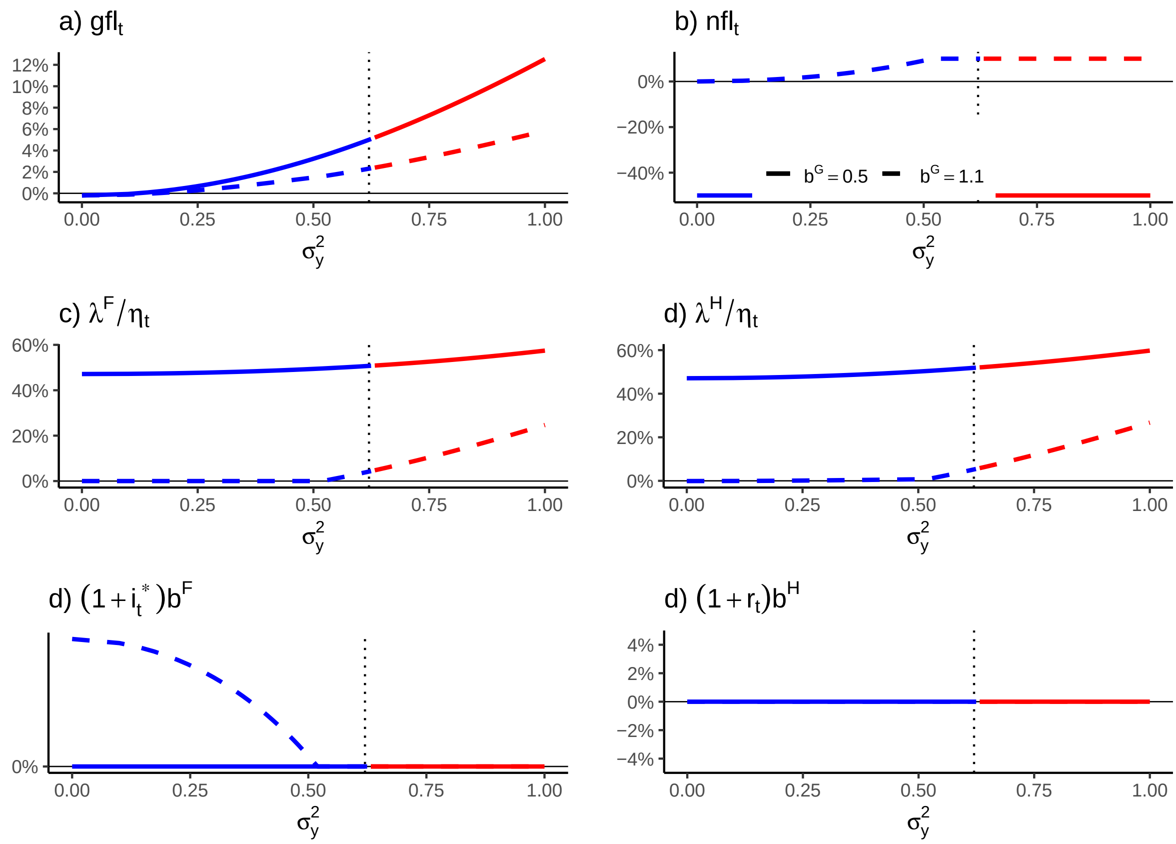
The derivative of $E_t X^*_{t+1}$ with respect to σ_y^2 is of the same sign as

$$-\rho + \Gamma\frac{2\Gamma\rho(1-\alpha b_t^G) + \chi\rho(\alpha+\rho)}{[2\Gamma+\rho(\alpha+\rho)\sigma_y^2]^2}$$

Therefore, $E_t X^*_{t+1}$ is decreasing in σ_y if Γ is not too large (hence (ii)).

E Additional Figures

Figure E.1: Comparative statics of σ_y^2 - continued



Notes: Baseline parameters : $\beta = 0.98, \chi = 0.002\,\Gamma = 0.5, \alpha = 0.6, \rho = 0.2$. We assume that $\bar{b}^H = \bar{b}^F = 0$.

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