A Macroeconomic Model of Central Bank Digital Currency*

Pascal Paul Mauricio Ulate Jing Cynthia Wu

February 2024

Abstract

We develop a quantitative New Keynesian DSGE model to study the introduction of a central bank digital currency (CBDC): digital government-backed money available to retail consumers. At the heart of our model are monopolistic banks with market power in deposit and loan markets. When CBDC is introduced, households benefit from the expansion of liquidity services and higher deposit rates as bank deposit market power is curtailed. However, deposits also flow out of the banking system and bank lending contracts. We assess this welfare trade-off for a wide range of economies that differ in their level of interest rates. We find substantial welfare gains of introducing CBDC, with an optimal CBDC interest rate that can be approximated by a simple rule of thumb: it equals the maximum between 0% and the policy rate minus 1%.

JEL codes: E3, E4, E5, G21, G51.
Keywords: Central bank digital currency, Banks, DSGE, Monetary policy.

*Paul and Ulate: Federal Reserve Bank of San Francisco, Wu: University of Notre Dame and NBER. We thank Pengfei Jia, Ashley Lannquist, Emi Nakamura, and Sanjay Singh for their useful comments and suggestions. We also thank Caroline Paulson for excellent research assistance. Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the FRBSF or the Federal Reserve System.

1 Introduction

The introduction of a central bank digital currency is one of the most far-reaching innovations that central banks have considered over the last decades. By 2023, 11 countries have officially adopted a CBDC and 19 of the G20 countries are advancing CBDC projects, most prominently the European Central Bank. The introduction of such a new currency can drastically change the financial landscape and raises a number of salient questions. First and foremost, is the introduction of a CBDC beneficial for an economy as a whole? Second, how should central banks set the interest rate on CBDC, and how does this rate depend on the state of an economy, in particular the level of interest rates? And third, how does the presence of a CBDC affect the conduct of monetary policy and the behavior of an economy over the business cycle? In this paper, we seek to answer these questions by proposing a new general equilibrium model that features a realistic banking sector and that is closely calibrated to empirical evidence.

To preview the key mechanisms, we start with a static partial equilibrium model of deposit intermediation. This simple framework has two important features. First, cash, deposits, and CBDC provide households with liquidity benefits and the three instruments are imperfectly substitutable. Second, banks are monopolistic and set the deposit rate as a variable markdown on the policy rate. As a result, banks' deposit market power and the competition between the three liquidity-providing instruments jointly determine the difference between the policy rate and the deposit rate, which is the deposit spread that banks charge.

Our static framework illustrates the following relations. In the absence of CBDC, the deposit spread rises with the level of the policy rate since banks gain market power as the rate on cash is fixed at zero (c.f., Drechsler et al., 2017). When CBDC is introduced, the deposit spread decreases since households value the liquidity benefits that CBDC provides and lower their deposit holdings. The deposit spread falls the most if the rate on CBDC is close to the policy rate since CBDC is a stronger competitor to deposits within that range. Thus, in an environment with a high policy rate and a large deposit spread, a CBDC that pays interest can be an important competitive force, lowering banks' deposit market power.

Besides the behavior of the deposit spread, the static framework also points to the key trade-off that determines the impact of introducing CBDC on welfare in general equilibrium. On one hand, households benefit from CBDC since it is a liquidity-providing instrument that they desire and because it provides competition to bank deposits which lowers deposit spreads. On the other hand, banks not only have to raise their deposit rates but also face deposit outflows, both lowering their profitability and decreasing bank intermediation capacity.

To fully explore this trade-off, we enrich the static framework with a set of features that are particularly relevant in this context: a banking sector that intermediates between deposit and loan markets, financial frictions that make bank capital slow-moving and determine credit supply, a corporate production sector, a bond market that can substitute for bank financing, and nominal price rigidities building on the New Keynesian tradition. We tightly calibrate the model to U.S. data and show that it successfully matches loan and bond spreads, as well as historical deposit rates for various levels of the policy rate.

We use the model as a laboratory to explore the effects of introducing CBDC and its role in monetary policy transmission. First, we investigate how the impact of CBDC introduction varies with the level of CBDC remuneration, that is, its interest rate. Interestingly, the welfare change displays an inverted U-shape. If CBDC pays a low interest rate, households hold a negligible amount of CBDC in their portfolios and banks' deposit market power is largely unaffected, limiting the potential gains from CBDC introduction. By contrast, if CBDC pays a high interest rate, households flock to CBDC, deposits pour out of the banking sector, and bank profitability and bank lending contract substantially. As a result, the welfare impact of CBDC turns negative as the bank disintermediation effect that leads to lower aggregate investment and output dominates the beneficial effects of CBDC. Thus, the model delivers a unique optimal CBDC rate. For our baseline economy that is calibrated to U.S. data, this rate is different from zero and lies at around 0.8%.

Second, instead of studying the introduction of CBDC for a specific economy, we analyze the effects of introducing CBDC in many economies that differ in the level of their steady-state policy rates. To start, we assess the introduction of a CBDC that pays zero interest as often envisioned by countries that plan to introduce CBDC. For a large range of negative as well as positive policy rates, we find positive welfare gains that are smaller in high interest rate economies where households would hold only small amounts of CBDC and bank deposit market power is barely challenged.

While encouraging, this exercise hides the fact that a remunerated CBDC can lead to substantially higher welfare gains. To explore this possibility, we determine the CBDC rate that maximizes welfare for each of these economies. For policy rates below 1%, the optimal CBDC rate is slightly negative and can even be higher than the policy rate. For policy rates above 1%, the optimal CBDC rate lies between 80 and 120 basis points below the policy rate. We show that this welfare-maximizing CBDC rate can be well approximated by a simple rule of thumb: it is the maximum between 0% and the steady-state policy rate minus 1%. The simplicity of this rule is appealing since it can be applied to many economies that differ substantially in their level of interest rates. Central banks can also easily communicate this remuneration scheme to households and avoid the political-economy concerns that can potentially arise when CBDC pays negative interest.

The introduction of a CDBC with such a remuneration scheme has far-reaching effects on the banking system. Particularly striking is how banks' deposit market power is curtailed in high interest rate environments. At a policy rate of 5%, banks charge a substantial deposit spread of around 2.5% in the absence of CBDC. If CBDC is introduced at its optimal rate, the deposit rate rises from around 2.5% to 4.3%, diminishing the deposit spread to only 70 basis points. In fact, for the range of policy rates between 2% and 7%, we find that the positive relation between the deposit spread and the level of the policy rate vanishes after the CBDC introduction, and that the deposit spread stabilizes around the aforementioned 70-basis-point level. These results connect with the intuition from our static framework: while cash is a weak competitor to deposits at high interest rates, a CBDC that pays interest can substantially curtail bank market power in deposit markets.

The scaling down of bank market power in deposit markets at high interest rates is also reflected in the welfare changes from a CBDC introduction across policy rates. For policy rates below 2%, we find positive but modest welfare gains of around 0.25%—measured as the multiplicative consumption-equivalent variation required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady states. However, this number increases in high interest rate environments. For example, for a policy rate of 6%, we find a sizeable welfare gain of around 1% using our welfare measure.

Finally, we explore the role of CBDC in potentially altering the response of an economy to typical business cycle innovations. Across a wide range of CBDC remuneration schemes, we find that the reactions of various macroeconomic indicators to standard monetary policy and technology shocks are remarkably similar. Thus, even though the introduction of a CBDC can lead to lasting welfare effects and changes in the financial landscape, responses to transitory shocks remain almost identical.

Related Literature. Our paper contributes to the new and rapidly emerging literature on the macroeconomics of CBDC.[1] In particular, our work is closely related to studies that examine the impact of CBDC on bank disintermediation in a macroeconomic framework.

Most existing studies base their analysis on a New Monetarist approach. For example, Keister and Sanches (2022) show that CBDC causes bank disintermediation as it crowds out bank deposits, leading to a decline in investment. However, they find that intro-

[1] See, e.g., Chapman et al. (2023), Infante et al. (2023), and Ahnert et al. (2022) for recent surveys.

ducing CBDC often raises welfare because it improves payment efficiency. Williamson (2022b) develops a model of banking and payments in which firms are subject to collateral constraints and CBDC is introduced through a narrow banking facility. He finds that CBDC can be welfare-improving as it uses safe assets more efficiently and helps to mitigate a capital over-accumulation problem. In contrast to these models with competitive banking, Andolfatto (2021) considers a model with monopolistic banks and finds that the introduction of a CBDC can increase a bank's deposit rate and thus increase deposit financing while not necessarily impacting bank lending. Chiu et al. (2023) use a micro-founded model of payments where banks engage in oligopolistic competition in the deposit market. They show that CBDC may crowd-in or crowd-out bank intermediation, depending on the interest rate paid on CBDC. Relative to these contributions, we consider a New Keynesian DSGE model with imperfect substitutability between bank deposits and CBDC, bank market power in deposit and loan markets, and where bank profitability matters for bank lending.

Up to this point, relatively few papers have studied the macroeconomic effects of introducing CBDC in a DSGE model of the type that is commonly used by central banks. Barrdear and Kumhof (2022) find that CBDC issuance of 30 percent of GDP against government bonds could lower the real interest rate and thus increase GDP by 3 percent. Most closely related to our work is the paper by Burlon et al. (2023), who find that the introduction of CBDC can lead to substantial welfare gains. In comparison, our model features bank market power in deposit markets, which gives rise to the endogenous deposit spread that we highlight, as well as nonbank lending through the bond market. As a result, our model allows for two additional channels through which CBDC can lead to relatively higher welfare gains.

The welfare gains of introducing CBDC may also be higher if the bank disintermediation effect is dampened, which may occur for the following two reasons. Using a banking industry equilibrium model, Whited et al. (2023) show that banks largely replace lost deposits with wholesale funding, such that bank lending only contracts by a fourth of the deposits lost. Similarly, Abad et al. (2023) find that banks mainly decrease their excess reserves when deposits leave as opposed to contracting their lending. In our framework, banks are able to replace deposits with wholesale funding and decrease their reserves.

Several other papers study optimal monetary policy and CBDC design. Brunnermeier and Niepelt (2019) formulate conditions under which a swap of private money for CBDC is irrelevant to economic allocation. Davoodalhosseini (2021) explores optimal monetary policy in a model where agents use cash and CBDC as payment instruments. Agur et al. (2022) consider the optimal design of CBDC in the presence of network effects. Closely related to our work, Niepelt (2023) studies the optimal quantity of CBDC in a standard growth and business cycle model where banks are monopsonists in deposit markets. He finds that the welfare-maximizing share of CBDC in payments generally exceeds that of deposits. In comparison, our framework features nominal rigidities, bank market power in loan markets, nonbank lending, and a role for bank profitability to determine credit supply.

The modeling differences that we highlight distinguish our paper from the literature and allow us to assess the quantitative importance of the mentioned channels following an introduction of a CBDC. One key contribution is to show that the welfare-maximizing CBDC rates for economies that differ in their levels of interest rates can be approximated by a simple rule of thumb. We further reveal that economies with higher policy rates can obtain larger welfare gains from introducing CBDC. That is because bank deposit market power—an important feature of our model—is reduced relatively more in such environments.

2 A Static Bank Deposit Model

How does the introduction of CBDC affect the deposit rate and its spread relative to the policy rate? And how does this relationship change with the level of the policy rate and the interest that CBDC pays? In this section, we present a static partial equilibrium model of deposit intermediation with monopolistic banks to answer these questions. This simple model facilitates analytical tractability and helps to build intuition for the results of the larger quantitative DSGE model that we discuss in Section 3.

2.1 Deposit Supply Functions

To start, we take the household's deposit supply schedule as given. Section 3 shows how such a schedule can be formally derived from the household optimization problem. The household has access to three liquidity-providing instruments: cash (m), aggregate deposits (d), and CBDC. Their returns are zero, i^d, and i^{cbdc}, respectively. The aggregate deposit supply function is

$$d = \gamma_d \left(\frac{1 + i^d}{1 + i^{\mathcal{L}}} \right)^{\theta} \mathcal{L}, \tag{2.1}$$

where θ is the elasticity of substitution between the three aggregate liquidity-providing instruments, γ_d is described below, and $\mathcal{L}$ is the real aggregate liquidity supplied by the household, which we take as given for now and endogenize in Section 3. Equation (2.1) specifies that deposit supply depends positively on the ratio of the gross deposit rate to the gross rate on liquid instruments, defined as

$$1 + i^{\mathcal{L}} = \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}. \tag{2.2}$$

The coefficients γ_m, γ_d, and γ_{cbdc} determine the importance of each of the instruments to the household due to exogenous non-interest-rate characteristics, and they satisfy $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Aggregate deposits d, in turn, are comprised of deposits in n individual banks, each of which is indexed by j. Bank j pays a deposit rate of i_j^d and faces an individual deposit supply function given by

$$d_j = \frac{1}{n}\left(\frac{1+i_j^d}{1+i^d}\right)^{\varepsilon^d} d, \tag{2.3}$$

where ε^d is the elasticity of substitution between different banks. Equation (2.3) indicates that the supply of deposits to bank j depends positively on the ratio of its gross deposit rate to the aggregate gross deposit rate, which is defined as

$$1 + i^d = \left(\sum_{j=1}^{n} \frac{1}{n}(1+i_j^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}. \tag{2.4}$$

2.2 Banks

At the beginning of the period, each individual bank is endowed with equity f_j and issues deposits d_j. The bank uses these funds to finance its holding of reserves h_j, which pay the policy rate i. For simplicity, reserves are the only asset that banks invest in, an assumption that we relax later. Bank j's balance sheet condition is therefore:

$$h_j = f_j + d_j. \tag{2.5}$$

The bank maximizes its end-of-period equity

$$\max_{i_j^d, d_j, h_j} \quad (1+i)h_j - (1+i_j^d)d_j,$$

subject to the deposit supply equations (2.1)-(2.4) and the balance sheet constraint (2.5). Each bank has some monopoly power, and it chooses the interest rate it pays on deposits, the amount of deposits it takes on, and how many reserves to hold. The first order condition for this bank problem is

$$1 + i_j^d \;=\; \frac{\epsilon_j^d}{\epsilon_j^d + 1}(1 + i), \tag{2.6}$$

where ϵ_j^d is the *endogenous* elasticity of deposits with respect to the deposit rate, that is, $\epsilon_j^d \equiv \partial \ln d_j / \partial \ln(1 + i_j^d)$; see Appendix A for derivations. Equation (2.6) highlights that bank j sets its deposit rate as a markdown on the policy rate. Assuming all banks are symmetric, we can express the endogenous elasticity as

$$\epsilon^d \;=\; \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n}(1 - \omega_{\mathcal{L}}^d), \tag{2.7}$$

where

$$\omega_{\mathcal{L}}^d = \frac{(1+i^d)d}{(1+i^{\mathcal{L}})\mathcal{L}} = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \tag{2.8}$$

is the endogenous share of liquidity that stems from deposits at the end of the period, which we label the "endogenous deposit share" for short. When cash, deposits, and CBDC pay the same interest rate, the endogenous share coincides with the exogenous share, $\omega_{\mathcal{L}}^d = \gamma_d$.

Equation (2.7) shows that the endogenous elasticity of bank deposits with respect to the deposit rate is a combination of two elasticities. With weight $(n-1)/n$, it simply reflects the exogenous elasticity ε^d with which depositors substitute across different banks. With the complementary weight $1/n$, it depends on how aggregate deposit supply reacts to changes in the aggregate deposit rate, which individual banks partially internalize because of their monopoly power and non-infinitesimal size.[2] Given that all banks face the same endogenous elasticity, equation (2.6) can be expressed as

$$\frac{i - i^d}{1 + i^d} = \frac{1}{\epsilon^d}, \tag{2.9}$$

where $(i - i^d)/(1 + i^d)$ represents the spread that banks make when they accept deposits

[2] Atkeson and Burstein (2008) derive a similar equation, but their focus is on the goods market, whereas we study bank deposits.

at rate i^d and keep them at the central bank earning the policy rate, normalized by $1 + i^d$. This deposit spread is solely determined by the endogenous deposit elasticity. Taken together, equations (2.2) and (2.7)-(2.9) form a system that determines i^d, ϵ^d, $\omega^d_{\mathcal{L}}$, and $i^{\mathcal{L}}$ simultaneously.

2.3 How Central Bank Interest Rates Affect Deposit Rates

We first inspect how interest rates controlled by the central bank, namely the rate on reserves (the policy rate) and the rate on CBDC, affect the deposit rate and the deposit spread.

Proposition 1.

1. *The deposit rate increases with the policy rate and the rate on CBDC.*
2. *The deposit spread increases with the policy rate but decreases with the rate on CBDC.*
3. *Deposits increase with the policy rate but decrease with the rate on CBDC.*

Proof: see Appendix A.2.

The result on the deposit rate is intuitive, it shows the spillover from the central bank's policy instruments to the rates that are relevant for banks and households. But why do the two rates have opposite effects on the deposit spread and the amount of deposits? Equations (2.7), (2.8), and (2.9) are the key expressions that capture the transmission mechanism. When the rate on reserves increases, banks pass a fraction of this higher rate to their depositors. A higher deposit rate increases the endogenous deposit share $\omega^d_{\mathcal{L}}$, which has two effects. First, a higher deposit share translates directly into more deposits given the aggregate liquidity being fixed. Second, a higher deposit share lowers the endogenous elasticity ϵ^d and increases the deposit spread. This mechanism is also present in Drechsler et al. (2017). With a higher policy rate, banks gain market power relative to alternative liquid instruments and therefore charge a higher spread. On the other hand, when the rate on CBDC increases, CBDC poses more competition to banks, the endogenous deposit share decreases, which decreases both deposits and the deposit spread.

2.4 Effects of Introducing CBDC

Next, we turn to the core question of interest: what happens to the deposit rate and the deposit spread when the central bank introduces CBDC? We capture the introduction of CBDC by changing the interest rate paid on CBDC from -100% to some higher percent that

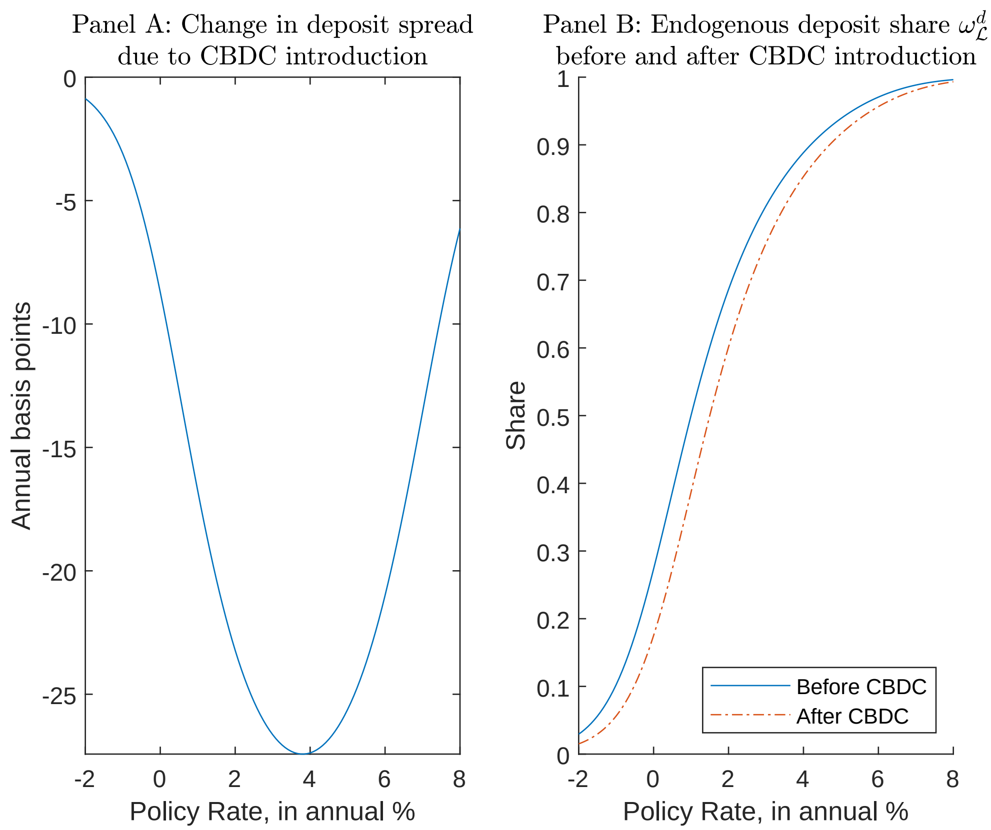

Figure 2.1: Panel A: Change in the deposit spread following the introduction of CBDC across different values of the policy rate. Panel B: Endogenous deposit share ($\omega^d_{\mathcal{L}}$) across different values of the policy rate before and after the introduction of CBDC. The figure uses the baseline calibration described in Section 4.

is roughly in the vicinity of 0%. We choose -100% as a starting point since that corresponds to the case where CBDC is not used at all in our larger DSGE model.

According to Proposition 1, the introduction of CBDC increases the deposit rate, decreases the deposit spread, and induces deposit outflow. Based on a calibration that corresponds to the one used in Section 3, Panel A of Figure 2.1 plots the change in the deposit spread when CBDC is introduced with a zero percent interest rate, as a function of the policy rate. Interestingly, it displays a U-shape. What is the intuition? Per equations (2.7), (2.8), and (2.9), it works through the endogenous elasticity via the endogenous deposit share $\omega^d_{\mathcal{L}}$, which is plotted in Panel B of Figure 2.1. When CBDC pays zero interest and the policy rate is high, CBDC and cash barely compete with deposits, and hence $\omega^d_{\mathcal{L}}$ is close to one regardless of whether CBDC exists or not. Therefore, the introduction of CBDC leaves $\omega^d_{\mathcal{L}}$ mostly unaffected and hence the deposit spread remains roughly unchanged. In the other extreme, for a fairly negative policy rate, deposits are undesirable compared to cash or CBDC. Therefore, $\omega^d_{\mathcal{L}}$ is close to zero regardless of the existence of

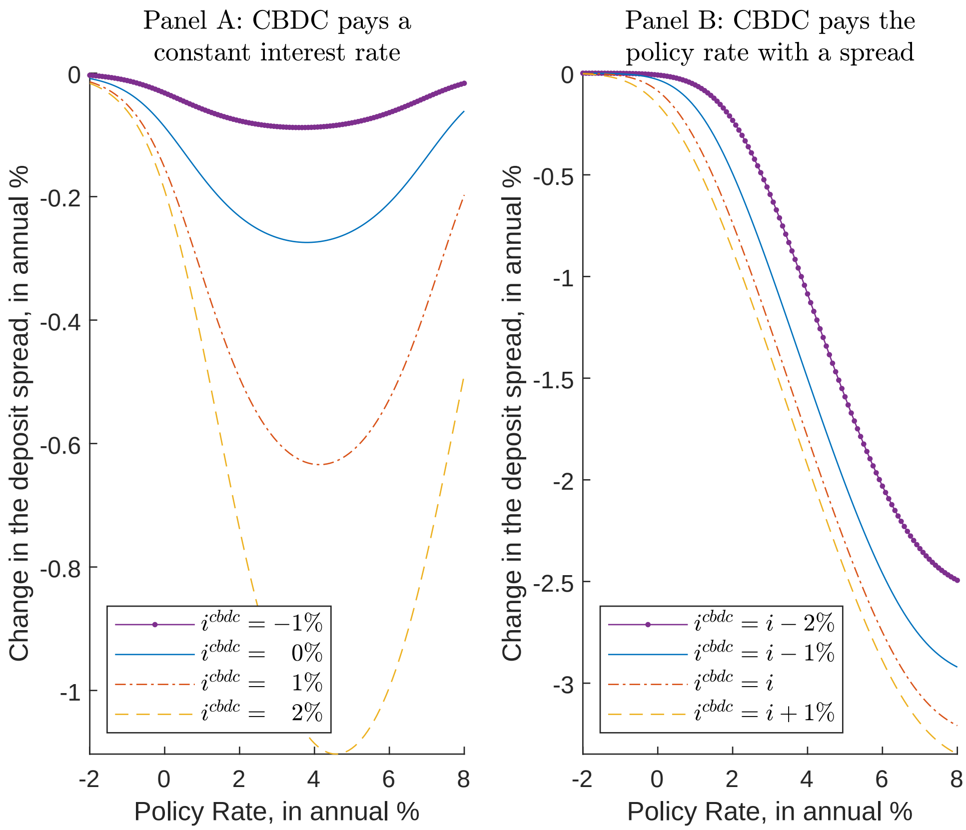

Figure 2.2: Change in the deposit spread following the introduction of CBDC, across different values of the policy rate, for different choices on the interest rate of CBDC (i^{cbdc}). Panel A depicts a CBDC that pays a constant interest rate, Panel B depicts a CBDC that pays the policy rate with a fixed spread.

CBDC.

Only when the policy rate is at intermediate levels, CBDC and deposits are close substitutes. In this case, introducing CBDC affects the endogenous share and hence the endogenous elasticity of deposits substantially. Therefore, the deposit spread drops the most for moderate levels of the policy rate.

The U-shape is not unique to a CBDC that pays a zero interest rate. Panel A of Figure 2.2 shows this shape holds as long as CBDC pays a constant interest rate. A higher interest rate on CBDC shifts the minimum of the curve towards the southeast: it increases the policy rate where CBDC introduction affects the deposit spread the most while also increasing the maximum change in the spread in absolute value.

Alternatively, when the interest rate on CBDC is pegged to the policy rate, the U-shape disappears, as shown in Panel B of Figure 2.2. In this case, the change in the deposit spread is a decreasing function of the policy rate. This occurs because CBDC becomes a more effective competitor to deposits the higher the policy rate is.

Thus, this simplified model already points to an important tradeoff of a potential CBDC introduction. While such a policy can benefit households and shield them from the monopolistic power of banks, it can lower deposit spreads and therefore affect commercial bank profitability negatively. In dynamic models where commercial bank equity is slow moving and relevant for lending, a fall in bank profitability can have a negative impact on the economy. In the following section, we embed the static model into a New Keynesian DSGE model to quantify this tradeoff and further study aggregate welfare effects.

3 The DSGE Model

In this section, we introduce a full fledged DSGE model for quantitative analyses. The key players in the model are a representative household, banks with monopoly power, a production sector, and a government.

The deposit side of the banking sector builds upon the ingredients laid out in Section 2. In addition, banks also issue corporate loans and face several operational costs. The household has access to four saving instruments: bonds, cash, bank deposits, and CBDC, where the last three instruments provide liquidity services with imperfect substitution.

The production sector consists of a representative intermediate good firm, a representative capital producer, monopolistically competitive retail firms, and a representative final good producer. The intermediate good firm purchases capital from the capital producer and combines it with labor from the household to produce an intermediate good. Its capital input is aggregated from two types of capital with non-unitary substitution: "non-pledgeable capital," which is financed by unsecured bond borrowing, and "pledgeable capital," which is financed through bank loans.

Retail firms face the standard Calvo price rigidity and transform the intermediate good into differentiated retail goods, which are then aggregated into a final good by the final good producer. The government includes a central bank that conducts monetary policy and a fiscal authority with a balanced budget.

3.1 Household

Setup. The household's lifetime utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t) - v(N_t) \right),$$

where β is the discount factor, C_t is consumption, and N_t is labor supply. The household's budget constraint is given by

$$P_tC_t + B_t + \Phi(\mathcal{L}_t)P_t \quad = \quad W_tN_t + AH_{t-1} + T_t,$$

where P_t is the aggregate price level, B_t are nominal bond holdings, W_t is the nominal wage, and $\Phi(\mathcal{L}_t)$ is described below. T_t captures transfers that are exogenous from the household's perspective, including net transfers from the government as well as profits from firms and banks. AH_{t-1} refers to "assets in hand" that the household enters period t with, given by

$$AH_{t-1} = M_{t-1} + (1 + i_{t-1})B_{t-1} + \sum_{j=1}^{n}(1 + i^d_{j,t-1})D_{j,t-1} + (1 + i^{cbdc}_{t-1})CBDC_{t-1}.$$

The household can save in cash (M_t), bonds (B_t), deposits with any of the n different commercial banks ($D_{j,t}$), and CBDC ($CBDC_t$) if available, where capital letters denote nominal terms. The associated net nominal returns for these instruments are zero, i_t, $i^d_{j,t}$, and i^{cbdc}_t, respectively. The variable $\mathcal{L}_t$ in the budget constraint aggregates the various liquidity-providing instruments (cash, deposits, and CBDC), defined as

$$\mathcal{L}_t \quad = \quad \left(\gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} + \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} + \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}}, \tag{3.1}$$

where lowercase letters denote real variables (e.g., $m_t = M_t/P_t$). The parameter θ is the elasticity of substitution between liquid instruments and $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Additionally, real deposits d_t are an aggregation of deposits in n banks:

$$d_t = \left(\sum_{j=1}^{n} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{\frac{\varepsilon^d}{\varepsilon^d+1}},$$

where $\sum_{j=1}^{n} \alpha_j = 1$ and $\varepsilon^d \geq \theta$. The fact that cash, deposits, and CBDC are not perfect substitutes within $\mathcal{L}_t$ captures the possibility that the household uses them for different types of transactions because of their different properties. For example, bank deposits and CBDC are useful for online transactions while cash is not; cash provides better anonymity than deposits and CBDC; cash and CBDC are government-backed while bank deposits are not necessarily insured; cash is more likely to be subject to theft. For these reasons, among others, the representative household might want to hold a combination of liquidity-providing instruments instead of simply holding the one with the highest return. A similar argument holds for deposits from different banks.[3]

Lastly, $\Phi(\mathcal{L}_t)$ captures a non-linear cost function of acquiring liquidity. Initially, when the household has few liquid instruments, the cost of acquiring liquidity is less than one-for-one, $\Phi(\mathcal{L}_t) < \mathcal{L}_t$, which reflects the convenience benefit of holding liquidity.[4] Eventually, when agents get "satiated" with liquidity services, it can be the case that $\Phi(\mathcal{L}_t) > \mathcal{L}_t$.[5] We choose to introduce $\Phi(\cdot)$ directly in the budget constraint for simplicity. However, as shown in Appendix B.2, one can obtain the same first-order conditions for the liquidity-providing instruments by allowing them to enter the utility function instead.[6]

Equilibrium Conditions. Our setup delivers convenient equilibrium conditions. First, the first-order conditions with respect to labor and bonds are the usual intra-temporal condition for labor supply and the Euler equation:

$$v'(N_t) = u'(C_t)\left(\frac{W_t}{P_t}\right), \tag{3.2}$$

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). \tag{3.3}$$

Next, the holding schedules of the liquidity-providing instruments are

$$m_t = \gamma_m\left(\frac{1}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.4}$$

$$d_t = \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.5}$$

$$cbdc_t = \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t. \tag{3.6}$$

These holding schedules are well-defined even for negative values of the interest rates on deposits, CBDC, or overall liquidity. The interest rate for liquidity and aggregate deposits

[3]Note that, unlike the traditional CES aggregator, the exponents within the $\mathcal{L}_t$ and d_t aggregators are greater than one instead of smaller than one. This occurs because these aggregators enter the budget constraint instead of the utility function. Therefore, they must be convex (instead of concave) to prevent the household from bunching its choice into a single liquidity-providing instrument or a single bank.

[4]Balloch and Koby (2019) use a related, but different, cost function of liquidity in the context of negative nominal interest rates in Japan.

[5]This "satiation" is similar to the one described in Rognlie (2016).

[6]This holds as long as one assumes a GHH-style non-separable utility function between consumption and liquidity as shown in Appendix B.2.

are defined as

$$1 + i_t^{\mathcal{L}} \equiv \left(\gamma_m + \gamma_d(1 + i_t^d)^{\theta+1} + \gamma_{cbdc}(1 + i_t^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \tag{3.7}$$

and

$$1 + i_t^d \equiv \left(\sum_{j=1}^{n} \alpha_j (1 + i_{j,t}^d)^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}.$$

Furthermore, the amount of deposits that the household supplies to an individual bank amounts to

$$d_{j,t} = \alpha_j \left(\frac{1 + i_{j,t}^d}{1 + i_t^d}\right)^{\varepsilon^d} d_t, \tag{3.8}$$

and the equilibrium condition for the aggregator $\mathcal{L}_t$ is as follows:

$$\frac{1 + i_t^{\mathcal{L}}}{1 + i_t} = \Phi'(\mathcal{L}_t). \tag{3.9}$$

Appendix B.1 provides details on the derivations of the equilibrium conditions. Note that equations (2.1)-(2.4) are a special case of the equilibrium conditions above. Besides being static, Section 2 imposes the symmetry restriction that $\alpha_j = 1/n \ \forall j$.

3.2 Intermediate Good Firm

The intermediate good firm uses labor and capital to produce intermediate output. The production function is Cobb-Douglas:

$$Y_t^m = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{3.10}$$

where $0 < \alpha < 1$, Y_t^m is the amount of intermediate output produced, A_t is productivity, and K_t is capital input. The intermediate good firm purchases capital from a capital producer and finances its purchases via two possible channels. It borrows from the bond market to finance capital that cannot be used as collateral, denoted non-pledgeable capital K_t^{NP}, which reflects the empirical observation that bond borrowing is typically unsecured (Schwert, 2020). Alternatively, the firm can borrow from banks to purchase pledgeable capital K_t^P that can be used as collateral. Aggregate capital is a CES combination of these two types:

$$K_t = \left((1-\psi)^{\frac{1}{\theta^k}} (K_t^{NP})^{\frac{\theta^k-1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} (K_t^P)^{\frac{\theta^k-1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k-1}}, \tag{3.11}$$

where θ^k captures the elasticity of substitution between the two types. K_t^P is itself an aggregate of the pledgeable capital financed by each of the n banks:

$$K_t^P = \left(\sum_{j=1}^{n} (\alpha_j^l)^{\frac{1}{\varepsilon^l}} (K_{j,t}^P)^{\frac{\varepsilon^l-1}{\varepsilon^l}} \right)^{\frac{\varepsilon^l}{\varepsilon^l-1}},$$

where ε^l is the loan elasticity of substitution among banks and α_j^l captures the exogenous importance of a particular bank in the loan portfolio, with $\sum_{j=1}^{n} \alpha^l = 1$ and $\varepsilon^l \geq \theta^k$.

Capital is predetermined. In period $t-1$, the intermediate good firm borrows from the bond market or banks in order to purchase capital for next period's production at price Q_{t-1}. At time t, it sells its depreciated capital stock $(1-\delta)$ back to the capital producer after production. Meanwhile, it pays back the lenders who charge different interest rates: bank j charges the loan rate $i_{j,t-1}^l$, while the bond market charges the risk-free rate i_{t-1} plus a spread ϱ. The intermediate good firm's period t profit is

$$\begin{aligned} \Pi_t^m &= P_t^m Y_t^m - W_t N_t + (1-\delta) Q_t \sum_{j=1}^{n} K_{j,t}^P + (1-\delta) Q_t K_t^{NP} \\ &- \sum_{j=1}^{n} (1 + i_{j,t-1}^l) Q_{t-1} K_{j,t}^P - (1 + i_{t-1} + \varrho) Q_{t-1} K_t^{NP}. \end{aligned}$$

The intermediate good firm maximizes the present value of profits (discounted using the household's stochastic discount factor) by choosing labor and capital inputs. The associated optimality conditions are given in Appendix B.3, and they depend on the real wage, as well as on the effective one-period user costs of aggregate capital, pledgeable capital, and non-pledgeable capital, which we denote with z_t, z_t^P, and z_t^{NP}, respectively.

3.3 Capital Good Producer

The capital producer faces the capital accumulation equation:

$$\left[K_{t+1}^P + \sum_{j=1}^{n} K_{j,t+1}^{NP} \right] = (1-\delta) \left[K_t^P + \sum_{j=1}^{n} K_{j,t}^{NP} \right] + I_t \left(1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) \right), \tag{3.12}$$

where the function $\Xi(\cdot)$ captures investment adjustment costs and satisfies $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) \geq 0$. The problem of the capital producer in period t is:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left[Q_{t+\tau} I_{t+\tau} \left(1 - \Xi \left(\frac{I_{t+\tau}}{I_{t+\tau-1}} \right) \right) - P_{t+\tau} I_{t+\tau} \right],$$

where $\Lambda_{t,t+\tau}$ denotes the household's stochastic discount factor for discounting nominal flows from $t + \tau$ back to t. The first order condition of the capital producer is given in Appendix B.4.

3.4 Banks

Bank's Problem. The liability side of the bank balance sheet is similar to the one in Section 2.2, while on the asset side we also consider the possibility of lending to the production sector. The nominal balance sheet constraint of bank j now takes the form

$$L_{j,t} + H_{j,t} = F_{j,t} + D_{j,t}, \tag{3.13}$$

where $L_{j,t}$ represents lending to the intermediate firm, $H_{j,t}$ are reserves issued by the central bank, $F_{j,t}$ is bank equity, and $D_{j,t}$ are household deposits, all in nominal terms.

Besides adding bank lending, we introduce three additional features.[7] First, in each period, a bank returns an exogenous fraction, $1 - \omega$, of its profits to the household as dividends and spends a fraction ς of its nominal net worth to operate the managerial side of the bank. This setup implies that bank equity is slow moving, i.e., it takes time to replenish after a shock. Second, a bank pays a quadratic cost, denoted by $\Psi(L_{j,t}/F_{j,t})$, when its loan-to-equity ratio, $L_{j,t}/F_{j,t}$, deviates from a target value. This cost captures the idea that regulators discourage banks from having high levels of leverage by imposing punishments when banks breach certain capital requirements, while market forces incentivize banks to avoid levels of leverage that are too low. Together, the previous two assumptions imply that a fall in bank profitability stemming from the introduction of CBDC can impact bank equity, which in turn can affect bank lending. Finally, banks face exogenous costs of issuing loans, μ^l, and obtaining deposits, μ^d, expressed per dollar of loan or deposit issued. These costs are used to match the deposit and lending spreads without having to necessarily assume their existence is solely due to the presence of monopoly power.

With the assumptions described in the previous paragraph, the nominal resources that

[7] These features are adopted from Ulate (2021).

bank j has available when entering period $t+1$ are given by

$$S_{j,t+1} = (1 + i^l_{j,t} - \mu^l)L_{j,t} + (1 + i_t)H_{j,t} - (1 + i^d_{j,t} + \mu^d)D_{j,t} - \varsigma F_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}.$$

These total resources have to be used either to pay dividends or as next-period equity:

$$S_{j,t+1} = F_{j,t+1} + DIV_{j,t+1},$$

where dividends $DIV_{j,t+1}$ are a fraction $1-\omega$ of a bank's profit $X_{j,t+1}$:

$$DIV_{j,t+1} = (1-\omega)X_{j,t+1},$$

and profits $X_{j,t+1}$ are, in turn, defined as

$$\begin{aligned} X_{j,t+1} \equiv\; & i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} \\ -\; & \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}. \end{aligned} \tag{3.14}$$

We define $X_{j,t+1}$ as the net profit before paying managerial costs but after adjusting for inflation $\pi_{t+1} \equiv P_{t+1}/P_t - 1$. The inflation adjustment is purely for convenience, because it delivers a clean and interpretable expression for the law of motion of real bank equity, which takes the form

$$\frac{F_{j,t+1}}{P_{t+1}} = \frac{F_{j,t}}{P_t}(1-\varsigma) + \omega\frac{X_{j,t+1}}{P_{t+1}}. \tag{3.15}$$

If $\omega = \varsigma = 0$, then a bank's real equity is constant. The larger ω is, the more bank equity depends on profits and the more volatile it becomes.

A bank seeks to maximize the present discounted value of future dividends that it returns to the household. Hence, bank j's problem is:

$$\max \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau+1} DIV_{j,t+\tau+1}.$$

As shown in Appendix B.5.1, the solution to the bank's problem can be broken down into a deposit and a loan sub-problem that we discuss next.

Deposit Sub-problem. The deposit sub-problem amounts to

$$\max_{i_{j,t}^d} (i_t - i_{j,t}^d - \mu^d) D_{j,t},$$

subject to the deposit supply schedule $D_{j,t}(i_{j,t}^d)$ of the household given by equation (3.8). Assuming that a bank takes the decisions of all other banks as given, it sets its deposit rate as follows:

$$1 + i_{j,t}^d = \frac{\epsilon_{j,t}^d}{\epsilon_{j,t}^d + 1}(1 + i_t - \mu^d). \tag{3.16}$$

Expression (3.16) shows that banks set their gross deposit rates as a markdown on the gross policy rate minus the cost of issuing deposits. The markdown is determined by $\epsilon_{j,t}^d$, the endogenous elasticity of bank j's deposits with respect to its deposit rate:

$$\epsilon_{j,t}^d \equiv \frac{\partial d_{j,t}}{\partial(1 + i_{j,t}^d)} \frac{1 + i_{j,t}^d}{d_{j,t}}.$$

As shown in Appendix B.5.2, for the case with identical banks, this endogenous elasticity takes the form:

$$\epsilon_t^d = \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1 - \omega_{\mathcal{L},t}^d)\theta + \omega_{\mathcal{L},t}^d \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1 + i_t^{\mathcal{L}})}\right]. \tag{3.17}$$

where $\omega_{\mathcal{L},t}^d$ is again the endogenous deposit share

$$\omega_{\mathcal{L},t}^d \equiv \frac{(1 + i_t^d)d_t}{(1 + i_t^{\mathcal{L}})\mathcal{L}_t} = \gamma_d \left(\frac{1 + i_t^d}{1 + i_t^{\mathcal{L}}}\right)^{\theta+1}. \tag{3.18}$$

Note that we can recover the expression in equation (2.7) from equation (3.17) when $\partial \ln \mathcal{L}_t / \partial \ln(1 + i_t^{\mathcal{L}}) = 0$, which is imposed in Section 2, where $\mathcal{L}$ is assumed to be constant. Thus, even in the larger model, the interpretation of ϵ_t^d remains similar: it is a weighted average of the exogenous elasticities ε^d and θ, as well as $\partial \ln \mathcal{L}_t / \partial \ln(1 + i_t^{\mathcal{L}})$, where the weights for the last two terms are endogenous and can vary with the introduction of CBDC.

Loan Sub-problem. The loan sub-problem of bank j is:

$$\max_{i^l_{j,t}} \; (i^l_{j,t} - i_t - \mu^l)L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t},$$

subject to the loan demand schedule of the intermediate firm and $L_{j,t} = Q_t K^P_{j,t+1}$. As opposed to the markdown in the deposit rate, each individual bank sets its gross loan rate as a markup on the cost-adjusted policy rate:

$$1 + i^l_{j,t} \;=\; \frac{\epsilon^l_{j,t}}{\epsilon^l_{j,t} - 1}\left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right], \tag{3.19}$$

where $\epsilon^l_{j,t}$ denotes (the negative of) the endogenous loan elasticity of $l_{j,t}$ with respect to $1 + i^l_{j,t}$:

$$\epsilon^l_{j,t} \;\equiv\; -\frac{\partial l_{j,t}}{\partial(1 + i^l_{j,t})}\frac{1 + i^l_{j,t}}{l_{j,t}}.$$

As shown in Appendix B.5.3, for the case of identical banks, this endogenous elasticity takes the form:

$$\epsilon^l_t \;=\; \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[\theta^k(1 - \omega^{K_P}_{K,t}) + \frac{\omega^{K_P}_{K,t}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1 + i^l_t}{1 + i_t}\frac{1}{z^P_t}. \tag{3.20}$$

where $\omega^{K_P}_{K,t}$ is the expenditure on pledgeable capital as a share of total capital expenditure

$$\omega^{K_P}_{K,t} \;\equiv\; \frac{z^P_t K^P_t}{z_t K_t} = \psi\left(\frac{z^P_t}{z_t}\right)^{1-\theta^k}. \tag{3.21}$$

Equations (3.19)-(3.21) provide some intuition on the response of loan spreads to the introduction of CBDC which disintermediates banks and thereby decreases $\omega^{K_P}_{K,t}$. If θ^k is greater than $1/(1-\alpha)$, then the introduction of CBDC increases ϵ^l_t and therefore lowers the loan spread.

Finally, we discuss the similarities and differences between equations (3.17) and (3.20). For both, the endogenous elasticity puts a weight $(n-1)/n$ on the exogenous elasticity (ε^d or ε^l). The remaining weight of $1/n$ is split between the two elasticities inside the square brackets: an elasticity of substitution (θ for deposits or θ^k for loans) and the elasticity of total liquidity ($\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t)$) or capital ($\partial \ln K / \partial \ln z = 1/(1-\alpha)$) with respect to its price.[8]

3.5 Retail Firms and Final Good Producer

The setup of retail firms and the final good producer follows the typical modeling approach in the New-Keynesian literature. A continuum of retail firms indexed by $s \in [0, 1]$ transform intermediate output Y_t^m into differentiated retail goods $Y_t(s)$, which are aggregated into a final good Y_t by the final good producer via a CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(s)^{\frac{\varphi-1}{\varphi}} ds \right)^{\frac{\varphi}{\varphi-1}},$$

where φ is the elasticity of substitution between the differentiated retail goods. The optimization problem of the final good producer implies the following demand function for good s:

$$Y_t(s) = \left(\frac{P_t(s)}{P_t} \right)^{-\varphi} Y_t,$$

where the price index is given by

$$P_t = \left(\int_0^1 P_t(s)^{1-\varphi} ds \right)^{\frac{1}{1-\varphi}}.$$

Each period, a retail firm is able to freely adjust its price with probability $1 - \gamma$ as in the Calvo setup, and it chooses the optimal reset price P_t^* to solve:

$$\max_{P_t^*} \quad \mathbb{E}_t \sum_{\tau=0}^{\infty} \gamma^\tau \beta^\tau \frac{u'(C_{t+\tau})}{u'(C_t)} \frac{P_t}{P_{t+\tau}} \left[P_t^* - P_{t+\tau}^m \right] Y_{t+\tau|t},$$

where $Y_{t+r|t}$ is the amount sold in period $t + \tau$ by a firm that last reset its price in period t. The conditions describing the optimal behavior of retail firms are given in Appendix B.6.

[8] A further difference between the two equations is that (3.20) features a term outside the curly bracket to reflect the fact that loan demand reacts to $z_{j,t}^P$ (which is a function of $1 + i_{j,t}^l$) instead of to $1 + i_{j,t}^l$ directly; see the intermediate firm problem in Section 3.2.

3.6 Government

Monetary policy is characterized by a Taylor rule with interest-rate smoothing:

$$i_t = (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon_t^i, \tag{3.22}$$

where $\bar{\imath}$ is the steady state nominal rate, $\rho_i \in [0,1]$ reflects interest rate inertia, and ϵ_t^i is an exogenous shock to monetary policy. Note that the policy rate is also the rate on reserves, which is the same as the return on bonds. For simplicity, we assume government spending is a constant fraction of output

$$G_t = gY_t. \tag{3.23}$$

We also assume that the government balances its budget period-by-period. Therefore, the lump sum transfers from the government to the household are given by the proceeds from seigniorage (covering cash, reserves, and CBDC) net of government expenditures.

3.7 Resource Constraint and Shocks

Output is divided between consumption, investment (for the two types of capital), government expenditure, and adjustment costs. The economy-wide resource constraint is thus given by

$$Y_t = C_t + I_t + G_t + \Gamma_t, \tag{3.24}$$

where Γ_t represents all additional costs:

$$\begin{aligned}\Gamma_t &= \mu^l \frac{L_{t-1}}{P_t} + \mu^d \frac{D_{t-1}}{P_t} + \varsigma \frac{F_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \varrho \frac{Q_{t-1}}{P_t} K_t^P \\ &+ \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t}.\end{aligned} \tag{3.25}$$

Finally, we assume that the technology process follows an AR(1):

$$A_t = A_{t-1}^{\rho_a} \exp(\epsilon_t^a). \tag{3.26}$$

The full set of dynamic equations characterizing the equilibrium of the model is given in Appendix B.8.

4 Calibration

We calibrate the model to the U.S. economy at a quarterly frequency. The parameters associated with the financial block are particularly important for the quantitative realism of the model. We lay out our calibration in four parts. First, we discuss parameters that are set externally or are relatively standard in the literature. Next, we collect parameters related to the deposit side of the model, followed by the ones associated with the loan side, and finally we discuss all other bank parameters. Table 4.1 lists the full set of parameters, their values, and calibration targets.

4.1 Non-bank Parameters

The quarterly discount factor, β, is set to 0.995, giving an annualized policy rate of 2%, which is consistent with the low interest rates that prevailed in the United States before the COVID-19 pandemic. We use the standard functional forms for $u(c)$ and $v(n)$:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \qquad v(n) = \chi \frac{n^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \tag{4.1}$$

and set the intertemporal elasticity of substitution, $1/\sigma$, and the Frisch elasticity, η, both to one. The former is consistent with balanced growth in our model, while the latter is consistent with the upper bound for macro elasticities in Chetty et al. (2011). The disutility from labor, χ, is chosen such that steady-state labor is normalized to one-third.

The capital income share, α, is one-third and the depreciation rate, δ, is 0.02 quarterly, or 8% annually. The functional form for the investment adjustment cost function is $\Xi(x) = \kappa_I/2 \cdot (x-1)^2$, as in Sims and Wu (2021), and the κ_I parameter is set to 2 as in that paper. We set the elasticity of substitution between differentiated retail goods, φ, to 6, which is consistent with a steady state markup of 20%. The Calvo parameter, γ, captures the probability for a retail firm to keep its price fixed and is set to the typical value of 0.75, implying an average duration between price updates of one year. The Taylor rule parameters are set to standard values: the persistence parameter, ρ_i, is 0.8 and the response to inflation, ψ_π, is 1.5. Finally, the ratio of government spending to GDP, g, is set to 0.2, roughly consistent with historical U.S. data.

Table 4.1: Calibration.

Param.	Value	Description	Target or source
Panel A. Non-bank			
β	0.9950	Discount factor	2% policy rate
χ	8.8487	Disutility of labor	One third S.S. labor
η	1.0000	Frisch elasticity	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
α	0.3333	Capital share	Standard
δ	0.0200	Depreciation rate	8% annual dep.
κ_I	2.0000	Investment adjustment cost	Sims and Wu (2021)
φ	6.0000	Elasticity of subs. b/t diff. goods	20% mark-up
γ	0.7500	Prob. of keeping prices fixed	One year duration
ψ_π	1.5000	Inflation coefficient, Taylor rule	Standard
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard
g	0.2000	Steady state G/Y	Standard
Panel B. Deposit side			
n	1.1685	Number of banks	Deposit rate target #1
γ_m	0.3005	Importance of cash in Liq.	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3990	Importance of deposits in Liq.	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3005	Importance of CBDC in Liq.	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
θ	554.21	E.o.S. between instruments in Liq.	Deposit rate target #2
ε^d	661.36	E.o.S. between banks in deposits	Deposit rate target #3
a	0.8764	Parameter in Liquidity function Φ	$\mathcal{L}/Y = 2.4$ quarterly
b	1.0700	Parameter in liquidity function Φ	Estimation
q	-0.1615	Parameter in Liquidity function Φ	S.S. relationship
μ^d	-0.20%	Cost of issuing deposits	Deposit rate target #4
Panel C. Loan side			
ψ	0.3000	Importance of pledgeable capital	Crouzet (2021)
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)
ε^l	40.013	E.o.S. between banks in loans	$i^l = i + \rho \Rightarrow \theta^k = f(\varepsilon^l)$
θ^k	5.0000	Subs. between NP and P capital	Feasible region
Panel D. Joint bank side			
ω	0.6780	Fraction staying in bank	$L/F = \nu$ in S.S.
ς	0.0474	Bank managerial cost	2.25% S.S. ROE
ν	9.0000	Loan-to-equity ratio target	Ulate (2021)
κ	0.0012	Cost of deviating from target ratio	Ulate (2021)

Notes: This table contains the parameter values used in the calibration, together with their description and their source or target. All interest rates are annualized.

4.2 Deposit Parameters

On the deposit side, it is important that our model matches empirical estimates for deposit rates (and therefore also deposit spreads) at different levels of the policy rate. Namely, we target four moments: (i) a deposit rate of 0% at a policy rate of 0.5% (taken from Ulate, 2021), (ii) a deposit rate of 0.75% at a policy rate of 2%, (iii) a deposit rate of 1.25% at a policy rate of 3%, and (iv) a deposit rate of 2% at a policy rate of 4.5%, where the last three targets are estimated from historical Ratewatch data.[9] We match these four moments by jointly calibrating n, the number of banks, θ, the elasticity of substitution between different liquidity-providing instruments, ε^d, the elasticity of substitution between banks in deposits, and μ^d, the cost of issuing deposits. This exercise yields estimates of $n = 1.16$, $\theta = 554$, $\varepsilon^d = 661$, and $\mu^d = -0.0020$ (20 basis points quarterly).

Two points are noteworthy about these estimates. First, our calibration requires a fairly low value of n. While this estimate is not an integer, and it is certainly lower than the actual number of U.S. banks, we do not intend it to be taken literally. Rather, it allows the model to match the relationship between the deposit rate and the policy rate while remaining parsimonious.[10] Second, the negative value of μ^d implies a "benefit" of issuing deposits instead of a cost, and the calibrated value is close to the one in Ulate (2021) of -0.0025. In reduced form, the negative μ^d could capture complementarities between deposit taking and lending, fees charged to depositors, or benefits of using a relatively stable source of funding (see also Abadi et al., 2022).[11]

The exogenous shares of cash, deposits, and CBDC (γ_m, γ_d, and γ_{cbdc}) are set to match two targets together with the model-implied restriction that $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. The first target is the pre-CBDC deposit-to-liquidity ratio $d/\mathcal{L}$ in steady state. We obtain an estimate for this ratio using historical data on checking deposits, savings deposits, as well as currency holdings, and constructing $\mathcal{L}$ based on equation (3.1) given our calibration for θ. For the sample 1975:Q1-2020:Q1, we find that it is approximately 0.8 on average. The second target is that cash and CBDC have roughly the same market share if CBDC pays

[9] We compute a historic deposit rate series that resembles the one in our model by using data from Ratewatch on checking and saving deposit rates and weighting those by the historical shares of such deposits based on data from the H.6 releases from the Federal Reserve Board of Governors (Sample: 2000:M1-2020:M4). Comparing the resulting series to the federal funds rate yields approximately the calibration targets between the policy rate and the deposit rate stated in the text.

[10] In particular, n is crucially related to the pass-through of the policy rate to the deposit rate, which is found to be less than unity. Drechsler et al. (2017), for example, document a pass-through of 0.39 among large banks and 0.46 on average (see pages 1821 and 1824 therein). In Appendix A.3, we obtain a closed-form expression for the pass-through of the policy rate to the deposit rate and show that the crucial parameter that governs this relation is the number of banks, n.

[11] Note that any further costs of operating the deposit franchise are incorporated in the managerial costs of operating the bank, ς, which are substantial in our baseline calibration.

no interest, as documented by surveys such as Bidder et al. (2024). Consequently, we set $\gamma_m = \gamma_{cbdc} = 0.3005$ and $\gamma_d = 0.3990$.

The cost of liquidity function is parameterized as $\Phi(\mathcal{L}) = a\mathcal{L}^b - q$. The elasticity parameter b is calibrated starting from the equilibrium condition (3.9),

$$\frac{1 + i_t^{\mathcal{L}}}{1 + i_t} = ab\mathcal{L}_t^{b-1}. \tag{4.2}$$

We proceed to take logs and subtract the resulting equation from its lagged counterpart, giving

$$s_t - s_{t-1} = (b - 1) \cdot [\ln(\mathcal{L}_t) - \ln(\mathcal{L}_{t-1})], \tag{4.3}$$

where $s_t \approx i_t^{\mathcal{L}} - i_t$. As described above, we construct a time series for $\mathcal{L}_t$ using equation (3.1). Similarly, we measure $i_t^{\mathcal{L}}$ based on equation (3.7) using a historic deposit rate series.[12] Finally, we estimate (4.3) for the sample 2000:M1-2020:M4, which is the maximum time span across all data series, and obtain $b = 1.07$.

Finally, the other parameters a and q inside the cost function for liquidity (Φ) are selected to match a liquidity-over-GDP ratio $\mathcal{L}/Y$ of 2.4 at the quarterly frequency, and the relationship that $\Phi(\cdot) = m + d + cbdc$ in steady state. This approach yields the estimates $a = 0.8764$ and $q = -0.1615$, respectively.

4.3 Loan Parameters

Next, we turn to parameters related to the loan side of the model. The parameter ψ governs the importance of pledgeable capital for aggregate capital in (3.11) and therefore pins down the share of bank borrowing. Crouzet (2021) shows that this share has declined to around 30% for the most recent years and we calibrate ψ accordingly.

For the costs of bank and bond borrowing, we obtain estimates from Schwert (2020) who compares bank loan rates and secondary bond quotes for the same firms on the same date. Schwert (2020) finds that loan and bond spreads are similar for investment grade firms. However, estimations yield that the average bond-implied loan spread should be around 50% of the average all-in-drawn spread of 2.8% since loans are less risky due to higher recovery rates in bankruptcy. Schwert (2020) associates the remaining premium to banks' market power in the loan market. To match these numbers, we assume that bond and loan spreads are the same in steady state, that is, $i^l = i + \varrho = 2.8\%$. However,

[12] We compute the deposit rate series as described in footnote 9.

banks face half of the costs of issuing credit compared with the bond market, resulting in $\varrho = 0.7\%$ for the costs of issuing bonds and $\mu^l = 0.35\%$ for the costs of issuing loans, both at the quarterly frequency.

Based on equations (3.20) and (3.21), the equivalence between bond and loan spreads in steady state implies the following relationship between ε^l and θ^k :

$$n(i + \varrho + \delta) = \left[(n-1)\varepsilon^l + \theta^k - \psi\left(\theta^k - \frac{1}{1-\alpha}\right)\right](\varrho - \mu^l), \tag{4.4}$$

where all other parameters apart from ε^l and θ^k are pinned down. Therefore, we can interpret the elasticity of substitution between bonds and loans, θ^k, as the remaining free parameter, and, conditional on that, back out ε^l from (4.4). While we lack an empirical target to pin down θ^k exactly, the model implies that it must lie in a feasible region between one and 11.8.[13] For our baseline specification, we choose $\theta^k = 5$ as a suggestive value roughly in the middle of the feasible set, and show robustness of our main results to alternative values in Appendix C.1.

4.4 Other Bank Parameters

Besides parameters related to the loan and deposit sides, a few other bank-related ones remain. The function for the cost of deviating from the target loan-to-equity ratio is parameterized as: $\Psi(L/F) = \kappa\nu x\left(\ln(L/F) - \ln\nu - 1\right) + \kappa\nu^2$, following Ulate (2021). The loan-to-equity target ratio, ν, and the cost of deviating from this ratio, κ, are also taken from that paper, matching a steady-state loan-to-equity ratio of nine and using cross-sectional relations between loan rates, loan amounts, and bank capital to obtain a value of κ of 12 basis points. We also check the robustness of our results across different values of κ in Appendix C.1.

Banks' managerial cost, ς, determines their profitability. Using Call Report data for commercial banks over 1984:Q1-2022:Q3, we find an average annualized return on assets close to one percent. Given the loan-to-equity ratio of nine, this implies a quarterly return-on-equity of 2.25% and we calibrate ς to match this moment in steady state. The fraction of bank profits that stay within the bank and that are not paid out as dividends, ω, is calibrated so that in the initial pre-CBDC steady state the loan-to-equity ratio L/F is equal to its target ν.

[13] The lower bound of one comes from the assumption that pledgeable and non-pledgeable capital are substitutes instead of complements. The upper bound for θ^k is actually ε^l, due to the nested-CES structure of the model. Given our remaining calibration and equation (4.4), this implies an upper bound for θ^k of 11.8.

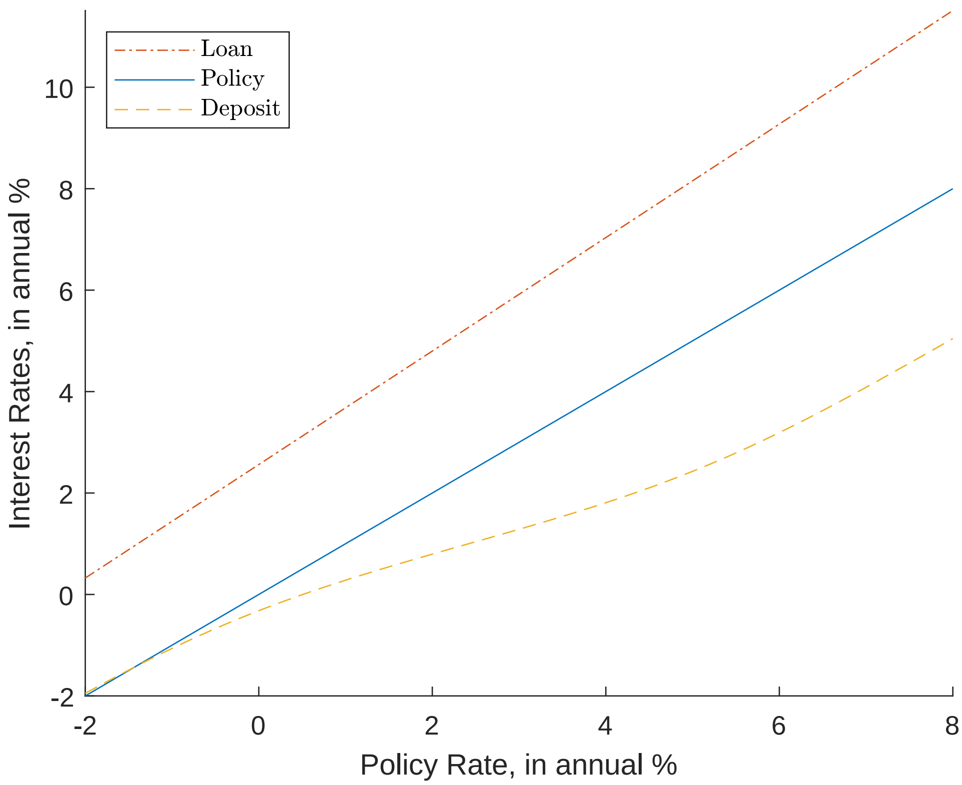

Figure 4.1: This figure shows the loan rate (dash-dot orange line) and the deposit rate (dashed yellow line) obtained in the baseline calibration of the model as a function of the policy rate. The policy rate is also plotted as the 45 degree line for comparison (solid blue line).

4.5 Loan and Deposit Spreads

To provide some intuition for the behavior of spreads in the calibrated model, Figure 4.1 displays the loan rate and the deposit rate for different levels of the policy rate (which is also shown as the 45-degree line). The loan spread ranges between 2.3% and 3.5%. It is larger for higher levels of the policy rate. That is because banks gain market power at higher policy rates, raising their profitability and market share relative to bonds, and increasing the endogenous loan elasticity and therefore their loan markup over the policy rate. We can see this mechanism from equations (3.19)-(3.21).

The deposit rate is below the policy rate for all positive values, but is close to the policy rate for rates below -1%. The deposit spread rises with higher levels of the policy rate and bank market power. However, this relation is nonlinear. For policy rates between -1% and 5%, the deposit spread widens substantially as the policy rate increases, as targeted by our calibration strategy over these values. The widening of the deposit spread becomes smaller when the policy rate is above 5% and stabilizes at higher policy rates.

This behavior of the deposit spread is consistent with the data even though we do not target deposit rates for such high levels of the policy rate in our calibration—providing an external validation for the empirical fit of the model.[14] Appendix A.3 provides further details on the behavior of the pass-through of the policy rate to the deposit rate in our model.

5 Implications of CBDC Introduction

In this section, we discuss the implications of CBDC introduction through the lens of our full DSGE model. First, we focus on comparing how the economy differs between an initial steady state where CBDC is not used and a final steady state where CBDC is available and we consider various remuneration schemes for CBDC. Throughout this section, we frequently refer to the "welfare change" from introducing CBDC, which is formally the multiplicative consumption-equivalent variation required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady states (see Appendix B.10 for details), in percent. Second, we also discuss how the economy responds to shocks around the pre-CBDC and various post-CBDC steady states.

5.1 CBDC Introduction for Different CBDC Rates

We first focus on our baseline calibration where the steady state policy rate is 2% and analyze outcomes of CBDC introduction for different levels of the interest rate paid on CBDC. Figure 5.1 shows changes in welfare, the deposit-to-GDP ratio, and the CBDC-to-GDP ratio across CBDC rates between -1% and 3% annually. As the rate paid on CBDC increases, the CBDC-to-GDP ratio increases and the deposit-to-GDP ratio decreases monotonically. Intuitively, as the interest rate paid on CBDC becomes more negative, the CBDC-to-GDP ratio tends to zero, since households do not want to use a very unattractive liquid instrument. In the limit, when the CBDC rate is -100% quarterly, households do not use CBDC at all, which corresponds to our pre-CBDC scenario.

Most importantly, the welfare change from CBDC introduction displays an inverted U-shape with respect to the interest rate paid on CBDC. It tends to zero for a very negative CBDC rate, becomes negative for very high CBDC rates, and achieves a positive maximum of approximately 27 basis points (of initial steady-state consumption) when the CBDC rate is approximately 0.8% per year. This welfare gain is higher than the one

[14] Such a behavior of deposit rates is reminiscent of deposit betas that are not constant but rise with higher market rates, as documented in Greenwald et al. (2023), for example.

of approximately 22 basis points when the CBDC rate is zero percent, an often-discussed remuneration level by central banks which consider the introduction of CBDC. Interestingly, the welfare-maximizing CBDC interest rate of approximately 0.8% per year is close to the deposit rate in steady state.

The impact of CBDC on welfare in our model depends on three different channels. First, CBDC can curtail commercial bank monopoly power and thereby increase the deposit rate that households get paid. Second, households like some of the characteristics that CBDC has to offer. For example, CBDC can be used for electronic transactions while it is also a direct liability of the central bank and thus not subject to bank runs. Such benefits are jointly captured in the model with a positive γ_{CBDC} (which is present even in the pre-CBDC steady state). Households therefore benefit when CBDC is introduced because it allows them to better distribute their usage across the available liquid instruments. Third, despite a higher deposit rate, some deposits flow out of the banking system when CBDC is introduced. The higher deposit rate and the reduced amount of deposits

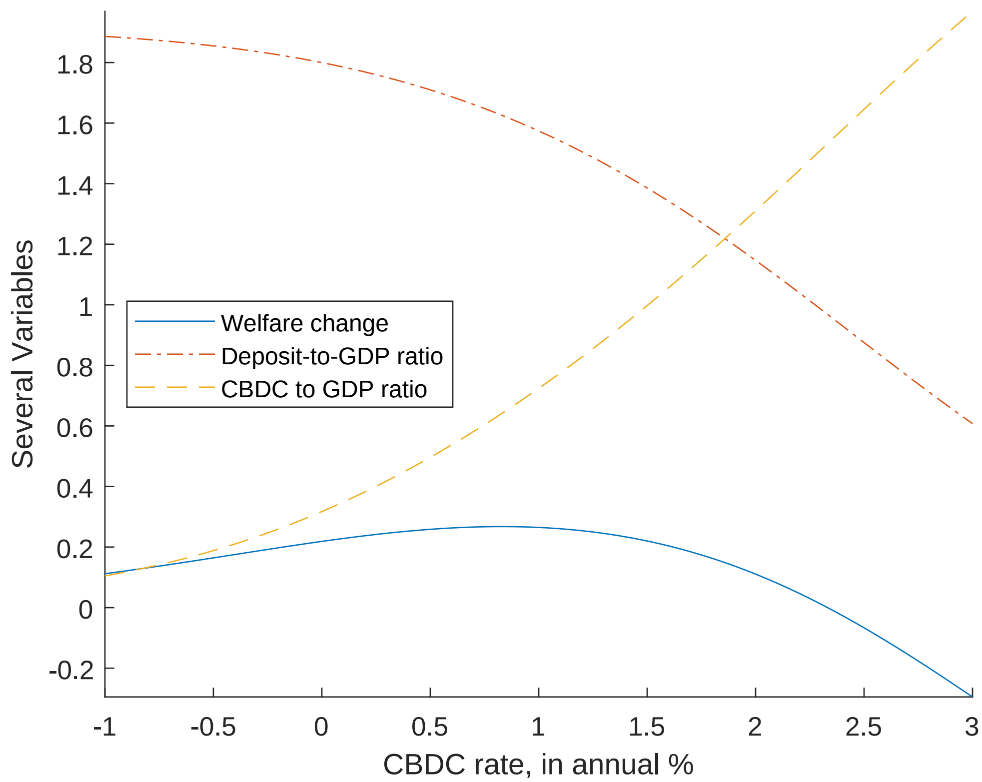

Figure 5.1: This figure displays the behavior of some important variables for different levels of the interest rate paid by CBDC. The welfare change (gain if positive, loss if negative) from CBDC introduction is in blue, the deposit-to-GDP ratio is in orange, and the CBDC-to-GDP ratio is in yellow.

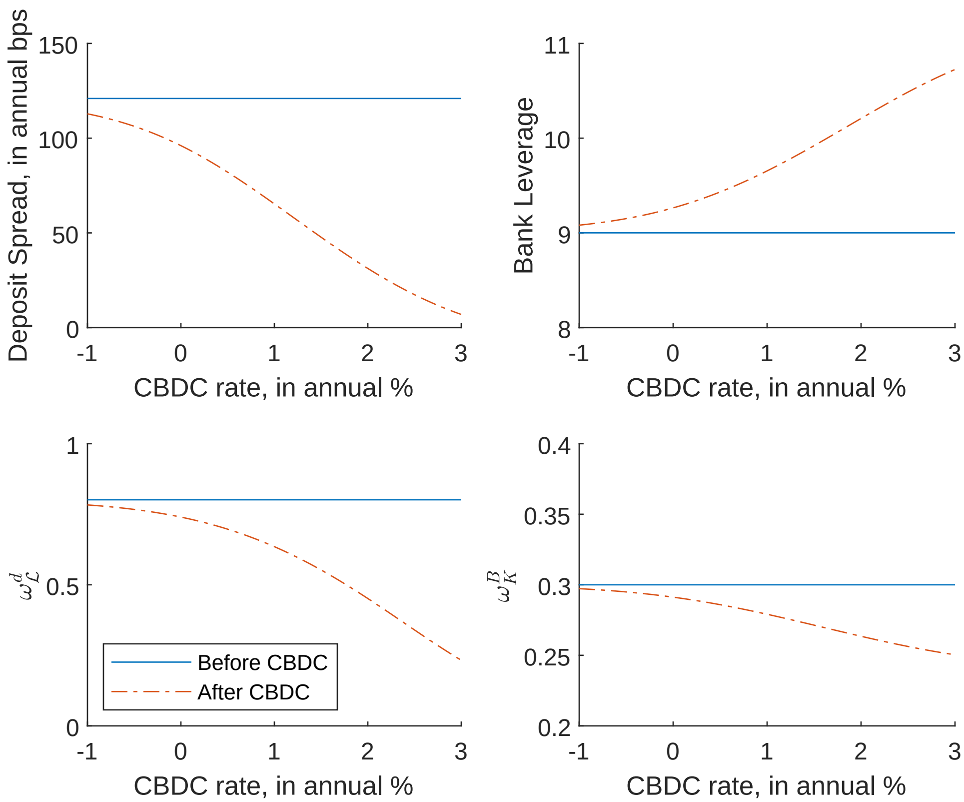

Figure 5.2: This figure shows different outcomes, before and after the introduction of CBDC, for different levels of the interest rate paid by CBDC.

imply that bank equity declines, which in turn reduces credit supply, raises the cost of capital for firms, and lowers welfare. For a discussion on intuition in depth, see Section 2.

For low and moderate levels of the CBDC rate, the first two channels described in the previous paragraph dominate the third one, leading to an increase in overall welfare due to CBDC introduction. However, for high levels of the CBDC rate, the bank disintermediation channel dominates, leading to a fall in overall welfare as observed by the right tail in Figure 5.1.

Figure 5.2 plots how some other variables of interest behave, before and after the introduction of CBDC, for different levels of the interest rate paid on CBDC. The deposit spread is 120 basis points before CBDC. It falls to 96 basis points when CBDC is introduced with a rate of 0%, but to 72 basis points when CBDC is introduced with the optimal rate of 0.8%. Bank leverage is around nine in the initial steady state, but increases when CBDC is introduced, a pattern that intensifies as the rate on CBDC increases. When bank leverage increases, banks charge a higher loan rate, which explains the negative welfare impact of a CBDC that pays a very high interest rate. Both the endogenous deposit share and the share of bank lending (0.8 and 0.3, respectively, in the pre-CBDC steady state)

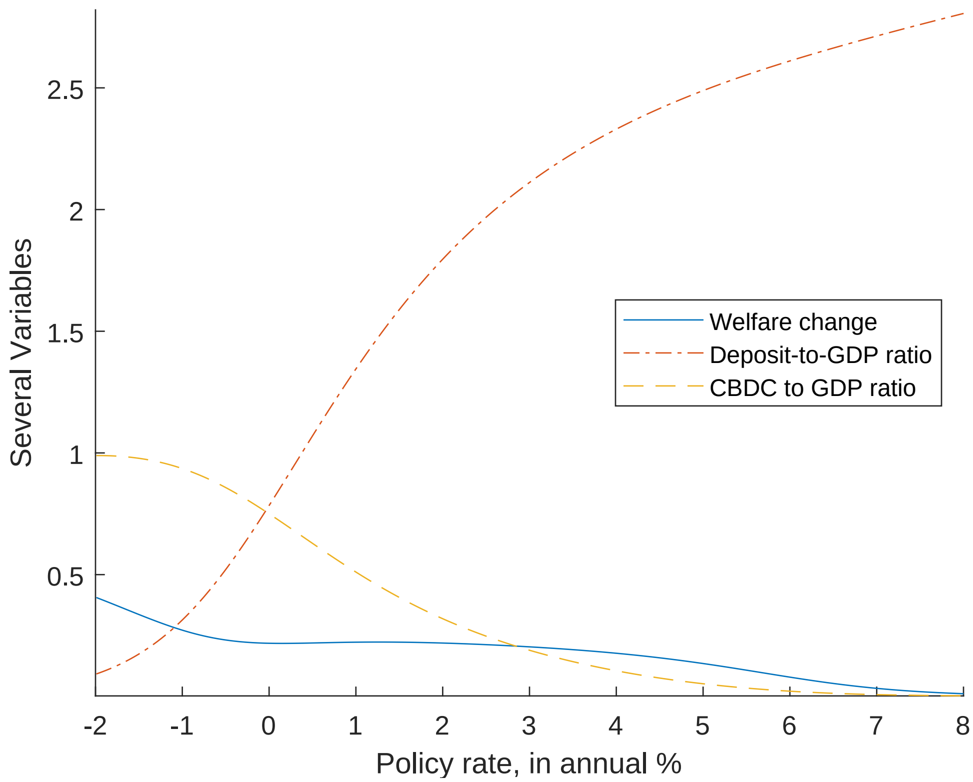

Figure 5.3: This figure displays the behavior of some important variables for different levels of the policy rate. The welfare change (gain if positive, loss if negative) from CBDC introduction is in blue, the deposit-to-GDP ratio is in orange, and the CBDC-to-GDP ratio is in yellow.

decrease with the introduction of CBDC, and fall more as the rate on CBDC increases.

5.2 CBDC Introduction for Different Policy Rates

Next, we change the nature of the exercise that we perform. Instead of analyzing CBDC introduction for a given steady-state policy rate but different levels of the interest rate on CBDC, we keep the interest rate on CBDC constant at 0%, as envisioned by many central banks which consider the introduction of CBDC, and change the steady-state level of the policy rate. We achieve the different steady-state levels of the policy rate by recalibrating the discount factor β, while keeping the rest of the parameters of our baseline calibration constant (however, our results are robust to recalibrating a larger set of parameters, see Appendix C.4).[15]

[15] Notice that the most important parameters in our model, namely the deposit-side banking parameters ε^d, θ, n, and μ^d, are calibrated to match deposit rates across levels of the policy rate. These parameters therefore do not need to be re-calibrated when the discount factor is changed. Appendix C.4 shows that

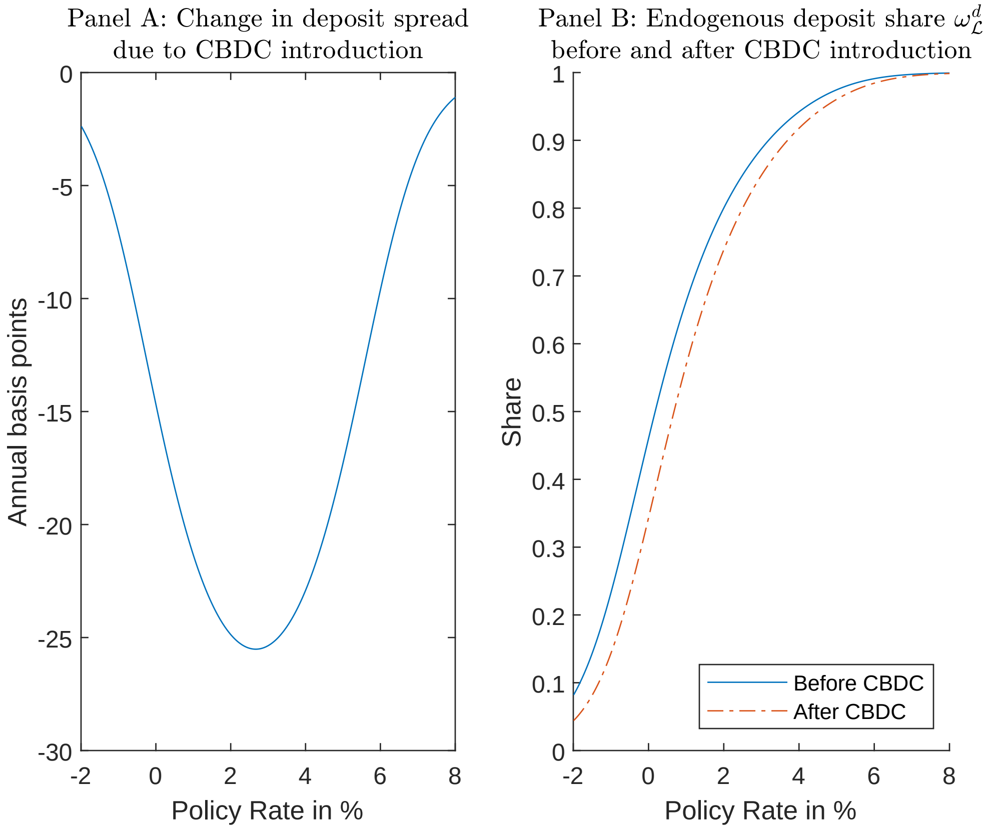

Figure 5.4: Panel A: Change in the deposit spread following the introduction of CBDC across different values of the policy rate. Panel B: Endogenous deposit share ($\omega^d_{\mathcal{L}}$) across different values of the policy rate before and after the introduction of CBDC. The figure uses the baseline calibration described in Section 4.

Figure 5.3 illustrates how several outcome variables of interest behave for steady-state policy rates between -2% and 8% annually. As in Figure 5.1, we consider the welfare change from CBDC introduction, the CBDC-to-GDP ratio, and the deposit-to-GDP ratio. As the steady-state policy rate increases, the CBDC-to-GDP ratio decreases monotonically, whereas the deposit-to-GDP ratio increases monotonically. Additionally, when the policy rate increases, the CBDC-to-GDP tends to zero, since households do not want to use a liquid instrument that pays relatively little compared to deposits. The welfare gains from CBDC introduction have an approximately monotonic shape: they roughly fall with the steady-state policy rate and tend to zero as the policy rate rises, precisely because CBDC is mostly unused in such a scenario.

Figure 5.4 replicates Figure 2.1 for the full DSGE model instead of the simple static model in Section 2. While some magnitudes change slightly due to various new ingre-

our results in this subsection and the next are robust to recalibrating additional parameters so that certain targets continue to be matched across different levels of the policy rate.

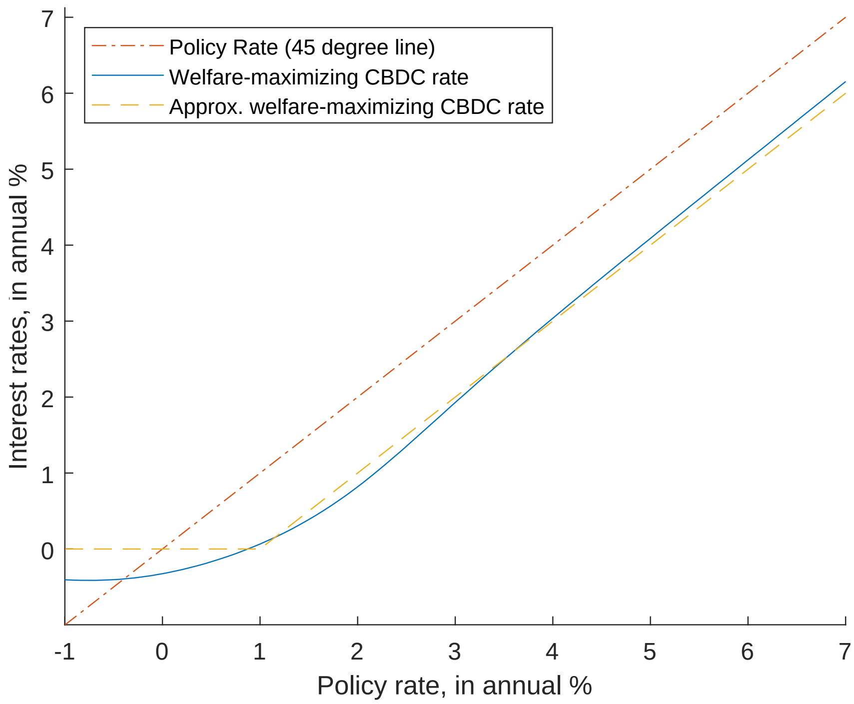

Figure 5.5: This figure displays the policy rate, in orange (in both axes, so it is the 45 degree line), the welfare-maximizing level of the CBDC rate, in blue, and an approximate welfare-maximizing rule of thumb rate which is the maximum between 0 and the policy rate minus 1%, in yellow.

dients and the general equilibrium nature of the model, the intuition carries over from Section 2. The deposit spread falls the most due to the introduction of CBDC for intermediate levels of the steady-state policy rate of approximately 2.7% annually. For very high or very low levels of the policy rate, the endogenous deposit share changes little with the introduction of CBDC and the deposit spread therefore does not react much.

5.3 Welfare-Maximizing CBDC Rate Across Policy Rates

In Section 5.1, we showed that, for our baseline steady-state policy rate of 2%, the welfare-maximizing level of the CBDC interest rate is around 0.8% per year. However, the effects of introducing a CBDC for a given interest rate also vary substantially depending on the steady-state level of the policy rate as shown in Section 5.2. Therefore, a natural question that emerges is: what is the CBDC interest rate that maximizes the welfare change of introducing CBDC for each level of the steady-state policy rate? Figure 5.5 displays the answer to this question. In orange, the policy rate is shown as the 45 degree line, and in blue, the welfare-maximizing CBDC rate is plotted.

Starting on the left, for negative levels of the policy rate, the welfare-maximizing

CBDC rate is negative and above the policy rate. The two cross at around -40 basis points annually. Subsequently, the welfare-maximizing CBDC rate is below the policy rate by roughly 1 percent annually. This welfare-maximizing CBDC rate as a function of the steady-state policy rate can be approximated fairly well by a rule of thumb CBDC rate that is the maximum between 0% and the policy rate minus 1%, as illustrated by the yellow line in Figure 5.5. While this approximate welfare-maximizing CBDC rate does not capture all the intricacies of the full welfare-maximizing CBDC rate (like being negative for negative levels of the policy rate), it is a rule of thumb that could easily be communicated by central banks and, in welfare terms, does almost as well as the welfare-maximizing rate, as we show below. This rule of thumb also has the benefit of avoiding negative rates on CBDC, which present a political economy concern for central banks due to the fear of the public that CBDC would be used to "expropriate their savings" with below-zero interest rates.

What is the intuition for the fact that the welfare-maximizing CBDC rate increases with the policy rate? The higher the policy rate, the higher the CBDC rate needs to be to take a given share of the liquid-instruments market and therefore to contain commercial-bank monopoly power by a given amount.

Figure 5.6 plots the deposit spread in the top row and the endogenous share of deposits in the bottom row—before and after the introduction of CBDC—across levels of the policy rate (on the x-axis) and for different CBDC remuneration schemes. In the left column, we present CBDCs that pay a constant interest rate, while in the right column, we present a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC rate, and a CBDC that pays the approximately welfare-maximizing rule of thumb rate described in Figure 5.5.

Importantly, for levels of the policy rate that are roughly above 2% per year, the welfare-maximizing policy rate achieves a stabilization of the deposit spread at around 70 basis points. Similarly, the endogenous deposit share is stabilized at around 65%. In contrast, a CBDC that pays a constant interest rate (regardless of the policy rate), can neither stabilize the deposit spread nor the deposit share, as visible from the left column of Figure 5.6. On the other hand, a CBDC that pays the policy rate reduces the deposit spread and the endogenous deposit share by too much relative to the welfare optimum.

Figure 5.7 plots the welfare change from introducing CBDC across different levels of the policy rate (on the x-axis). The different lines represent the alternative CBDC remuneration schemes considered in Figure 5.6. As expected, the welfare change from the welfare-maximizing CBDC rate is the envelope of the other lines. Echoing the message from Figure 5.5, the line for the optimal CBDC rate touches the line for a constant CBDC

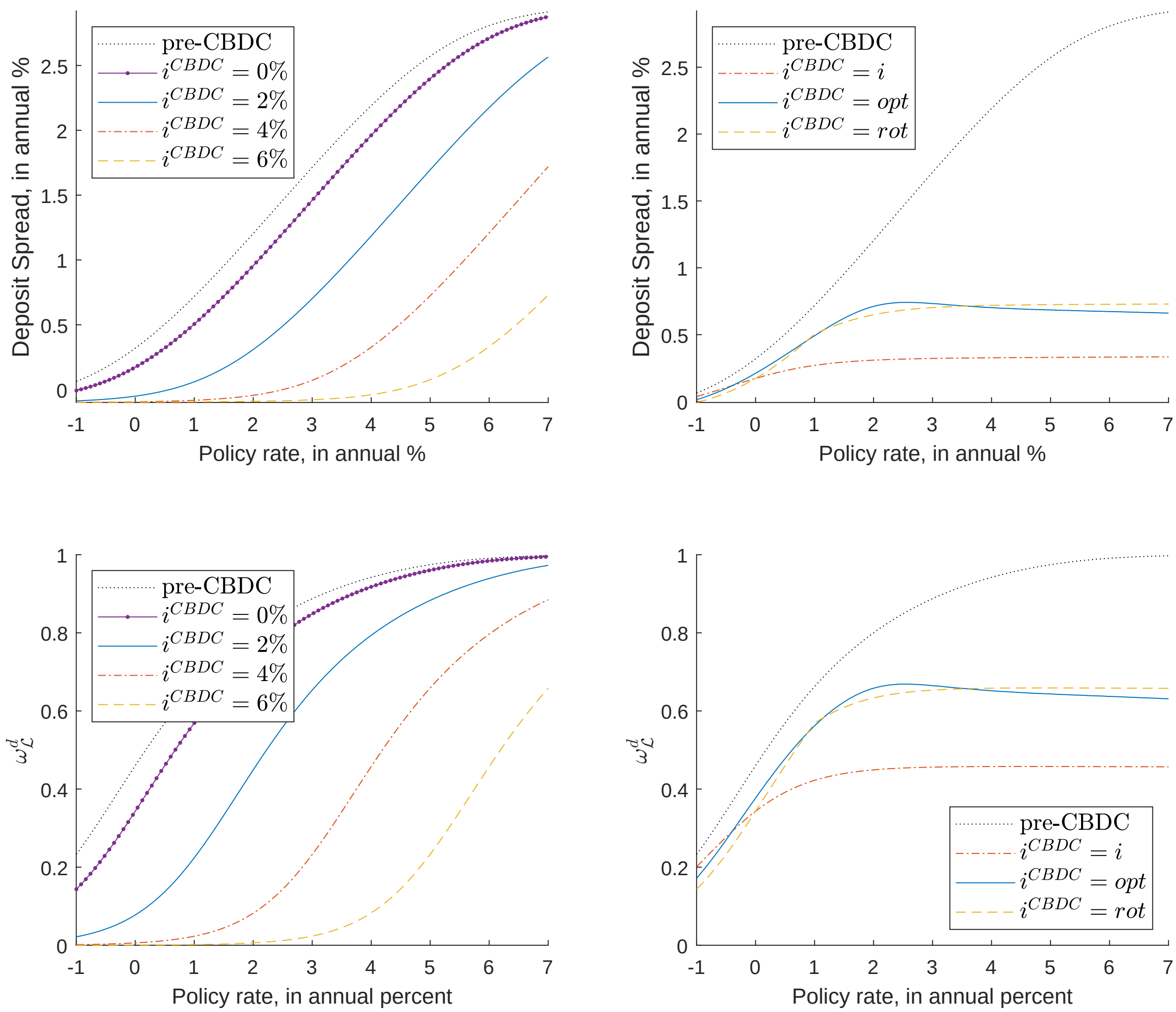

Figure 5.6: This figure shows the deposit spread (in the top row) and the endogenous share of deposits (bottom row), before and after the introduction of CBDC, across levels of the policy rate (in the x-axis), for different CBDC remuneration schemes. On the left column, we present CBDCs that pay a constant interest rate, while on the right column, we present a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC interest rate for each level of the policy rate, and a CBDC that pays the approximately welfare-maximizing rule of thumb (denoted "rot") rate described in Figure 5.5.

rate at a level of the policy rate that is around 1% higher (e.g., the line for the optimal rate touches the line for $i^{CBDC} = 4\%$ at a policy rate of roughly 5%). Interestingly, the welfare change of the rule of thumb rate is almost identical to the one of the welfare-maximizing rate. In contrast, a CBDC that pays the policy rate is only optimal when the policy rate is about -0.4% because that is the point at which the welfare-maximizing policy rate intersects the policy rate in Figure 5.5.

Finally, Figure 5.8 plots the change in the deposit spread from the introduction of

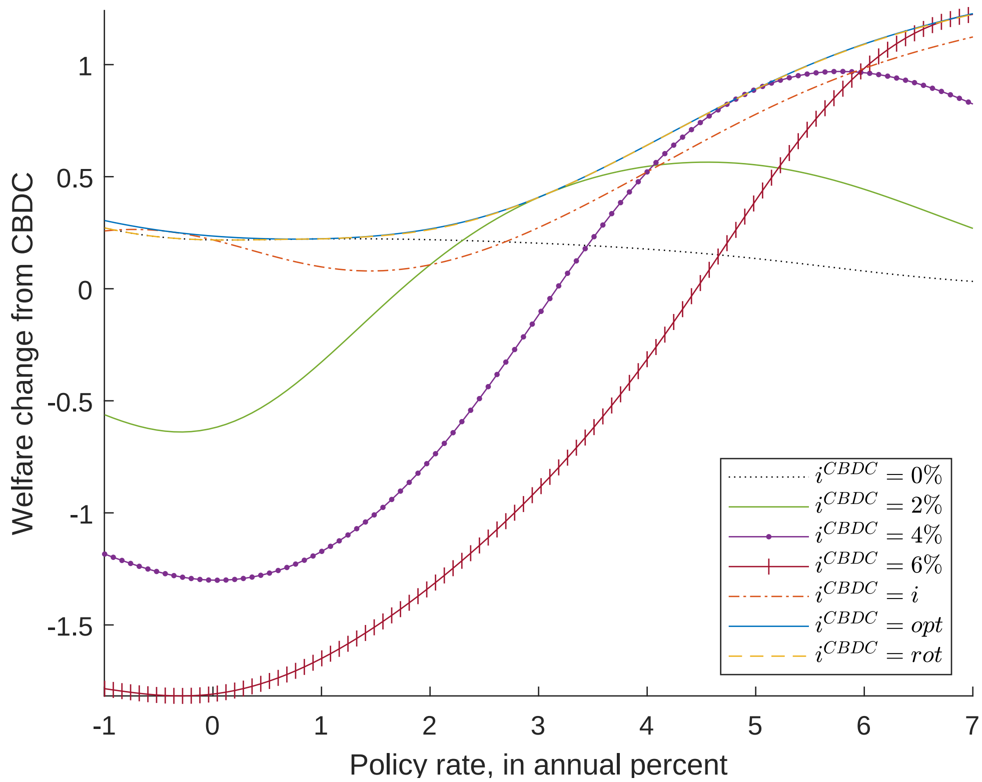

Figure 5.7: This figure shows the welfare change from the introduction of CBDC, across levels of the policy rate (in the x-axis), for different CBDC remuneration schemes.

CBDC across levels of the policy rate for the same CBDC remuneration schemes considered thus far. Note that the constant CBDC rates display a U-shape like the ones discussed in the left panel of Figure 2.2 (or Figure 5.4). By contrast, CBDCs that pay the policy rate, the welfare-maximizing CBDC rate, or the rule of thumb CBDC rate have downward sloping lines like the ones discussed in the right panel of Figure 2.2. However, a CBDC that pays the policy rate decreases the deposit spread by substantially more than a CBDC that pays the welfare-maximizing rate. In turn, bank disintermediation is stronger and the welfare change is lower.

5.4 Responses to Monetary Policy Shocks

Having already examined how the economy reacts to the introduction of CBDC by comparing the initial pre-CBDC steady state with the final post-CBDC steady state, we now turn to analyzing how the two economies differ in their response to transitory shocks around the respective steady states. We focus on impulse responses to a monetary policy

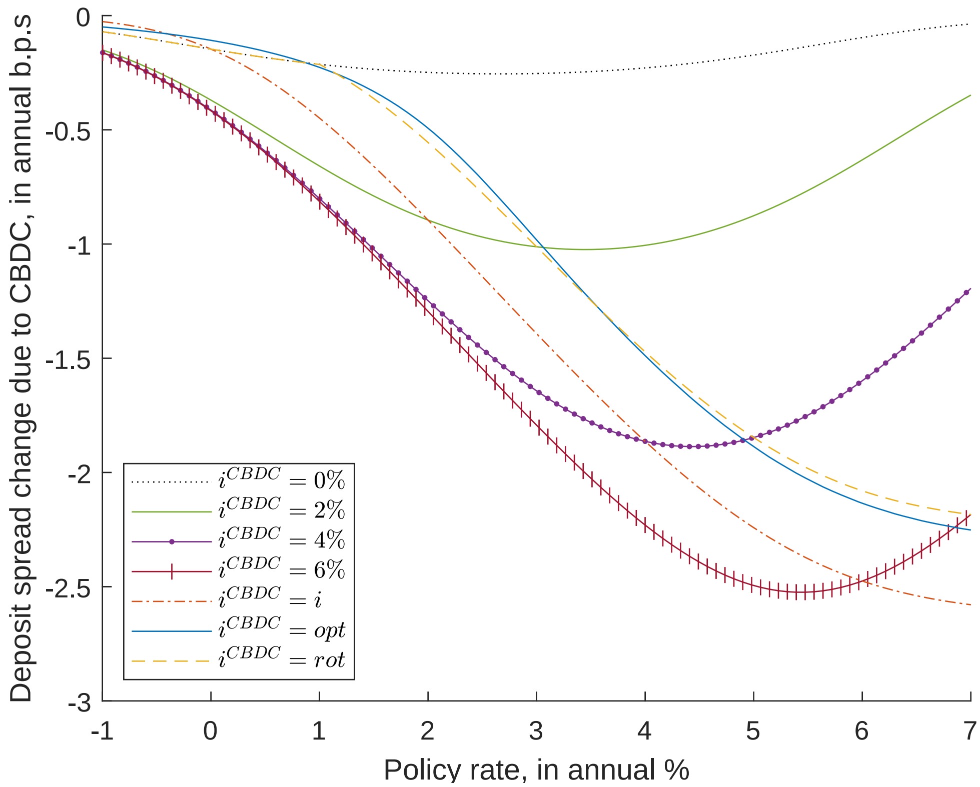

Figure 5.8: This figure plots the change in the deposit spread from the introduction of CBDC across levels of the policy rate (in the x-axis), for different CBDC remuneration schemes.

shock in this section, but our main findings also apply to technology shocks as illustrated in Appendix C.3.

Figure 5.9 depicts impulse responses to a 50 basis point expansionary monetary policy shock for different CBDC remuneration schemes. The dotted black line shows the pre-CBDC case which we compare to the following cases: (i) a CBDC that pays a constant interest rate of 0%, (ii) a CBDC that pays a constant rate at the welfare-maximizing level of roughly 0.8% annually for a policy rate of 2%, (iii) a CBDC that pays the policy rate minus one percent, corresponding to the approximated rule of thumb CBDC rate, and (iv) a CBDC that pays the policy rate.

Even though these regimes have significantly different welfare implications, the impulse responses are remarkably similar. These results indicate that the introduction of CBDC and the choice of CBDC remuneration scheme do not have a substantial impact on the response of the economy to a transitory shock.

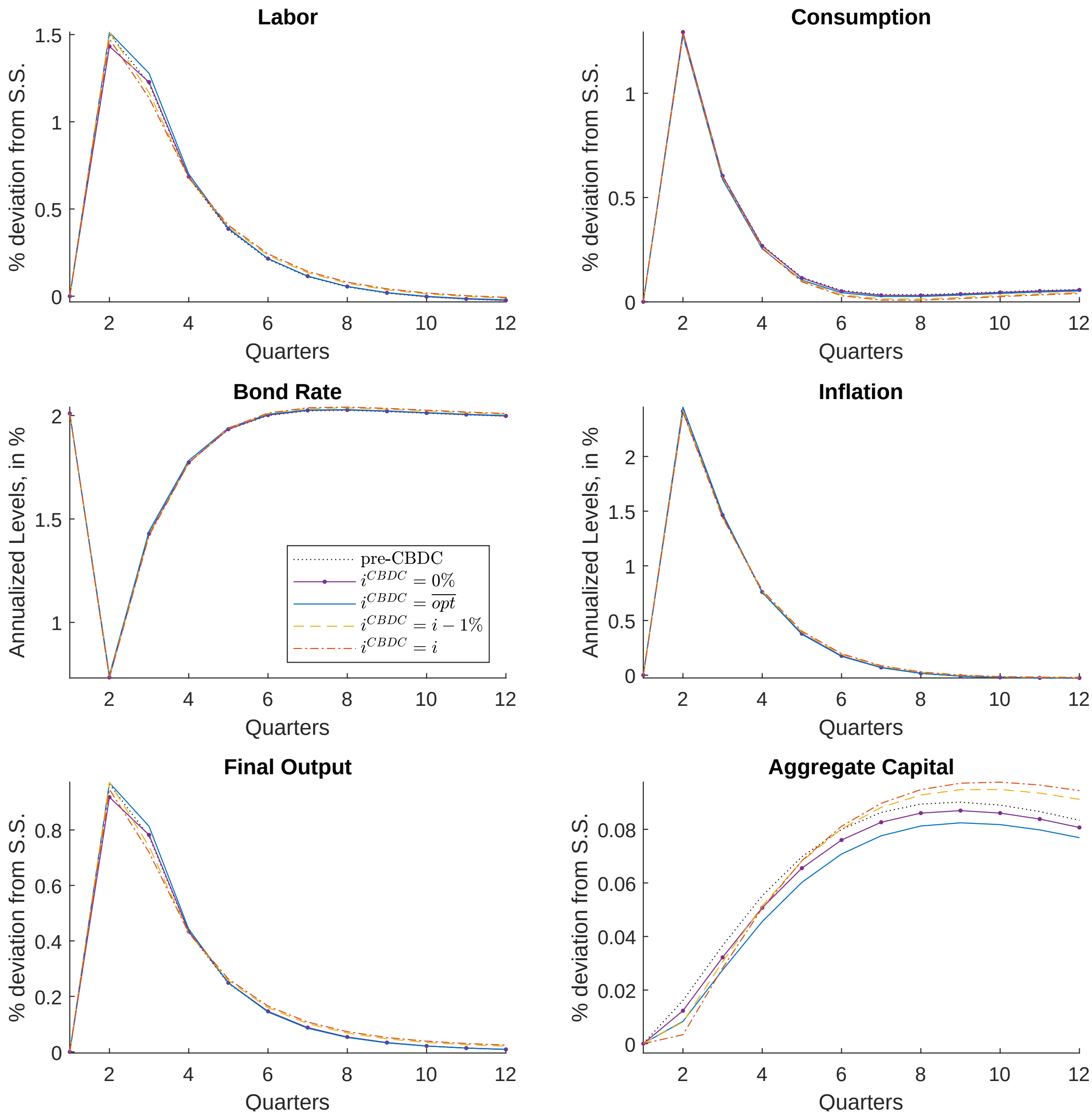

Figure 5.9: This figure depicts the IRFs to a 50 basis points expansionary monetary policy shock, for different CBDC remuneration schemes.

6 Conclusion

Many countries around the world are currently considering the introduction of a central bank digital currency and debate what the effects on their economies might be. Since practical experience with CBDC remains scarce, policy-makers turn to analysis based on theoretical economic models for insights. Our paper provides such guidance and delivers a practical message that can be applied to various economies around the world.

We develop a New Keynesian DSGE model to assess the introduction of a CBDC.

Three competing channels determine the welfare effects in our model. On the positive side, households benefit from the introduction of a CBDC in two ways. First, they value the expansion of liquidity services that the new saving instrument provides. Second, households receive higher deposit rates since CBDC competes with bank deposits which reduces bank deposit market power. On the negative side, banks face deposit outflows and cut their lending, which in turn reduces aggregate investment and output.

We assess this welfare trade-off for a wide range of economies that differ in their level of interest rates. We find substantial welfare improvements of introducing CBDC if countries follow a simple rule that determines the rate of interest on CBDC: it pays the maximum between 0% and the policy rate minus 1%. The simplicity of this rule is appealing since it can easily be communicated to the public and avoids political economy concerns related to paying negative rates on CBDC. Interestingly, we also find that the introduction of a CBDC is most beneficial for economies with high interest rates. In such environments, banks have substantial market power in deposit markets which is sharply reduced once a CBDC is introduced.

Finally, we close with a potential avenue for future research. Our model abstracts from the impact of CBDC on financial instability. In times of financial distress, uninsured depositors may withdraw their funds from banks and convert them to CBDC, which may in turn exacerbate the financial turmoil. A line of research studies the implications of CBDC on financial stability. Theoretical work along this line often builds on the traditional Diamond and Dybvig (1983) model, and includes Fernandez-Villaverde et al. (2021), Schilling et al. (2020), Williamson (2022a), Keister and Monnet (2022), and Bidder et al. (2024) among others. A salient path for succeeding analyses is to integrate financial crises into our DSGE-framework to study how the introduction and remuneration of CBDC affects the frequency and severity of crises.

References

ABAD, J., G. NUNO, AND C. THOMAS (2023): "CBDC and the operational framework of monetary policy," *Working paper*.

ABADI, J., M. K. BRUNNERMEIER, AND Y. KOBY (2022): "The Reversal Interest Rate," Working Papers 22-28, Federal Reserve Bank of Philadelphia.

AGUR, I., A. ARI, AND G. DELL'ARICCIA (2022): "Designing central bank digital currencies," *Journal of Monetary Economics*, 125, 62–79.

AHNERT, T., K. ASSENMACHER, P. HOFFMANN, A. LEONELLO, C. MONNET, AND D. PORCELLACCHIA (2022): "The economics of central bank digital currency," *ECB Working Paper*.

ANDOLFATTO, D. (2021): "Assessing the Impact of Central Bank Digital Currency on Private Banks," *Economic Journal*, 131, 525–540.

ATKESON, A. AND A. BURSTEIN (2008): "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, 98, 1998–2031.

BALLOCH, C. AND Y. KOBY (2019): "Low Rates and Bank Loan Supply: Theory and Evidence from Japan," Mimeo.

BARRDEAR, J. AND M. KUMHOF (2022): "The macroeconomics of central bank issued digital currencies," .

BIDDER, R., T. P. JACKSON, AND M. ROTTNER (2024): "CBDC and banks: Disintermediating fast and slow," Mimeo.

BRUNNERMEIER, M. K. AND D. NIEPELT (2019): "On the equivalence of private and public money," *Journal of Monetary Economics*, 106, 27–41.

BURLON, L., C. MONTES-GALDON, M. A. MUÑOZ, AND F. SMETS (2023): "The optimal quantity of CBDC in a bank-based economy," *AEJ:Macro*.

CHAPMAN, J., J. CHIU, M. DAVOODALHOSSEINI, J. JIANG, F. RIVADENEYRA, AND Y. ZHU (2023): "Central Bank Digital Currencies and Banking: Literature Review and New Questions," *Bank of Canada Staff Discussion Paper*.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *American Economic Review*, 101, 471–475.

Chiu, J., S. M. Davoodalhosseini, J. Jiang, and Y. Zhu (2023): "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment," *Journal of Political Economy*, 131, 1213–1248.

Crouzet, N. (2021): "Credit Disintermediation and Monetary Policy," *IMF Economic Review*, 69, 23–89.

Davoodalhosseini, S. M. (2021): "Central bank digital currency and monetary policy," *Journal of Economic Dynamics and Control*, 104150.

Diamond, D. W. and P. H. Dybvig (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

Drechsler, I., A. Savov, and P. Schnabl (2017): "The Deposits Channel of Monetary Policy," *The Quarterly Journal of Economics*, 132, 1819–1876.

——— (2021): "Banking on Deposits: Maturity Transformation without Interest Rate Risk," *The Journal of Finance*, 76, 1091–1143.

Fernandez-Villaverde, J., D. Sanches, L. Schilling, and H. Uhlig (2021): "Central Bank Digital Currency: Central Banking For All?" *Review of Economic Dynamics*, 41, 225–242.

Greenwald, E., S. Schulhofer-Wohl, and J. Younger (2023): "Deposit Convexity, Monetary Policy and Financial Stability," *Federal Reserve Bank of Dallas Working Paper 2315*.

Greenwood, J., Z. Hercowitz, and G. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Infante, S., K. Kim, A. Orlik, A. Silva, and R. Tetlow (2023): "The Macroeconomic Implications of CBDC: A Review of the Literature," *Finance and Economics Discussion Series 2023-072. Washington: Board of Governors of the Federal Reserve System*.

Keister, T. and C. Monnet (2022): "Central bank digital currency: Stability and information," *Journal of Economic Dynamics and Control*, 142.

KEISTER, T. AND D. R. SANCHES (2022): "Should Central Banks Issue Digital Currency?" *The Review of Economic Studies*, 90, 404–431.

NIEPELT, D. (2023): "Money and Banking with Reserves and CBDC," *Journal of Finance, forthcoming*.

ROGNLIE, M. (2016): "What Lower Bound? Monetary Policy with Negative Interest Rates," *Mimeo*.

SCHILLING, L., J. FERNÁNDEZ-VILLAVERDE, AND H. UHLIG (2020): "Central Bank Digital Currency: When Price and Bank Stability Collide," NBER Working Papers 28237, National Bureau of Economic Research, Inc.

SCHWERT, M. (2020): "Does Borrowing from Banks Cost More than Borrowing from the Market?" *The Journal of Finance*, 75, 905–947.

SIMS, E. AND J. C. WU (2021): "Evaluating Central Banks' tool kit: Past, present, and future," *Journal of Monetary Economics*, 118, 135–160.

ULATE, M. (2021): "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates," *American Economic Review*, 111, 1–40.

WHITED, T. M., Y. WU, AND K. XIAO (2023): "Will Central Bank Digital Currency Disintermediate Banks?" IHS Working Paper Series 47, Institute for Advanced Studies.

WILLIAMSON, S. (2022a): "Central bank digital currency and flight to safety," *Journal of Economic Dynamics and Control*, 142.

——— (2022b): "Central Bank Digital Currency: Welfare and Policy Implications," *Journal of Political Economy*, 130, 2829–2861.

Appendix A Solving the Static Bank Model

First, substitute the balance sheet condition (2.5) into the objective function and write d_j as an implicit function of $1+i_j^d$, then the bank's problem becomes

$$\max_{i_j^d} \quad (1+i)(f_j+d_j)-(1+i_j^d)d_j,$$

Take the first order condition with respect to $1+i_j^d$

$$-d_j+\left((1+i)-(1+i_j^d)\right)\epsilon_j^d\frac{d_j}{1+i_j^d}$$

where $\epsilon_j^d \equiv \frac{\partial \ln d_j}{\partial \ln(1+i_j^d)}$. Rearrange, we obtain (2.6).

Next, we solve ϵ_j^d using (2.3):

$$\begin{aligned}
\epsilon_j^d &= \frac{\partial d_j}{\partial(1+i_j^d)}\frac{1+i_j^d}{d_j} \\
&= d_j\frac{\varepsilon^d}{1+i_j^d}\frac{1+i_j^d}{d_j}-d_j\frac{\varepsilon^d}{1+i^d}\frac{\partial(1+i^d)}{\partial(1+i_j^d)}\frac{1+i_j^d}{d_j}+d_j\frac{1}{d}\frac{\partial d}{\partial(1+i^d)}\frac{\partial(1+i^d)}{\partial(1+i_j^d)}\frac{1+i_j^d}{d_j} \\
&= \varepsilon^d-\varepsilon^d\frac{\partial(1+i^d)}{\partial(1+i_j^d)}\frac{1+i_j^d}{1+i^d}+\frac{\partial d}{\partial(1+i^d)}\frac{1+i^d}{d}\frac{\partial(1+i^d)}{\partial(1+i_j^d)}\frac{1+i_j^d}{1+i^d} \\
&= \varepsilon^d-\varepsilon^d\frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)}+\frac{\partial \ln d}{\partial \ln(1+i^d)}\frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)}. \qquad \text{(A.1)}
\end{aligned}$$

We then solve the elasticities. We define the elasticity of the aggregate deposit rate w.r.t. an individual deposit, and solve it using (2.4) and then (2.3).

$$\omega_d^{d_j} \equiv \frac{\partial \ln(1+i^d)}{\partial \ln(1+i_j^d)}=\frac{1}{n}\left(\frac{1+i_j^d}{1+i^d}\right)^{\varepsilon^d+1}=\frac{(1+i_j^d)d_j}{(1+i^d)d}. \qquad \text{(A.2)}$$

We can interpret it as the share of bank j's deposit.

We solve the elasticity of total deposit w.r.t the deposit rate using (2.1):

$$\frac{\partial \ln d}{\partial \ln(1+i^d)} = \theta\left(1-\frac{\partial \ln(1+i^{\mathcal{L}})}{\partial \ln(1+i^d)}\right)\equiv\theta\left(1-\omega_{\mathcal{L}}^d\right), \qquad \text{(A.3)}$$

where the last equality is by definition and we can solve it further using (2.2) and then (2.1):

$$\omega_{\mathcal{L}}^d \equiv \frac{\partial \ln(1+i^{\mathcal{L}})}{\partial \ln(1+i^d)}=\gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}=\frac{(1+i^d)d}{(1+i^{\mathcal{L}})\mathcal{L}}$$

Substitute (A.2) and (A.3) into (A.1), and we obtain

$$\epsilon_j^d = (1-\omega_d^{d_j})\varepsilon^d + \omega_d^{d_j}(1-\omega_{\mathcal{L}}^d)\theta. \tag{A.4}$$

When all banks are identical, in a symmetric equilibrium, they all pay the same deposit rate $i_j^d = i^d$, face the same elasticity $\epsilon_j^d = \epsilon^d$, and obtain one n-th of total deposit. Consequently, $\omega_d^{d_j} = 1/n$, and we obtain equation (2.7).

Once symmetry across banks has been imposed in the model of Section 2, the equilibrium system for the determination of the endogenous deposit rate is composed of equation (2.6) for the representative bank, the definition of $\omega_{\mathcal{L}}^d$ in equation (2.8), the definition of the liquidity interest rate in equation (2.2), as well as the equation for the behavior of the endogenous deposit markdown (2.7). Reproducing those here, we have the following equilibrium system of equations:

$$\begin{aligned} 1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i) \\ \omega_{\mathcal{L}}^d &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \\ 1+i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\ \epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n}(1-\omega_{\mathcal{L}}^d) \end{aligned}$$

Introduce the third into the second and simplify in order to obtain:

$$\begin{aligned} 1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i) \\ \omega_{\mathcal{L}}^d &= \frac{\gamma_d}{\gamma_m\left(\frac{1}{1+i^d}\right)^{\theta+1} + \gamma_d + (1-\gamma_m-\gamma_d)\left(\frac{1+i^{cbdc}}{1+i^d}\right)^{\theta+1}} \\ \epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n}(1-\omega_{\mathcal{L}}^d) \end{aligned}$$

This is a system of three equations in three endogenous variables $(i^d, \omega_{\mathcal{L}}^d, \epsilon^d)$ and several exogenous variables $(i, i^{cbdc}, \gamma_m, \gamma_d, n, \varepsilon^d)$. The system is implicit and cannot be solved in closed form. Therefore, we apply the implicit function theorem to determine how changes in exogenous variables affect the endogenous variables. First, we show that in a special case, there is a known solution to the system that we can apply the implicit function theorem around.

Appendix A.1 A Special Case

In the special case where cash and CBDC pay zero interest rate, we can solve in close form for the level of the policy rate where the deposit rate reaches zero percent. In this case, the equilibrium equations are:

$$\begin{aligned} i^d &= i^{\mathcal{L}} = 0 \\ \omega_{\mathcal{L}}^d &= \gamma_d \\ \epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n}(1-\gamma_d) \end{aligned}$$

$$1 = \frac{\epsilon^d}{\epsilon^d + 1}(1+i)$$

Which from the last equation allows us to obtain the required level of the policy rate for this to be an equilibrium:

$$\begin{aligned}
\frac{\epsilon^d + 1}{\epsilon^d} &= 1 + i \\
\frac{1}{\epsilon^d} &= i \\
i_{i^d=0} &= \frac{n}{\varepsilon^d(n-1) + \theta(1-\gamma_d)}
\end{aligned}$$

This is useful because we know that there is an actual solution to the system of equations that we can then approximate the system around (this is technically a requirement for the implicit function theorem). It is also useful to know what is the level of the policy rate where the deposit rate becomes zero, both before and after the introduction of CBDC.

Appendix A.2 Implicit Function Theorem Application

Denote with x all the exogenous variables and with y the three endogenous ones, simplify the notation of $\omega^d_{\mathcal{L}}$ to just ω, and define:

$$\begin{aligned}
F_1(x,y) &= 1 + i^d - \frac{\epsilon^d}{\epsilon^d + 1}(1+i) \\
F_2(x,y) &= \omega - \frac{\gamma_d}{\frac{\gamma_m + (1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d} \\
F_3(x,y) &= \epsilon^d - \frac{n-1}{n}\varepsilon^d - \frac{\theta}{n}(1-\omega).
\end{aligned}$$

Then we can apply the implicit function theorem to our system of equations that can be represented by $F(x,y) = 0$. We can write the matrix of derivatives of the F's w.r.t. the endogenous variables as:

$$D_y F = \begin{bmatrix} \frac{\partial F_1}{\partial i^d} & \frac{\partial F_1}{\partial \omega} & \frac{\partial F_1}{\partial \epsilon^d} \\ \frac{\partial F_2}{\partial i^d} & \frac{\partial F_2}{\partial \omega} & \frac{\partial F_2}{\partial \epsilon^d} \\ \frac{\partial F_3}{\partial i^d} & \frac{\partial F_3}{\partial \omega} & \frac{\partial F_3}{\partial \epsilon^d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ b & 1 & 0 \\ 0 & c & 1 \end{bmatrix},$$

where:

$$\begin{aligned}
a &= -\frac{1+i}{(\epsilon^d+1)^2} < 0 \\
b &= -\frac{\gamma_d(1+\theta)(1+i^d)^{-\theta-2}\left[\gamma_m + (1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}\right]}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} < 0 \\
c &= \frac{\theta}{n} > 0
\end{aligned}$$

The determinant of D_yF is $1+abc$, which is positive because of the signs of a, b and c. Moreover, we can also calculate the inverse of D_yF (using the transpose of the matrix of cofactors divided by the determinant):

$$(D_yF)^{-1} = \frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix},$$

We can also write:

$$D_xF = \begin{bmatrix} \frac{\partial F_1}{\partial i} & \frac{\partial F_1}{\partial i^{cbdc}} & \frac{\partial F_1}{\partial \gamma_m} & \frac{\partial F_1}{\partial \gamma_d} & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial \varepsilon^d} \\ \frac{\partial F_2}{\partial i} & \frac{\partial F_2}{\partial i^{cbdc}} & \frac{\partial F_2}{\partial \gamma_m} & \frac{\partial F_2}{\partial \gamma_d} & \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial \varepsilon^d} \\ \frac{\partial F_3}{\partial i} & \frac{\partial F_3}{\partial i^{cbdc}} & \frac{\partial F_3}{\partial \gamma_m} & \frac{\partial F_3}{\partial \gamma_d} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial \varepsilon^d} \end{bmatrix} = \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix},$$

where:

$$\begin{aligned}
e &= \frac{\gamma_d(1-\gamma_m-\gamma_d)(1+\theta)(1+i^{cbdc})^{\theta}(1+i^d)^{-\theta-1}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} > 0 \\
f &= \frac{\gamma_d\frac{1-(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} \lesseqgtr 0 \\
g &= -\frac{\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d-\gamma_d\left(1-\frac{(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}\right)}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} \\
&= -\frac{\frac{\gamma_m+(1-\gamma_m)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}+\gamma_d\right)^2} < 0
\end{aligned}$$

Notice that f has the opposite sign of i^{cbdc}. That is, if i^{cbdc} is positive, then f is negative, if $i^{cbdc}=0$, then $f=0$, and if i^{cbdc} is negative then f is positive. We can use the implicit function theorem to write:

$$\begin{aligned}
D_xy &= \begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varepsilon^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varepsilon^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varepsilon^d} \end{bmatrix} \\
&= -(D_yF)^{-1}D_xF \\
&= -\frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix} \\
&= -\frac{1}{1+abc}\begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & ace & acf & acg & a\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & a\left(1-\frac{1}{n}\right) \\ b\frac{\epsilon^d}{\epsilon^d+1} & e & f & g & -ab\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & ab\left(\frac{1}{n}-1\right) \\ -bc\frac{\epsilon^d}{\epsilon^d+1} & -ce & -cf & -cg & -\frac{\varepsilon^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix}.
\end{aligned}$$

Knowing the sign of all the letters ($a < 0, b < 0, c > 0, e > 0, f \lesseqgtr 0, g < 0$) we can sign these derivatives:

$$\begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varepsilon^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varepsilon^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varepsilon^d} \end{bmatrix} = \begin{bmatrix} + & + & ? & - & + & + \\ + & - & ? & + & + & + \\ - & + & ? & - & + & + \end{bmatrix},$$

where we required $\varepsilon^d > \theta(1-\omega)$ to sign the fifth column, but this requirement is less stringent that $\varepsilon^d > \theta$ which we should assume anyway (more substitutability in the inner nest than the outer nest, saying that banks are more substitutable with each other than deposits are substitutable with cash and CBDC). The signs of the third column are -, + , - if $i^{cbdc} > 0$, all zero if $i^{cbdc} = 0$, and +, - , + if $i^{cbdc} < 0$.

Appendix A.3 Pass-Through of the Policy Rate to the Deposit Rate

In this appendix, we analyze the pass-through of the policy rate to the deposit rate and how that depends on parameters, and whether this pass-through has a minimum, what that minimum is, and how it depends on parameters. Relative to the static model in Section 2, we introduce the μ^d cost of issuing deposits that we adopt in the full model, and we also allow total liquidity to be endogenous as in the full model, denoting $\varepsilon^{\mathcal{L}} \equiv (\partial \ln \mathcal{L})/(\partial \ln(1+i^{\mathcal{L}}))$. To start, notice that, in the pre-CBDC scenario where $i^{cbdc} = 0$ and where cash pays zero percent, the deposit rate depends on just four equations (we ignore the time subscripts for notational convenience and because all variables are dated the same):

$$(1+i^{\mathcal{L}})^{\theta+1} = \gamma_m + \gamma_d(1+i^d)^{\theta+1} \tag{A.5}$$

$$\omega^d_{\mathcal{L}} = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \tag{A.6}$$

$$\epsilon^d = \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n} - \frac{\omega^d_{\mathcal{L}}}{n}\left(\theta - \varepsilon^{\mathcal{L}}\right) \tag{A.7}$$

$$1+i^d = \frac{\epsilon^d}{\epsilon^d+1}(1+i-\mu^d) \tag{A.8}$$

We can combine all of these into a single equation in i and i^d and then use the implicit function theorem to compute the derivative of i^d w.r.t. i, and then we can see how this object (which is the pass-through) behaves. Start with equation (A.8) and simplify:

$$\begin{aligned} (\epsilon^d+1)(1+i^d) &= \epsilon^d(1+i-\mu^d) \\ 1+i^d &= \epsilon^d(i-\mu^d-i^d) \end{aligned} \tag{A.9}$$

Then, introduce equation (A.7) and simplify:

$$n(1+i^d) = \left[(n-1)\varepsilon^d + \theta - \omega^d_{\mathcal{L}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i-\mu^d-i^d) \tag{A.10}$$

Furthermore, introduce equation (A.6) and simplify:

$$n(1+i^d) = \left[(n-1)\varepsilon^d + \theta - \frac{\gamma_d(1+i^d)^{\theta+1}}{(1+i^{\mathcal{L}})^{\theta+1}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i-\mu^d-i^d) \tag{A.11}$$

Finally, introduce equation (A.5) and simplify:

$$n(1+i^d) \quad = \quad \left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i - \mu^d - i^d) \tag{A.12}$$

Notice that this is an equation in the variables i and i^d and the parameters n, ε^d, θ, γ_m, γ_d, $\varepsilon^{\mathcal{L}}$, and μ^d. Now we write:

$$F(i^d, i) = n(1+i^d) - \left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right](i - \mu^d - i^d) \tag{A.13}$$

So, the equilibrium equation for i^d as a function of i can be written as:

$$F(i^d, i) = 0 \tag{A.14}$$

Then, if the assumptions of the implicit function theorem are satisfied, we know that:

$$\frac{di^d}{di} = -\frac{F_i}{F_{i^d}} \tag{A.15}$$

Since this is the derivative of i^d w.r.t. to i, it has the interpretation of the pass-through of the policy rate to the deposit rate, which is an important object in papers like Drechsler et al. (2017, 2021). Notice also that if we want the second derivative of i^d w.r.t. i we can also use the implicit function theorem for this:

$$\frac{d^2 i^d}{di^2} = \frac{2F_i F_{i^d} F_{ii^d} - F_{ii}F_{i^d}^2 - F_{i^d i^d}F_i^2}{F_{i^d}^3} \tag{A.16}$$

We want to study if there is a value of the policy rate for which $\frac{d^2 i^d}{di^2} = 0$, and then we can obtain the value of the pass-through, $\frac{di^d}{di}$, at that value of the policy rate, to obtain the minimum pass-through and evaluate how it depends on parameters.

Start with F_i:

$$F_i = -\left[(n-1)\varepsilon^d + \theta - \frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varepsilon^{\mathcal{L}}\right)\right] \equiv -\aleph(i^d) < 0 \tag{A.17}$$

Where the Aleph function denotes the expression inside the brackets which is a function of i^d only. Notice that $\aleph(i^d) > 0$ everywhere. Then compute F_{i^d}:

$$F_{i^d} = n + \aleph(i^d) - \aleph'(i^d)(i - \mu^d - i^d) \tag{A.18}$$

And notice that:

$$\aleph'(i^d) \quad = \quad -\left(\theta - \varepsilon^{\mathcal{L}}\right)\frac{d}{di^d}\left[\frac{1}{1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\right]$$

$$= \quad -\left(\theta-\varepsilon^{\mathcal{L}}\right)\frac{(\theta+1)\frac{\gamma_m}{\gamma_d}(1+i^d)^{-\theta-2}}{\left(1+\frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}\right)^2}<0 \tag{A.19}$$

Notice then than $F_{i^d} > 0$ so we can apply the implicit function theorem safely. We also know that $\frac{di^d}{di}$ is positive, so that pass-through is always positive (this is one of the things proved in proposition 1 of the paper in Section 2, but that was with exogenous total liquidity, while here liquidity is endogenous, and there we had the possibility of CBDC, whereas here we are necessarily in the pre-CBDC scenario). Now lets investigate the second derivative of i^d w.r.t. i and when it is zero. Notice that $F_{ii} = 0$. Given this, we know that $\frac{d^2i^d}{di^2} = 0$ iff:

$$F_i F_{i^d i^d} \quad = \quad 2F_{i^d}F_{ii^d} \tag{A.20}$$

Using the expressions above, we can re-write this as:

$$-\aleph(i^d)\left[\aleph'(i^d)-\aleph''(i^d)(i-\mu^d-i^d)+\aleph'(i^d)\right]=2\left[n+\aleph(i^d)-\aleph'(i^d)(i-\mu^d-i^d)\right](-\aleph'(i^d)) \tag{A.21}$$

Simplifying, we get:

$$-\aleph(i^d)\left[2-\frac{\aleph''(i^d)}{\aleph'(i^d)}(i-\mu^d-i^d)\right] \quad = \quad -2\left[n+\aleph(i^d)-\aleph'(i^d)(i-\mu^d-i^d)\right]$$

$$2n \quad = \quad \aleph(i^d)(i-\mu^d-i^d)\left(2\frac{\aleph'(i^d)}{\aleph(i^d)}-\frac{\aleph''(i^d)}{\aleph'(i^d)}\right) \tag{A.22}$$

Using the equilibrium condition $F(i^d, i) = 0$, which can be re-written as $n(1+i^d) = \aleph(i^d)(i-\mu^d-i^d)$, we can write the previous equation as:

$$\frac{2}{1+i^d}\frac{\aleph(i^d)}{\aleph'(i^d)} \quad = \quad 2-\frac{\aleph''(i^d)\aleph(i^d)}{(\aleph'(i^d))^2} \tag{A.23}$$

Next, we have to calculate $\aleph''(i^d)$. We first write $\aleph(i^d)$ as a function that depends on constants a and b (these are unrelated to the a and b parameters inside the Φ function that we use in the full model) and another function of i^d denoted $y(i^d)$:

$$\aleph(i^d) \quad = \quad a-by(i^d)^{-1} \tag{A.24}$$

Where $a = (n-1)\varepsilon^d + \theta$, $b = \theta - \varepsilon^{\mathcal{L}}$, and $y(i^d)$ is defined as follows:

$$y(i^d) \quad = \quad 1+g(1+i^d)^h \tag{A.25}$$

Where $g = \gamma_m/\gamma_d$ and $h = -(\theta+1)$. This seems convoluted, but it will make computing derivatives much easier. First, notice that:

$$\aleph'(i^d) \quad = \quad by(i^d)^{-2}y'(i^d)$$

$$\aleph''(i^d) \quad = \quad -2by(i^d)^{-3}(y'(i^d))^2+by(i^d)^{-2}y''(i^d) \tag{A.26}$$

Hence:

$$\frac{\aleph(i^d)\aleph''(i^d)}{(\aleph'(i^d))^2} = \left(\frac{ay(i^d)}{b} - 1\right)\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2} - 2\right) \tag{A.27}$$

And:

$$\frac{\aleph(i^d)}{\aleph'(i^d)} = \frac{ay(i^d)^2}{by'(i^d)} - \frac{y(i^d)}{y'(i^d)} = \frac{y(i^d)}{y'(i^d)}\left(\frac{ay(i^d)}{b} - 1\right) \tag{A.28}$$

With this, equation (A.23) can be written as:

$$\begin{aligned} \frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)}\left(\frac{ay(i^d)}{b} - 1\right) &= 2 - \left(\frac{ay(i^d)}{b} - 1\right)\left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2} - 2\right) \\ \frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)} &= \frac{2b}{ay(i^d) - b} - \left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2} - 2\right) \end{aligned} \tag{A.29}$$

Next, notice that:

$$\begin{aligned} y(i^d) &= 1 + g(1+i^d)^h \\ y'(i^d) &= gh(1+i^d)^{h-1} \\ y''(i^d) &= gh(h-1)(1+i^d)^{h-2} \end{aligned} \tag{A.30}$$

So:

$$\begin{aligned} \frac{y''(i^d)y(i^d)}{(y'(i^d))^2} &= (1+g(1+i^d)^h)\frac{gh(h-1)(1+i^d)^{h-2}}{g^2h^2(1+i^d)^{2h-2}} \\ &= \frac{(h-1)}{gh(1+i^d)^h} + 1 - \frac{1}{h} \end{aligned} \tag{A.31}$$

Hence:

$$\frac{y''(i^d)y(i^d)}{(y'(i^d))^2} - 2 = \frac{(h-1)}{gh(1+i^d)^h} - 1 - \frac{1}{h} \tag{A.32}$$

With this, equation (A.29) can be written as:

$$\begin{aligned} \frac{2}{1+i^d}\frac{y(i^d)}{y'(i^d)} &= \frac{2b}{ay(i^d) - b} - \left(\frac{y(i^d)y''(i^d)}{(y'(i^d))^2} - 2\right) \\ 1 + h + (1-h)g(1+i^d)^h &= \frac{2bhg(1+i^d)^h}{a + ag(1+i^d)^h - b} \end{aligned} \tag{A.33}$$

For convenience, notice that $a - b = (n-1)\varepsilon^d + \theta - \theta + \varepsilon^{\mathcal{L}} = (n-1)\varepsilon^d + \varepsilon^{\mathcal{L}} \equiv k$, so we get:

$$1 + h + (1-h)g(1+i^d)^h = \frac{2bhg(1+i^d)^h}{k + ag(1+i^d)^h} \tag{A.34}$$

Use the definition of $h = -\theta - 1$ to simplify:

$$\begin{aligned} (2+\theta)g(1+i^d)^h - \theta &= \frac{2bhg(1+i^d)^h}{k+ag(1+i^d)^h} \\ k(2+\theta)g(1+i^d)^h + a(2+\theta)g^2(1+i^d)^{2h} - \theta k - \theta a g(1+i^d)^h &= 2bhg(1+i^d)^h \end{aligned} \tag{A.35}$$

So, we can finally simplify this into a quadratic equation in $z = g(1+i^d)^h$:

$$a(2+\theta)z^2 + [k(2+\theta) - \theta a - 2bh]\, z - \theta k = 0 \tag{A.36}$$

Simplify the middle coefficient:

$$\begin{aligned} k(2+\theta) - \theta a - 2bh &= 2(a-b) + \theta(a-b) - \theta a + 2b(\theta+1) \\ &= 2a + b\theta \end{aligned} \tag{A.37}$$

With this, we can express the quadratic equation just in terms of a, b, and θ:

$$a(2+\theta)z^2 + (2a+b\theta)z - \theta(a-b) = 0 \tag{A.38}$$

The discriminant for this quadratic equation is:

$$\begin{aligned} \Delta &= (2a+b\theta)^2 + 4a\theta(2+\theta)(a-b) \\ &= (2a\theta + 2a - b\theta)^2 \end{aligned} \tag{A.39}$$

Therefore, the two solutions are:

$$z_{1,2} = \frac{-(2a+b\theta) \pm (2a\theta + 2a - b\theta)}{2a(2+\theta)} \tag{A.40}$$

The correct solution is the one with the plus (the one with the minus would lead to $1 + i^d$ being negative, which would lead to an extremely negative i^d that is implausible), so we get:

$$\begin{aligned} z^* &= \frac{2a\theta + 2a - b\theta - 2a - b\theta}{2a(2+\theta)} \\ &= \frac{a\theta - b\theta}{2a + a\theta} \end{aligned} \tag{A.41}$$

Since we know the minimizer z^* in close form, we can use it to obtain the minimizer i^{d*} in closed form as well:

$$\begin{aligned} z^* &= g(1+i^d)^h \\ i^{d*} &= \left(\frac{z^*}{g}\right)^{\frac{1}{h}} - 1 \end{aligned} \tag{A.42}$$

We want to obtain the value of the pass-through at this pass-through minimizer i^{d*}. Notice that since

$y^* = 1 + z^*$, then we get:

$$y^* = 1 + \frac{a\theta - b\theta}{2a + a\theta} = \frac{2a + 2a\theta - b\theta}{2a + a\theta} \tag{A.43}$$

And since $\aleph^* = a - b(y^*)^{-1}$, then we get:

$$\begin{aligned} \aleph^* &= a - b\frac{2a + a\theta}{2a + 2a\theta - b\theta} \\ &= \frac{2a(1+\theta)(a-b)}{2a(1+\theta) - b\theta} \end{aligned} \tag{A.44}$$

And then the pass-through at the minimizer is:

$$\begin{aligned} \left(\frac{di^d}{di}\right)^* &= -\frac{F_i^*}{F_{i^d}^*} = \frac{\aleph^*}{n + \aleph^* - (\aleph')^*(i - \mu^d - i^d)} \\ &= \frac{\aleph^*}{n + \aleph^* - (\aleph')^*(i - \mu^d - i^d)} \end{aligned} \tag{A.45}$$

Recall that $n(1 + i^d) = \aleph(i^d)(i - \mu^d - i^d)$, so we can rewrite the previous expression as:

$$\begin{aligned} \left(\frac{di^d}{di}\right)^* &= \frac{\aleph^*}{n + \aleph^* - (\aleph')^* \frac{n(1+i^d)}{\aleph^*}} \\ &= \frac{(\aleph^*)^2}{n\aleph^* + (\aleph^*)^2 - (\aleph')^* n(1 + i^d)} \end{aligned} \tag{A.46}$$

Recall that:

$$(\aleph')^* = b(y^*)^{-2}(y')^* \tag{A.47}$$

Then relate y' to y using the equations in (A.30):

$$\begin{aligned} y'(i^d) &= gh(1 + i^d)^{h-1} \\ (y')^* &= \frac{h(y^* - 1)}{1 + i^d} \end{aligned} \tag{A.48}$$

Introducing this into our equation for the minimum pass-through, we get:

$$\left(\frac{di^d}{di}\right)^* = \frac{(\aleph^*)^2}{n\aleph^* + (\aleph^*)^2 + b(y^*)^{-2}(\theta + 1)(y^* - 1)n} \tag{A.49}$$

Compute the inverse of the pass-through for convenience:

$$\begin{aligned} \left(\left(\frac{di^d}{di}\right)^*\right)^{-1} &= \frac{n}{\aleph^*} + 1 + \frac{b}{(y^*)^2}\frac{\theta + 1}{(\aleph^*)^2} n z^* \\ &= 1 + \frac{n}{a - b} + \frac{nb\theta^2}{4a(1+\theta)(a-b)} \end{aligned} \tag{A.50}$$

Finally, substituting what a and b are, we obtain an expression for inverse minimum pass-through as a function of just four parameter values n, ε^d, $\varepsilon^{\mathcal{L}}$, and θ:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varepsilon^d + \varepsilon^{\mathcal{L}}} + \frac{n(\theta - \varepsilon^{\mathcal{L}})\theta^2}{4[(n-1)\varepsilon^d + \theta](1+\theta)[(n-1)\varepsilon^d + \varepsilon^{\mathcal{L}}]} \tag{A.51}$$

This is an exact expression for the (inverse) minimum pass-through as a function of four relevant parameters n, ε^d, $\varepsilon^{\mathcal{L}}$, and θ. This tells us that the inverse minimum pass-through is always between 1 and infinity, so the minimum pass-through is always between 0 and 1. It is easy to see that when $n \to \infty$ then the inverse minimum pass-through tends to $1 + \frac{1}{\varepsilon^d}$.

The expression for the pass-through tells us that a higher $\varepsilon^{\mathcal{L}}$ always increases the minimum pass-through while a higher θ decreases the minimum pass-through. Therefore, if one intended to find the parameter values that lower the minimum pass-through, one would pick the lowest possible $\varepsilon^{\mathcal{L}}$ and the highest possible θ. However, we also require $0 \leq \varepsilon^{\mathcal{L}} \leq \theta \leq \varepsilon^d$, so in order to obtain the lowest possible minimum pass-through w.r.t. $\varepsilon^{\mathcal{L}}$ and θ one can pick $\varepsilon^{\mathcal{L}} = 0$ and $\theta = \varepsilon^d$. In this case, the expression for the inverse minimum pass-through is:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varepsilon^d} + \frac{\varepsilon^d}{4(1+\varepsilon^d)(n-1)} \tag{A.52}$$

While this expression depends both on n and ε^d, if n is too big, then there are no reasonable values of ε^d that can achieve a minimum pass-through of 50% or lower. Therefore, the model requires a low n to be able to match a low minimum pass-through.

Appendix B Details on the Full Model

Appendix B.1 The Household's Problem

The Bellman equation for the household's problem is given by:

$$V_t(AH_{t-1}) = \max_{C_t, N_t, M_t, \{D_{j,t}\}_{j=1}^n, CBDC_t, B_t} \left\{ u(C_t) - v(N_t) + \beta \mathbb{E}_t(V_{t+1}(AH_t)) \right\}.$$

We can express C_t as:

$$C_t = \frac{W_t N_t + AH_{t-1} + T_t - B_t - \Phi(\mathcal{L}_t) P_t}{P_t},$$

with this definition we can write the Bellman equation as a function of 4 individual choice variables and n deposit choices (the $D_{j,t}$). The first order conditions are:

$$0 = u'(C_t)\left(\frac{W_t}{P_t}\right) - v'(N_t)$$
$$0 = u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t}\frac{1}{P_t}\right) + \beta(1+i_t^m)\mathbb{E}_t\left(V'_{t+1}(AH_t)\right)$$

$$
\begin{aligned}
0 &= u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}}\frac{1}{P_t}\right) + \beta(1+i^d_{j,t})\mathbb{E}_t\left(V'_{t+1}(AH_t)\right) \\
0 &= u'(C_t)\left(-\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}\frac{1}{P_t}\right) + \beta(1+i^{cbdc}_t)\mathbb{E}_t\left(V'_{t+1}(AH_t)\right) \\
0 &= u'(C_t)\left(-\frac{1}{P_t}\right) + \beta(1+i_t)\mathbb{E}_t\left(V'_{t+1}(AH_t)\right).
\end{aligned}
$$

The Benveniste-Scheinkman condition is:

$$
V'_t(AH_{t-1}) = \frac{u'(C_t)}{P_t},
$$

moving this condition one period forward and introducing it into the F.O.C.'s we can rewrite them as:

$$
\begin{aligned}
v'(N_t) &= u'(C_t)\frac{W_t}{P_t} && \text{(B.1)} \\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t}\frac{1}{P_t} &= \beta(1+i^m_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}}\frac{1}{P_t} &= \beta(1+i^d_{j,t})\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}\frac{1}{P_t} &= \beta(1+i^{cbdc}_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right) \\
\frac{u'(C_t)}{P_t} &= \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). && \text{(B.2)}
\end{aligned}
$$

The first condition is the intratemporal condition for labor supply and the fifth one is the Euler equation. The second, third, and fourth deal with the demand for cash, deposits, and CBDC respectively.

We will first aggregate the individual demands for the deposits of each of the n banks into an aggregate deposit demand. If we introduce the fifth F.O.C. into the third, we obtain:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}} = \frac{1+i^d_{j,t}}{1+i_t}.
$$

The derivative of aggregate deposits w.r.t. an individual deposit is:

$$
\begin{aligned}
\frac{\partial d_t}{\partial d_{j,t}} &= \frac{\varepsilon^d}{\varepsilon^d+1}\left(\sum_{j=1}^{n}\alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{-\frac{1}{\varepsilon^d+1}} \alpha_j^{-\frac{1}{\varepsilon^d}}\frac{\varepsilon^d+1}{\varepsilon^d} d_{j,t}^{\frac{1}{\varepsilon^d}} \\
&= \left(d_t^{\frac{\varepsilon^d+1}{\varepsilon^d}}\right)^{-\frac{1}{\varepsilon^d+1}} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{1}{\varepsilon^d}} = \alpha_j^{-\frac{1}{\varepsilon^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varepsilon^d}}.
\end{aligned}
$$

Introducing this into the F.O.C. for deposits we get:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\alpha_j^{-\frac{1}{\varepsilon^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varepsilon^d}} = \frac{1+i^d_{j,t}}{1+i_t},
$$

raise this to the power of $\varepsilon^d + 1$, multiply by α_j, and then add over banks:

$$\begin{aligned}
\left(\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\right)^{\varepsilon^d+1} \alpha_j^{-\frac{\varepsilon^d+1}{\varepsilon^d}} \left(\frac{d_{j,t}}{d_t}\right)^{\frac{\varepsilon^d+1}{\varepsilon^d}} &= \frac{(1+i^d_{j,t})^{\varepsilon^d+1}}{(1+i_t)^{\varepsilon^d+1}} \\
\left(\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\right)^{\varepsilon^d+1} \left(\frac{1}{d_t}\right)^{\frac{\varepsilon^d+1}{\varepsilon^d}} \sum_{j=1}^{n} \alpha_j^{-\frac{1}{\varepsilon^d}} d_{j,t}^{\frac{\varepsilon^d+1}{\varepsilon^d}} &= \frac{\sum_{j=1}^{n} \alpha_j (1+i^d_{j,t})^{\varepsilon^d+1}}{(1+i_t)^{\varepsilon^d+1}} \\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1+i^d_t}{1+i_t},
\end{aligned}$$

where we have defined:

$$1+i^d_t = \left(\sum_{j=1}^{n} \alpha_j (1+i^d_{j,t})^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}}.$$

Using the equation $\Phi'(\mathcal{L}_t)(\partial\mathcal{L}_t/\partial d_t) = (1+i^d_t)/(1+i_t)$, we can turn the F.O.C. for individual deposits into:

$$d_{j,t} = \alpha_j \left(\frac{1+i^d_{j,t}}{1+i^d_t}\right)^{\varepsilon^d} d_t.$$

Once we have "aggregated up" deposits, we can turn to the decision between the three liquid savings instruments, where we have the following three F.O.C.s:

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{1+i^m_t}{1+i_t} \tag{B.3}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} = \frac{1+i^d_t}{1+i_t} \tag{B.4}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} = \frac{1+i^{cbdc}_t}{1+i_t}. \tag{B.5}$$

The derivative of liquidity w.r.t. real money balances is:

$$\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{\theta}{\theta+1}\left(\mathcal{L}_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}-1} \gamma_m^{-\frac{1}{\theta}} \frac{\theta+1}{\theta} m_t^{\frac{1}{\theta}} = \mathcal{L}_t^{-\frac{1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{1}{\theta}}.$$

Similar expressions are available for $\partial\mathcal{L}_t/\partial d_t$ and $\partial\mathcal{L}_t/\partial cbdc_t$. We can write demands as:

$$\begin{aligned}
\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{1}{\theta}} &= \frac{1+i^m_t}{1+i_t} \\
\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{1}{\theta}} &= \frac{1+i^d_t}{1+i_t} \\
\Phi'(\mathcal{L}_t)\mathcal{L}_t^{-\frac{1}{\theta}} \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{1}{\theta}} &= \frac{1+i^{cbdc}_t}{1+i_t}.
\end{aligned}$$

Raise all of these to the power of $\theta + 1$ and multiply by an appropriate constant:

$$\begin{aligned}
\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}}\gamma_m^{-\frac{1}{\theta}}m_t^{\frac{\theta+1}{\theta}} &= \gamma_m\frac{(1+i_t^m)^{\theta+1}}{(1+i_t)^{\theta+1}} \\
\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}}\gamma_d^{-\frac{1}{\theta}}d_t^{\frac{\theta+1}{\theta}} &= \gamma_d\frac{(1+i_t^d)^{\theta+1}}{(1+i_t)^{\theta+1}} \\
\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}}\gamma_{cbdc}^{-\frac{1}{\theta}}cbdc_t^{\frac{\theta+1}{\theta}} &= \gamma_{cbdc}\frac{(1+i_t^{cbdc})^{\theta+1}}{(1+i_t)^{\theta+1}},
\end{aligned}$$

by adding these three we get:

$$\Phi'(\mathcal{L}_t)^{\theta+1}\mathcal{L}_t^{-\frac{\theta+1}{\theta}}\mathcal{L}_t^{\frac{\theta+1}{\theta}} = \frac{(1+i_t^{\mathcal{L}})^{\theta+1}}{(1+i_t)^{\theta+1}},$$

where the aggregate interest rate for liquidity takes the form:

$$1+i_t^{\mathcal{L}} \equiv \left(\gamma_m(1+i_t^m)^{\theta+1} + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}}. \tag{B.6}$$

This finally allows us to write a simple demand equation for overall liquidity:

$$\frac{1+i_t^{\mathcal{L}}}{1+i_t} = \Phi'(\mathcal{L}_t). \tag{B.7}$$

And we can write the demand for each instrument as:

$$m_t = \gamma_m\left(\frac{1+i_t^m}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t \tag{B.8}$$

$$d_t = \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t \tag{B.9}$$

$$cbdc_t = \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t. \tag{B.10}$$

Appendix B.2 Alternative Setup: Liquidity in Utility

In the baseline model, liquid instruments are demanded by the household because of the non-linear cost function $\Phi(\mathcal{L}_t)$ in the budget constraint. This leads to the set of tractable holding schedules in (3.4)-(3.6). This appendix provides an alternative setup where we introduce liquidity into the utility function to achieve the same holding schedules. Assuming all banks are symmetric, we focus only on the aggregate deposits.

Assume the household has the following utility function:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(U(C_t,\mathcal{L}_t)-v(N_t)),$$

while keeping the budget constraint standard:

$$P_tC_t + B_t + M_t + D_t + CBDC_t = W_tN_t + AH_{t-1} + T_t,$$

where

$$AH_{t-1} = (1+i_{t-1})B_{t-1} + (1+i^{m}_{t-1})M_{t-1} + (1+i^{d}_{t-1})D_{t-1} + (1+i^{cbdc}_{t-1})CBDC_{t-1}.$$

is the same as in the main text. The liquidity instruments inside $\mathcal{L}_t$ now have a one-for-one cost in the budget constraint, but they enter the utility function.

The first order conditions are

$$\begin{aligned}
v'(N_t) &= U_C(C_t,\mathcal{L}_t)\frac{W_t}{P_t} \\
\frac{U_C(C_t,\mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{P_t}\frac{\partial \mathcal{L}_t}{\partial m_t} + \beta(1+i^m_t)\mathbb{E}_t\left(\frac{U_C(C_{t+1},\mathcal{L}_{t+1})}{P_{t+1}}\right) \\
\frac{U_C(C_t,\mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{P_t}\frac{\partial \mathcal{L}_t}{\partial d_t} + \beta(1+i^d_t)\mathbb{E}_t\left(\frac{U_C(C_{t+1},\mathcal{L}_{t+1})}{P_{t+1}}\right) \\
\frac{U_C(C_t,\mathcal{L}_t)}{P_t} &= \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{P_t}\frac{\partial \mathcal{L}_t}{\partial cbdc_t} + \beta(1+i^{cbdc}_t)\mathbb{E}_t\left(\frac{U_C(C_{t+1},\mathcal{L}_{t+1})}{P_{t+1}}\right) \\
\frac{U_C(C_t,\mathcal{L}_t)}{P_t} &= \beta(1+i_t)\mathbb{E}_t\left(\frac{U_C(C_{t+1},\mathcal{L}_{t+1})}{P_{t+1}}\right)
\end{aligned}$$

Introducing the last equation (Euler equation) into the second to fourth ones and simplifying, we obtain:

$$\frac{i_t - i^m_t}{1+i_t} = \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{U_C(C_t,\mathcal{L}_t)}\frac{\partial \mathcal{L}_t}{\partial m_t} \tag{B.11}$$

$$\frac{i_t - i^d_t}{1+i_t} = \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{U_C(C_t,\mathcal{L}_t)}\frac{\partial \mathcal{L}_t}{\partial d_t} \tag{B.12}$$

$$\frac{i_t - i^{cbdc}_t}{1+i_t} = \frac{U_{\mathcal{L}}(C_t,\mathcal{L}_t)}{U_C(C_t,\mathcal{L}_t)}\frac{\partial \mathcal{L}_t}{\partial cbdc_t}. \tag{B.13}$$

Let's assume a non-separable utility function similar to Greenwood et al. (1988), with the utility of C and $\mathcal{L}$ taking the following form:

$$U(C_t,\mathcal{L}_t) = \frac{(C_t + \xi(\mathcal{L}_t))^{1-\sigma} - 1}{1-\sigma},$$

then the marginal utilities with respect to consumption and $\mathcal{L}$ take the following forms:

$$\begin{aligned}
U_{\mathcal{L}}(C_t,\mathcal{L}_t) &= (C_t + \xi(\mathcal{L}_t))^{-\sigma}\xi'(\mathcal{L}_t) \\
U_C(C_t,\mathcal{L}_t) &= (C_t + \xi(\mathcal{L}_t))^{-\sigma}.
\end{aligned}$$

Hence, (B.11)-(B.13) become

$$\frac{i_t - i^m_t}{1+i_t} = \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} \tag{B.14}$$

$$\frac{i_t - i^d_t}{1+i_t} = \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} \tag{B.15}$$

$$\frac{i_t - i^{cbdc}_t}{1+i_t} = \xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}. \tag{B.16}$$

These equations are convenient, because they do not contain wealth effects in the demand for cash, deposits, and CBDC. This result comes from the GHH-type non-separable utility function.

Next, we assume the ξ function takes the following form:

$$\xi(\mathcal{L}_t(m_t, d_t, cbdc_t)) = m_t + d_t + cbdc_t - \Phi(\mathcal{L}_t(m_t, d_t, cbdc_t)),$$

Taking its derivatives with respect to money, aggregate deposits, and CBDC,

$$\begin{aligned}
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} \\
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} \\
\xi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} &= 1 - \Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t},
\end{aligned}$$

and (B.14)-(B.16) become

$$\begin{aligned}
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} &= \frac{1 + i_t^m}{1 + i_t} \\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1 + i_t^d}{1 + i_t} \\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} &= \frac{1 + i_t^{cbdc}}{1 + i_t}.
\end{aligned}$$

These equations are identical to equations (B.3)-(B.5), which then lead to the holding schedules (3.4)-(3.6); for further derivations, see Appendix B.1.

Appendix B.3 The Intermediate Good Firm's Problem

The Bellman equation of the intermediate good firm is:

$$V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP}) = \max_{N_t, \{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP}} \left\{ \Pi_t^m + \mathbb{E}_t(\Lambda_{t,t+1} V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP}) \right\},$$

where

$$\begin{aligned}
\Pi_t^m &= P_t^m Y_t^m - W_t N_t + (1-\delta) Q_t \sum_{j=1}^n K_{j,t}^P + (1-\delta) Q_t K_t^{NP} \\
&- \sum_{j=1}^n (1 + i_{j,t-1}^l) Q_{t-1} K_{j,t}^P - (1 + i_{t-1} + \varrho) Q_{t-1} K_t^{NP} \\
Y_t^m &= A_t K_t^\alpha N_t^{1-\alpha} \\
K_t &= \left((1-\psi)^{\frac{1}{\theta^k}} (K_t^{NP})^{\frac{\theta^k - 1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} (K_t^P)^{\frac{\theta^k - 1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k - 1}} \\
K_t^P &= \left(\sum_{j=1}^n (\alpha_j^l)^{\frac{1}{\varepsilon^l}} (K_{j,t}^P)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} \right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}},
\end{aligned}$$

and $\Lambda_{t,t+1}$ is the stochastic discount factor that the household uses to discount nominal cash flows between $t+1$ and t. The derivatives of K_t w.r.t. to non-pledgeable and the different components of pledgeable capital are:

$$\begin{aligned}
\frac{\partial K_t}{\partial K_t^{NP}} &= (1-\psi)^{\frac{1}{\theta^k}} \left(\frac{K_t}{K_t^{NP}}\right)^{\frac{1}{\theta^k}} \\
\frac{\partial K_t}{\partial K_{j,t}^P} &= \frac{\partial K_t}{\partial K_t^P}\frac{\partial K_t^P}{\partial K_{j,t}^P} = \psi^{\frac{1}{\theta^k}} \left(\frac{K_t}{K_t^P}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\epsilon^l}} \left(\frac{K_t^P}{K_{j,t}^P}\right)^{\frac{1}{\epsilon^l}}.
\end{aligned}$$

The F.O.C.'s w.r.t. labor, non-pledgeable, and all the individual types of pledgeable capital are then:

$$\begin{aligned}
0 &= (1-\alpha)P_t^m \frac{Y_t^m}{N_t} - W_t \\
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{\partial V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP})}{\partial K_{t+1}^{NP}}\right) \\
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{\partial V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP})}{\partial K_{j,t+1}^{P}}\right).
\end{aligned}$$

The Benveniste-Scheinkman conditions are:

$$\begin{aligned}
\frac{\partial V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP})}{\partial K_t^{NP}} &= \alpha(1-\psi)^{\frac{1}{\theta^k}} P_t^m \frac{Y_t^m}{K_t}\left(\frac{K_t}{K_t^{NP}}\right)^{\frac{1}{\theta^k}} + (1-\delta)Q_t - Q_{t-1}(1+i_{t-1}+\varrho) \\
\frac{\partial V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP})}{\partial K_{j,t}^{P}} &= \alpha\psi^{\frac{1}{\theta^k}} P_t^m \frac{Y_t^m}{K_t}\left(\frac{K_t}{K_t^{P}}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\epsilon^l}} \left(\frac{K_t^P}{K_{j,t}^P}\right)^{\frac{1}{\epsilon^l}} + (1-\delta)Q_t - Q_{t-1}(1+i_{j,t-1}^l).
\end{aligned}$$

Moving these forward one period and introducing them in the capital F.O.C.s we get:

$$\begin{aligned}
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\left(\alpha(1-\psi)^{\frac{1}{\theta^k}} P_{t+1}^m \frac{Y_{t+1}^m}{K_{t+1}}\left(\frac{K_{t+1}}{K_{t+1}^{NP}}\right)^{\frac{1}{\theta^k}} + (1-\delta)Q_{t+1} - Q_t(1+i_t+\varrho)\right)\right) \\
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\left(\alpha\psi^{\frac{1}{\theta^k}} P_{t+1}^m \frac{Y_{t+1}^m}{K_{t+1}}\left(\frac{K_{t+1}}{K_{t+1}^{P}}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\epsilon^l}} \left(\frac{K_{t+1}^P}{K_{j,t+1}^P}\right)^{\frac{1}{\epsilon^l}} + (1-\delta)Q_{t+1} - Q_t(1+i_{j,t}^l)\right)\right).
\end{aligned}$$

Using the fact that $\Lambda_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}$, the Euler equation, and denoting the intermediate variable $\Theta_t \equiv \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_{t+1}^m}{P_{t+1}}\frac{Y_{t+1}^m}{K_{t+1}}\right)$ we obtain:

$$\begin{aligned}
\frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) &= \Theta_t(1-\psi)^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K_{t+1}^{NP}}\right)^{\frac{1}{\theta^k}} \\
\frac{Q_t}{P_t}\frac{1+i_{j,t}^l}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) &= \Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K_{t+1}^{P}}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\epsilon^l}}\left(\frac{K_{t+1}^P}{K_{j,t+1}^P}\right)^{\frac{1}{\epsilon^l}}.
\end{aligned}$$

We manipulate the second equation (of which there are a total of n versions, one for each bank), raising it to the power of $1-\varepsilon^l$, multiplying by α_j^l, and then adding over all the n equations, to obtain:

$$\sum_{j=1}^{n} \alpha_j^l (z_{j,t}^P)^{1-\varepsilon^l} = \left(\Theta_t \psi^{\frac{1}{\theta^k}} \left(\frac{K_{t+1}}{K_{t+1}^P} \right)^{\frac{1}{\theta^k}} \right)^{1-\varepsilon^l} \sum_{j=1}^{n} (\alpha_j^l)^{\frac{1}{\varepsilon^l}} \left(\frac{K_{j,t+1}^P}{K_{t+1}^P} \right)^{\frac{\varepsilon^l-1}{\varepsilon^l}},$$

where,

$$z_{j,t}^P \equiv \frac{Q_t}{P_t} \frac{1+i_{j,t}^l}{1+i_t} - (1-\delta)\mathbb{E}_t \left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \right).$$

Defining:

$$z_t^P \equiv \left(\sum_{j=1}^{n} \alpha_j^l (z_{j,t}^P)^{1-\varepsilon^l} \right)^{\frac{1}{1-\varepsilon^l}},$$

we can rewrite the previous expression as:

$$z_t^P = \Theta_t \psi^{\frac{1}{\theta^k}} \left(\frac{K_{t+1}}{K_{t+1}^P} \right)^{\frac{1}{\theta^k}}.$$

We can also write demand for the individual pledgeable capital of bank j as:

$$K_{j,t+1}^P = \alpha_j^l \left(\frac{z_{j,t}^P}{z_t^P} \right)^{-\varepsilon^l} K_{t+1}^P.$$

Defining:

$$z_t^{NP} \equiv \frac{Q_t}{P_t} \frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t \left(\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \right),$$

we then have two aggregate conditions for K_t^{NP} and K_t^P that we can rewrite as:

$$\begin{aligned} \Theta_t (1-\psi)^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{NP})^{-\frac{1}{\theta^k}} &= z_t^{NP} \\ \Theta_t \psi^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{P})^{-\frac{1}{\theta^k}} &= z_t^{P}. \end{aligned}$$

Raise these to the power of $1-\theta^k$ and multiply by ψ in the top one and $(1-\psi)$ in the bottom one to obtain:

$$\begin{aligned} \left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}} \right)^{1-\theta^k} (1-\psi)^{\frac{1}{\theta^k}} (K_{t+1}^{NP})^{\frac{\theta^k-1}{\theta^k}} &= (1-\psi) \left(z_t^{NP} \right)^{1-\theta^k} \\ \left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}} \right)^{1-\theta^k} \psi^{\frac{1}{\theta^k}} (K_{t+1}^{P})^{\frac{\theta^k-1}{\theta^k}} &= \psi (z_t^P)^{1-\theta^k}. \end{aligned}$$

Adding both of the previous equations we get:

$$\left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}}\right)^{1-\theta^k} K_{t+1}^{\frac{\theta^k-1}{\theta^k}} = z_t^{1-\theta^k},$$

where:

$$z_t \equiv \left(\psi(z_t^P)^{1-\theta^k} + (1-\psi)(z_t^{NP})^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}}$$

The previous equation for determining aggregate K_t as a function of z_t can then be simplified to:

$$\Theta_t = z_t$$

With this, the F.O.C.'s for pledgeable and non-pledgeable capital can also be expressed as:

$$\begin{aligned}(1-\psi)^{\frac{1}{\theta^k}} z_t K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{NP})^{-\frac{1}{\theta^k}} &= z_t^{NP} \\ \psi^{\frac{1}{\theta^k}} z_t K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{P})^{-\frac{1}{\theta^k}} &= z_t^P,\end{aligned}$$

which can be rearranged to:

$$\begin{aligned}K_{t+1}^{NP} &= (1-\psi)\left(\frac{z_t^{NP}}{z_t}\right)^{-\theta^k} K_{t+1} \\ K_{t+1}^{P} &= \psi\left(\frac{z_t^{P}}{z_t}\right)^{-\theta^k} K_{t+1},\end{aligned}$$

the usual CES expressions.

Appendix B.4 The Capital Producer

We assume that even though non-pledgeable and pledgeable capital are financed differently by intermediate good firms (one by borrowing from banks and the other by borrowing in bonds), they are produced by the same representative capital producer that treats them indistinguishably, so they have the same price of capital Q_t and there is a single investment adjustment cost. It would be straightforward to augment the model to have two different prices of capital. Denote:

$$K_t^S = K_t^{NP} + \sum_{j=1}^{n} K_{j,t}^P.$$

The representative capital producer sells $Q_t K_{t+1}^S$ dollars worth of new capital, buys $(1-\delta)Q_t K_t^S$ dollars worth of used capital, and additionally pays I_t dollars in order to increase capital from K_t^S to K_{t+1}^S. New capital K_{t+1}^S is obtained from K_t^S and I_t as follows:

$$K_{t+1}^S = (1-\delta)K_t^S + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right)$$

With these elements, the nominal period-t profits of the capital good producer are:

$$\Pi_t^K = Q_t K_{t+1}^S - (1-\delta) Q_t K_t^S - P_t I_t,$$

which, using the previous equation for K_{t+1}^S, can be expressed as:

$$\Pi_t^K = Q_t I_t \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) - P_t I_t,$$

where the function $\Xi(\cdot)$ captures investment adjustment costs. The problem of the capital producer in period t is:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left[Q_{t+\tau} I_{t+\tau} \left(1 - \Xi\left(\frac{I_{t+\tau}}{I_{t+\tau-1}}\right)\right) - P_{t+\tau} I_{t+\tau} \right],$$

where $\Lambda_{t,t+\tau}$ is the household's nominal stochastic discount factor for discounting nominal flows from $t+\tau$ back to t. The F.O.C. is:

$$0 = Q_t \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) - Q_t \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} I_{t+1} \Xi'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}}{I_t^2} - P_t.$$

Which we rewrite as:

$$1 = \frac{Q_t}{P_t}\left[1 - \Xi\left(\frac{I_t}{I_{t-1}}\right) - \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \mathbb{E}_t \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

The $\Xi(\cdot)$ function satisfies $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) \geq 0$.

Appendix B.5 The Bank's Problem

Appendix B.5.1 Separation

Recall that the bank's problem is given by:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s+1} DIV_{j,t+s+1}.$$

As discussed in the main text, banks do not independently optimize their dividend distribution but instead take as given that a fraction $(1-\omega)$ of "profits" $X_{j,t+1}$ are distributed as dividends. The Bellman equation for the bank's problem is:

$$V(F_{j,t}, \Omega_t) = \max_{i_{j,t}^d, D_{j,t}, i_{j,t}^l, L_{j,t}} \mathbb{E}\left\{\beta\Lambda DIV_{j,t+1} + \beta\Lambda V(F_{j,t+1}, \Omega_{t+1})\right\},$$

where Ω_t denotes the aggregate state variables that influence the value of being a bank in period t. The maximization problem is subject to the deposit supply schedule, loan demand schedule, as well as:

$$\begin{aligned} DIV_{j,t+1} &= (1-\omega) X_{j,t+1} \\ F_{j,t+1} &= F_{j,t}(1-\varsigma)(1+\pi_{t+1}) + \omega X_{j,t+1} \end{aligned}$$

$$X_{j,t+1} = i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}$$

The F.O.C. w.r.t. i^d_j yields the following:

$$0 = \mathbb{E}\left\{\beta\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} + \beta\Lambda\frac{\partial V(F_{j,t+1},\Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}\right\}$$

Since $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}$ is deterministic (known in period t), it can exit the expectation operator and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} = 0$, which is equivalent to maximizing $(i_t - \mu^d - i^d_{j,t})D_{j,t}$ w.r.t. $i^d_{j,t}$ subject to the deposit supply schedule $D_{j,t}(i^d_{j,t})$.

Similarly, the F.O.C. w.r.t. i^l_j yields the following:

$$0 = \mathbb{E}\left\{\beta\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} + \beta\Lambda\frac{\partial V(F_{j,t+1},\Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}\right\}$$

Since $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}$ is also deterministic, it can exit the expectation operator as well, and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} = 0$, which is equivalent to maximizing

$$(i^l_{j,t} - \mu^l - i_t)L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}$$

w.r.t. $i^l_{j,t}$ subject to the loan demand schedule $L_{j,t}(i^l_{j,t})$.

The reason the deposit and loan problems can be neatly separated, is because banks can always use their reserves $H_{j,t}$ to borrow or lend any excess funds to the central bank, so they always optimize their loan and deposit franchises separately. If there was a constraint like $H_{j,t} \geq 0$, then there are certain circumstances under which the deposit and loan franchises interact and the maximization problem cannot be neatly separated into the two subproblems.

Appendix B.5.2 Deposits

A bank that maximizes $(i_t - i^d_{j,t} - \mu^d)D_{j,t}$ has the following F.O.C.:

$$0 = -D_{j,t} + ((1 + i_t - \mu^d) - (1 + i^d_{j,t}))\frac{\partial D_{j,t}}{\partial d_{j,t}}\frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})}$$

Denote with $\epsilon^d_{j,t}$ the endogenous elasticity of $d_{j,t}$ w.r.t. $(1 + i^d_{j,t})$:

$$\epsilon^d_{j,t} \equiv \frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})}\frac{1 + i^d_{j,t}}{d_{j,t}}$$

Then we can write the previous F.O.C. as:

$$\begin{aligned}
1 &= \epsilon^d_{j,t}((1+i_t-\mu^d)-(1+i^d_{j,t}))\frac{1}{1+i^d_{j,t}} \\
1+i^d_{j,t} &= \epsilon^d_{j,t}(1+i_t-\mu^d)-\epsilon^d_{j,t}(1+i^d_{j,t}) \\
1+i^d_{j,t} &= \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t}+1}(1+i_t-\mu^d) \qquad \text{(B.17)}
\end{aligned}$$

Now lets obtain $\epsilon^d_{j,t}$. This is not trivial because both $1+i^d_t$ and d_t depend on $1+i^d_{j,t}$. Lets compute the elasticity of the aggregate deposit rate w.r.t. one individual deposit rate:

$$\begin{aligned}
1+i^d_t &= \left(\sum_{j=1}^{n}\alpha_j(1+i^d_{j,t})^{\varepsilon^d+1}\right)^{\frac{1}{\varepsilon^d+1}} \\
\frac{\partial(1+i^d_t)}{\partial(1+i^d_{j,t})} &= \frac{1}{\varepsilon^d+1}\left(\sum_{j=1}^{n}\alpha_j(1+i^d_{j,t})^{\varepsilon^d+1}\right)^{-\frac{\varepsilon^d}{\varepsilon^d+1}}\alpha_j(\varepsilon^d+1)(1+i^d_{j,t})^{\varepsilon^d} \\
&= (1+i^d_t)^{-\varepsilon^d}\alpha_j(1+i^d_{j,t})^{\varepsilon^d} \\
&= \alpha_j\left(\frac{1+i^d_{j,t}}{1+i^d_t}\right)^{\varepsilon^d} = \frac{d_{j,t}}{d_t} \\
\frac{\partial(1+i^d_t)}{\partial(1+i^d_{j,t})}\frac{1+i^d_{j,t}}{1+i^d_t} &= \alpha_j\left(\frac{1+i^d_{j,t}}{1+i^d_t}\right)^{\varepsilon^d+1} = \frac{(1+i^d_{j,t})d_{j,t}}{(1+i^d_t)d_t} \equiv \omega^{d_j}_{d,t} \qquad \text{(B.18)}
\end{aligned}$$

where $\omega^{d_j}_{d,t}$ is the share of gross interest spending on deposits of bank j at time t. Now lets compute the elasticity of d_t w.r.t. $(1+i^d_t)$:

$$\begin{aligned}
d_t &= \gamma_d\left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta}\mathcal{L}_t \\
\ln d_t &= \ln\gamma_d+\theta\ln(1+i^d_t)-\theta\ln(1+i^{\mathcal{L}}_t)+\ln\mathcal{L}_t \\
\frac{\partial\ln d_t}{\partial\ln(1+i^d_t)} &= \theta-\theta\frac{\partial\ln(1+i^{\mathcal{L}}_t)}{\partial\ln(1+i^d_t)}+\frac{\partial\ln\mathcal{L}_t}{\partial\ln(1+i^{\mathcal{L}}_t)}\frac{\partial\ln(1+i^{\mathcal{L}}_t)}{\partial\ln(1+i^d_t)} \\
&= \theta\left(1-\frac{\partial\ln(1+i^{\mathcal{L}}_t)}{\partial\ln(1+i^d_t)}\right)+\frac{\partial\ln\mathcal{L}_t}{\partial\ln(1+i^{\mathcal{L}}_t)}\frac{\partial\ln(1+i^{\mathcal{L}}_t)}{\partial\ln(1+i^d_t)}
\end{aligned}$$

The elasticity of $1+i^{\mathcal{L}}_t$ w.r.t. $1+i^d_t$ is:

$$\begin{aligned}
1+i^{\mathcal{L}}_t &= \left(\gamma(1+i^m_t)^{\theta+1}+\delta(1+i^d_t)^{\theta+1}+\eta(1+i^{cbdc_t})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{\partial(1+i^{\mathcal{L}}_t)}{\partial(1+i^d_t)} &= \left(1+i^{\mathcal{L}}_t\right)^{-\theta}\gamma_d(1+i^d_t)^{\theta} = \frac{d_t}{\mathcal{L}_t} \\
\frac{\partial(1+i^{\mathcal{L}}_t)}{\partial(1+i^d_t)}\frac{1+i^d_t}{1+i^{\mathcal{L}}_t} &= \gamma_d\left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta+1} = \frac{(1+i^d_t)d_t}{(1+i^{\mathcal{L}}_t)\mathcal{L}_t} \equiv \omega^d_{\mathcal{L},t}
\end{aligned}$$

With all of these things we can write:

$$
\begin{aligned}
\ln d_{j,t} &= \ln \alpha_j + \varepsilon^d \ln(1+i^d_{j,t}) - \varepsilon^d \ln(1+i^d_t) + \ln d_t \\
\frac{\partial \ln d_{j,t}}{\partial \ln(1+i^d_{j,t})} &= \varepsilon^d - \varepsilon^d \frac{\partial \ln(1+i^d_t)}{\partial \ln(1+i^d_{j,t})} + \frac{\partial \ln d_t}{\partial \ln(1+i^d_t)} \frac{\partial \ln(1+i^d_t)}{\partial \ln(1+i^d_{j,t})} \\
\epsilon^d_{j,t} &= (1-\omega^{d_j}_{d,t})\varepsilon^d + \omega^{d_j}_{d,t}\left[(1-\omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t}\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)}\right]
\end{aligned}
$$

If all banks are identical they all pay the same deposit rate ($i^d_{j,t} = i^d_t$), all face the same elasticity ϵ^d_t, and they all obtain one n-th of total deposits (i.e. $\omega^{d_j}_{d,t} = 1/n$), and the expression becomes:

$$
\begin{aligned}
\epsilon^d_t &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t}\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)}\right] \\
&= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n} + \frac{1}{n}\omega^d_{\mathcal{L},t}\left(\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)} - \theta\right)
\end{aligned} \tag{B.19}
$$

Appendix B.5.3 Loans

The F.O.C. of the loan sub-problem (w.r.t. $i^l_{j,t}$) is:

$$
\begin{aligned}
0 &= L_{j,t} + \left\{[1+i^l_{j,t}] - \left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right]\right\}\frac{\partial L_{j,t}}{\partial l_{j,t}}\frac{\partial l_{j,t}}{\partial(1+i^l_{j,t})} \\
1 + i^l_{j,t} &= \frac{\epsilon^l_{j,t}}{\epsilon^l_{j,t}-1}\left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right]
\end{aligned} \tag{B.20}
$$

where $\epsilon^l_{j,t}$ denotes the (negative of the) elasticity of $l_{j,t}$ w.r.t. $(1+i^l_{j,t})$:

$$
\epsilon^l_{j,t} \equiv -\frac{\partial l_{j,t}}{\partial(1+i^l_{j,t})}\frac{1+i^l_{j,t}}{l_{j,t}}
$$

Now lets obtain an expression for $\epsilon^l_{j,t}$ as a function of the other variables in the model. We know the following things:

$$
\begin{aligned}
z_t &= \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P^m_{t+1}}{P_{t+1}}A_{t+1}N^{1-\alpha}_{t+1}\right)K^{\alpha-1}_{t+1} \\
z_t &= \left(\psi(z^P_t)^{1-\theta^k} + (1-\psi)(z^{NP}_t)^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}} \\
z^P_t &= \left(\sum_{j=1}^{n}\alpha^l_j(z^P_{j,t})^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}} \\
z^P_{j,t} &\equiv \frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right)
\end{aligned}
$$

$$
\begin{aligned}
l_{j,t} &= \alpha_j^l \left(\frac{z_{j,t}^P}{z_t^P}\right)^{-\varepsilon^l} l_t \\
l_t &= \frac{Q_t}{P_t}\psi\left(\frac{z_t^P}{z_t}\right)^{-\theta^k} K_{t+1}
\end{aligned}
$$

Lets first compute the elasticity of z_t^P w.r.t. one individual $z_{j,t}^P$:

$$
\begin{aligned}
\frac{\partial z_t^P}{\partial z_{j,t}^P} &= \frac{1}{1-\varepsilon^l}\left(\sum_{j=1}^{n}\alpha_j^l (z_{j,t}^P)^{1-\varepsilon^l}\right)^{\frac{1}{1-\varepsilon^l}-1} \alpha_j^l(1-\varepsilon^l)(z_{j,t}^P)^{-\varepsilon^l} \\
&= \alpha_j^l\left(\frac{z_{j,t}^P}{z_t^P}\right)^{-\varepsilon^l} = \frac{l_{j,t}}{l_t} \\
\frac{\partial z_t^P}{\partial z_{j,t}^P}\frac{z_{j,t}^P}{z_t^P} &= \alpha_j^l\left(\frac{z_{j,t}^P}{z_t^P}\right)^{1-\varepsilon^l} = \frac{z_{j,t}^P l_{j,t}}{z_t^P l_t} \equiv \omega_{l,t}^{l_j}
\end{aligned}
$$

Now, we compute the elasticity of l_t w.r.t. $(1+i_t^l)$. For simplicity, we assume that all banks take the real price of capital Q_t/P_t as given, as well as all aggregate variables that are not explicitly related to capital. Then, we have:

$$
\begin{aligned}
\frac{\partial \ln K_t}{\partial \ln z_t} &= \frac{1}{\alpha-1} \\
\frac{\partial \ln l_t}{\partial \ln z_t^P} &= -\theta^k + \theta^k\frac{\partial \ln z_t}{\partial \ln z_t^P} + \frac{\partial \ln K_t}{\partial \ln z_t}\frac{\partial \ln z_t}{\partial \ln z_t^P}
\end{aligned}
$$

The elasticity of z_t w.r.t. z_t^P is:

$$
\begin{aligned}
\frac{\partial z_t}{\partial z_t^P} &= z_t^{\theta^k}\psi(z_t^P)^{-\theta^k} = \frac{l_t}{K_t} \\
\frac{\partial \ln z_t}{\partial \ln z_t^P} &= \frac{\partial z_t}{\partial z_t^P}\frac{z_t^P}{z_t} = \psi\left(\frac{z_t^P}{z_t}\right)^{1-\theta^k} = \frac{l_t z_t^P}{K_t z_t} \equiv \omega_{K,t}^{K_{NP}} \qquad \text{(B.21)}
\end{aligned}
$$

We also need the elasticity of $z_{j,t}^P$ w.r.t. $(1+i_{j,t}^l)$:

$$
\begin{aligned}
\frac{\partial z_{j,t}^P}{\partial(1+i_{j,t}^l)} &= \frac{Q_t}{P_t}\frac{1}{1+i_t} \\
\frac{\partial \ln z_{j,t}^P}{\partial \ln(1+i_{j,t}^l)} &= \frac{\partial z_{j,t}^P}{\partial(1+i_{j,t}^l)}\frac{(1+i_{j,t}^l)}{z_{j,t}^P} = \frac{Q_t}{P_t}\frac{1+i_{j,t}^l}{1+i_t}\frac{1}{z_{j,t}^P}
\end{aligned}
$$

With all of these things we can write:

$$
\begin{aligned}
\ln l_{j,t} &= \ln\alpha_j^l - \varepsilon^l \ln z_{j,t}^P + \varepsilon^l \ln z_t^P + \ln l_t \\
\frac{\partial \ln l_{j,t}}{\partial \ln(1+i_{j,t}^l)} &= \left[-\varepsilon^l + \varepsilon^l\frac{\partial z_t^P}{\partial z_{j,t}^P} + \frac{\partial \ln l_t}{\partial \ln z_{j,t}^P}\right]\frac{\partial \ln z_{j,t}^P}{\partial \ln(1+i_{j,t}^l)}
\end{aligned}
$$

$$
\begin{aligned}
-\epsilon^l_{j,t} &= \left[-\varepsilon^l(1-\omega^{l_j}_{l,t}) + \frac{\partial \ln l_t}{\partial \ln z^P_t}\frac{\partial \ln z^P_t}{\partial \ln z^P_{j,t}}\right]\frac{\partial \ln z^P_{j,t}}{\partial \ln(1+i^l_{j,t})} \\
\epsilon^l_{j,t} &= \left[\varepsilon^l(1-\omega^{l_j}_{l,t}) + \omega^{l_j}_{l,t}\left(\theta^k(1-\omega^{K_{NP}}_{K,t}) + \frac{\omega^{K_{NP}}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t}\frac{1}{z^P_{j,t}}
\end{aligned}
$$

If all banks are identical, they all charge the same loan rate ($i^l_{j,t} = i^l_t$), face the same elasticity ϵ^l_t, and obtain one n-th of total loans (i.e. $\omega^{l_j}_{l,t} = 1/n$), and the expression becomes:

$$
\epsilon^l_t = \left[\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left(\theta^k(1-\omega^{K_{NP}}_{K,t}) + \frac{\omega^{K_{NP}}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}\frac{1}{z^P_t} \tag{B.22}
$$

Appendix B.6 The Retailer's Problem

Recall that the retailer's problem is:

$$
\max_{P^*_t} \ \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)} \frac{P_t}{P_{t+r}} \left[P^*_t - P^m_{t+r}\right] Y_{t+r|t}.
$$

Notice that $Y_{t+r|t}$, the amount sold in period $t+r$ by a firm that last reset its price in period t, is defined as:

$$
Y_{t+r|t} \equiv \left(\frac{P^*_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r}.
$$

Hence, its derivative with respect to the optimal reset price is given by:

$$
\frac{\partial Y_{t+r|t}}{\partial P^*_t} = -\varphi \frac{Y_{t+r|t}}{P^*_t}.
$$

The F.O.C. w.r.t. to the optimal reset price is then given by:

$$
\begin{aligned}
0 &= \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)} \frac{P_t}{P_{t+r}} \left[Y_{t+r|t} - \varphi(P^*_t - P^m_{t+r})\frac{Y_{t+r|t}}{P^*_t}\right] \\
&= \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r}\left[P^*_t(1-\varphi) + \varphi P^m_{t+r}\right].
\end{aligned}
$$

Define

$$
\begin{aligned}
\Gamma^1_t &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P^m_{t+r} \\
\Gamma^2_t &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}}\right)^{-\varphi} Y_{t+r} P^*_t.
\end{aligned}
$$

With this notation we can write the F.O.C. as:

$$
\varphi \Gamma^1_t = (\varphi - 1)\Gamma^2_t. \tag{B.23}
$$

We can also characterize Γ_t^1 recursively as:

$$\begin{aligned}\Gamma_t^1 &= \frac{u'(C_t)}{P_t}Y_tP_t^m + \mathbb{E}_t\sum_{r=1}^{\infty}\gamma^r\beta^r\frac{u'(C_{t+r})}{P_{t+r}}\left(\frac{P_t}{P_{t+r}}\right)^{-\varphi}Y_{t+r}P_{t+r}^m \\ &= u'(C_t)\frac{P_t^m}{P_t}Y_t + \gamma\beta\mathbb{E}_t\left(\frac{P_t}{P_{t+1}}\right)^{-\varphi}\Gamma_{t+1}^1. \end{aligned} \tag{B.24}$$

Similarly, for Γ_t^2 we have:

$$\begin{aligned}\Gamma_t^2 &= \frac{u'(C_t)}{P_t}Y_tP_t^* + \mathbb{E}_t\sum_{r=1}^{\infty}\gamma^r\beta^r\frac{u'(C_{t+r})}{P_{t+r}}\left(\frac{P_t}{P_{t+r}}\right)^{-\varphi}Y_{t+r}P_t^* \\ &= u'(C_t)\frac{P_t^*}{P_t}Y_t + \gamma\beta\mathbb{E}_t\frac{P_t^*}{P_{t+1}^*}\left(\frac{P_t}{P_{t+1}}\right)^{-\varphi}\Gamma_{t+1}^2. \end{aligned} \tag{B.25}$$

From the definition of the price index we can easily derive an equation for its evolution in terms of the real optimal reset price:

$$1 = (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\varphi} + \gamma\left(\frac{P_{t-1}}{P_t}\right)^{1-\varphi}. \tag{B.26}$$

Additionally, the aggregate demand for intermediate inputs is the integral over all retail firms:

$$Y_t^m = \int_0^1 Y_t(s)ds = \int_0^1\left(\frac{P_t(s)}{P_t}\right)^{-\varphi}Y_tds = Y_tv_t^p, \tag{B.27}$$

where v_t^p is an index of price dispersion that evolves as follows:

$$\begin{aligned} v_t^p &= \int_0^1\left(\frac{P_t(s)}{P_t}\right)^{-\varphi}ds \\ &= \gamma\left(\frac{P_{t-1}}{P_t}\right)^{-\varphi}v_{t-1}^p + (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\varphi}. \end{aligned} \tag{B.28}$$

Equations (B.23)-(B.28) are the ones given in the text as describing the optimal behavior of retail firms.

Appendix B.7 Resource Constraint

In this appendix, we derive the aggregate resource constraint of the model economy. Notice that the aggregate nominal profits of retail firms are:

$$\Pi_t^R = P_tY_t - P_t^mY_t^m$$

This is necessarily the case because on aggregate they sell all output in the economy at price P_t, so they make revenue of P_tY_t, and they buy all intermediate inputs in the economy at price P_t^m, so they have costs of $P_t^mY_t^m$. The dividends distributed by the bank in period t are:

$$\Pi_t^B = (1-\omega)X_t$$

Intermediate good firms (in the case with symmetric commercial banks) have nominal profits of:

$$\Pi_t^m = P_t^m Y_t^m - W_t N_t + Q_t(1-\delta)K_t^P + Q_t(1-\delta)K_t^{NP} - Q_{t-1}(1+i_{t-1}^l)K_t^P - Q_{t-1}(1+i_{t-1}+\varrho)K_t^{NP}$$

Capital producers (in the case with symmetric commercial banks) have nominal profits of:

$$\Pi_t^K = Q_t\left[K_{t+1}^{NP} + K_{t+1}^P - (1-\delta)\left(K_t^{NP} + K_t^P\right)\right] - P_t I_t$$

We also need the following equations:

$$\begin{aligned}
K_{t+1}^{NP} + K_{t+1}^P &= (1-\delta)\left[K_t^{NP} + K_t^P\right] + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) \\
Tr_t &= M_t - M_{t-1} + H_t - (1+i_{t-1})H_{t-1} + CBDC_t - (1+i_{t-1}^{cbdc})CBDC_{t-1} - P_t G_t \\
B_t &= Q_t K_{t+1}^{NP} \\
L_t &= Q_t K_{t+1}^P \\
D_t + F_t &= H_t + L_t \\
F_t &= (1+i_{t-1})F_{t-1} - (1-\omega)X_t - \varsigma F_{t-1} + (i_{t-1}^l - \mu^l - i_{t-1})L_{t-1} \\
&+ (i_{t-1} - \mu^d - i_{t-1}^d)D_{t-1} - \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)F_{t-1}
\end{aligned}$$

Start with the budget constraint of the households:

$$\begin{aligned}
P_t C_t &= W_t N_t - P_t\Phi(\mathcal{L}_t) - B_t + (1+i_{t-1})B_{t-1} + (1+i_{t-1}^m)M_{t-1} \\
&+ (1+i_{t-1}^d)D_{t-1} + (1+i_{t-1}^{cbdc})CBDC_{t-1} + Tr_t + \Pi_t^R + \Pi_t^B + \Pi_t^m + \Pi_t^K \\
&= P_t Y_t - P_t G_t - P_t\Phi(\mathcal{L}_t) - Q_t K_{t+1}^{NP} + (1+i_{t-1})Q_{t-1}K_t^{NP} + (1+i_{t-1}^d)D_{t-1} + M_t \\
&+ H_t - (1+i_{t-1})H_{t-1} + CBDC_t + (1-\omega)X_t + Q_t(1-\delta)K_t^P + Q_t(1-\delta)K_t^{NP} \\
&- Q_{t-1}(1+i_{t-1}^l)K_t^P - Q_{t-1}(1+i_{t-1}+\varrho)K_t^{NP} + \Pi_t^K \\
&= P_t Y_t - P_t G_t - Q_t(K_{t+1}^{NP} - (1-\delta)K_t^{NP}) - P_t\Phi(\mathcal{L}_t) + M_t + CBDC_t - \varrho Q_{t-1}K_t^{NP} \\
&+ (1+i_{t-1}^d)D_{t-1} + D_t + F_t - L_t - (1+i_{t-1})(D_{t-1} + F_{t-1} - L_{t-1}) + (1-\omega)X_t \\
&+ Q_t(1-\delta)K_t^P - Q_{t-1}(1+i_{t-1}^l)K_t^P + \Pi_t^K \\
&= P_t Y_t - P_t G_t - P_t I_t - P_t\Phi(\mathcal{L}_t) + M_t + CBDC_t + D_t - \varrho Q_{t-1}K_t^{NP} \\
&- \varsigma F_{t-1} - \mu^l L_{t-1} - \mu^d D_{t-1} - \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)F_{t-1}
\end{aligned}$$

Which finally implies:

$$\begin{aligned}
Y_t &= C_t + G_t + I_t + \Phi(\mathcal{L}_t) - \frac{M_t + CBDC_t + D_t}{P_t} + \varrho\frac{Q_{t-1}}{P_t}K_t^{NP} \\
&+ \varsigma\frac{F_{t-1}}{P_t} + \mu^l\frac{L_{t-1}}{P_t} + \mu^d\frac{D_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t}
\end{aligned}$$

Which is equivalent to equations (3.24)-(3.25) in the main text.

Appendix B.8 Equilibrium Equations

We assume the following functional forms for $v(\cdot)$, $u(\cdot)$, $\Psi(\cdot)$, $\Phi(\cdot)$ and $\Xi(\cdot)$:

$$\begin{aligned}
v(x) &= \chi \frac{x^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \\
u(x) &= \frac{x^{1-\sigma}-1}{1-\sigma} \\
\Psi(x) &= \kappa\nu x\left(\ln x - \ln\nu - 1\right) + \kappa\nu^2 \\
\Phi(x) &= ax^b - q \\
\Xi(x) &= \frac{\kappa_I}{2}\left(x-1\right)^2.
\end{aligned}$$

The derivatives then are:

$$\begin{aligned}
v'(x) &= \chi x^{\frac{1}{\eta}} \\
u'(x) &= x^{-\sigma} \\
\Psi'(x) &= \kappa\nu\left(\ln x - \ln\nu\right) \\
\Phi'(x) &= abx^{b-1} \\
\Xi(x) &= \frac{\kappa_I}{2}\left(x-1\right)^2 \\
\Xi'(x) &= \kappa_I\left(x-1\right) \\
\Xi''(x) &= \kappa_I.
\end{aligned}$$

The function Ψ is not exactly quadratic, but it has several useful properties described in Ulate (2021). Furthermore, its second order approximation around the steady state is:

$$\Psi(x) \approx^2 \frac{\kappa}{2}\left(x-\nu\right)^2,$$

which is the quadratic form that has been traditionally used in the literature.

We reiterate the equilibrium equations here according to their sector. Households (7 equations):

$$\begin{aligned}
\chi N_t^{\frac{1}{\eta}} &= C_t^{-\sigma}\frac{W_t}{P_t} \\
1 &= \beta(1+i_t)\mathbb{E}_t\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\right) \\
\frac{1+i_t^{\mathcal{L}}}{1+i_t} &= ab\mathcal{L}_t^{b-1} \\
(1+i_t^{\mathcal{L}})^{\theta+1} &= \gamma_m + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1} \\
m_t &= \gamma_m\left(\frac{1}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t \\
d_t &= \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t
\end{aligned}$$

$$cbdc_t = \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t.$$

Intermediate good firms (8 equations):

$$\begin{aligned}
Y_t^m &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= (1-\alpha)\frac{P_t^m}{P_t}\frac{Y_t^m}{N_t} \\
z_t &= \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_{t+1}^m}{P_{t+1}}\frac{Y_{t+1}^m}{K_{t+1}}\right) \\
z_t &= \left(\psi(z_t^P)^{1-\theta^k} + (1-\psi)(z_t^{NP})^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}} \\
z_t^P &= \frac{Q_t}{P_t}\frac{1+i_t^l}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) \\
z_t^{NP} &= \frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) \\
K_{t+1}^P &= \psi\left(\frac{z_t^P}{z_t}\right)^{-\theta^k} K_{t+1} \\
K_{t+1}^{NP} &= (1-\psi)\left(\frac{z_t^{NP}}{z_t}\right)^{-\theta^k} K_{t+1}.
\end{aligned}$$

Capital producers (2 equations):

$$\begin{aligned}
K_{t+1}^{NP} + K_{t+1}^P &= (1-\delta)\left[K_t^{NP} + K_t^P\right] + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right) \\
1 &= \frac{Q_t}{P_t}\left[1 - \Xi\left(\frac{I_t}{I_{t-1}}\right) - \Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\
&+ \mathbb{E}_t\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2.
\end{aligned}$$

Banks (10 equations):

$$\begin{aligned}
\omega_{\mathcal{L},t}^d &= \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta+1} \\
\epsilon_t^d &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega_{\mathcal{L},t}^d)\theta + \frac{\omega_{\mathcal{L},t}^d}{b-1}\right] \\
1+i_t^d &= \frac{\epsilon_t^d}{\epsilon_t^d+1}(1+i_t-\mu^d) \\
\omega_{K,t}^{K_{NP}} &= \psi\left(\frac{z_t^P}{z_t}\right)^{1-\theta^k} \\
\epsilon_t^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[(1-\omega_{K,t}^{K_{NP}})\theta^k + \frac{\omega_{K,t}^{K_{NP}}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1+i_t^l}{1+i_t}\frac{1}{z_t^P} \\
1+i_t^l &= \frac{\epsilon_t^l}{\epsilon_t^l-1}\left[1+i_t+\mu^l+\kappa\nu\left(\ln\left(\frac{L_t}{F_t}\right)-\ln(\nu)\right)\right]
\end{aligned}$$

$$
\begin{aligned}
\frac{L_t}{P_t} &= \frac{Q_t}{P_t} K_{t+1}^P \\
\frac{X_t}{P_t} \frac{P_t}{P_{t-1}} &= i_{t-1} \frac{F_{t-1}}{P_{t-1}} + (i_{t-1}^l - \mu^l - i_{t-1}) \frac{L_{t-1}}{P_{t-1}} + (i_{t-1} - \mu^d - i_{t-1}^d) \frac{D_{t-1}}{P_{t-1}} \\
&- \Psi \left(\frac{L_{t-1}}{F_{t-1}} \right) \frac{F_{t-1}}{P_{t-1}} - \frac{F_{t-1}}{P_{t-1}} (1 - \varsigma) \pi_t \\
\frac{F_t}{P_t} &= \frac{F_{t-1}}{P_{t-1}} (1 - \varsigma) + \omega \frac{X_t}{P_t} \\
\frac{H_t}{P_t} &= \frac{F_t}{P_t} + \frac{D_t}{P_t} - \frac{L_t}{P_t}.
\end{aligned}
$$

Retail firms (6 equations):

$$
\begin{aligned}
1 &= (1 - \gamma) \left(\frac{P_t^*}{P_t} \right)^{1-\varphi} + \gamma \left(\frac{P_{t-1}}{P_t} \right)^{1-\varphi} \\
\varphi \Gamma_t^1 &= (\varphi - 1) \Gamma_t^2 \\
\Gamma_t^1 &= C_t^{-\sigma} \frac{P_t^m}{P_t} Y_t + \gamma \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \right)^{-\varphi} \Gamma_{t+1}^1 \\
\Gamma_t^2 &= C_t^{-\sigma} \frac{P_t^*}{P_t} Y_t + \gamma \beta \mathbb{E}_t \frac{P_t^* / P_t}{P_{t+1}^* / P_{t+1}} \left(\frac{P_t}{P_{t+1}} \right)^{1-\varphi} \Gamma_{t+1}^2 \\
Y_t^m &= Y_t v_t^p \\
v_t^p &= \gamma \left(\frac{P_{t-1}}{P_t} \right)^{-\varphi} v_{t-1}^p + (1 - \gamma) \left(\frac{P_t^*}{P_t} \right)^{-\varphi}.
\end{aligned}
$$

Others (5 equations):

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t + \Gamma_t \\
\Gamma_t &= \mu^l \frac{L_{t-1}}{P_t} + \mu^d \frac{D_{t-1}}{P_t} + \varsigma \frac{F_{t-1}}{P_t} + \Psi \left(\frac{L_{t-1}}{F_{t-1}} \right) \frac{F_{t-1}}{P_t} + \varrho \frac{Q_{t-1}}{P_t} K_t^{NP} \\
&+ \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t} \\
i_t &= (1 - \rho_i) \left(\bar{\imath} + \psi_\pi (\pi_t - \overline{\pi}) \right) + \rho_i i_{t-1} + \epsilon_t^i \\
A_t &= A_{t-1}^{\rho_a} \exp(\epsilon_t^a) \\
G_t &= g Y_t.
\end{aligned}
$$

Plus a value for the interest rate on CBDC (this would be -100% in the pre-CBDC scenario, but something like $i_t^{cbdc} = 0$ or $i_t^{cbdc} = i - 1$ in the post-CBDC scenario).

Appendix B.9 Steady State

In steady state, we have $Q/P = 1$, $P^* = P$, $v^p = 1$, $Y^M = Y$, $P^m/P = \frac{\varphi - 1}{\varphi}$, and $i = \bar{\imath}$, we can also get rid of the investment equations. This way we can drop all the 6 equations for the retailers, 2 for the intermediate good firms, 2 for the capital producers, and 6 of the "others", to simplify the steady state system from 39 equations to 24:

$$
\chi N^{\frac{1}{\eta}} = C^{-\sigma} \frac{W}{P}
$$

$$
\begin{aligned}
\frac{1}{\beta} - 1 &= i \\
\frac{1+i^{\mathcal{L}}}{1+i} &= ab\mathcal{L}^{b-1} \\
1+i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d (1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
m &= \gamma_m \left(\frac{1}{1+i^{\mathcal{L}}}\right)^{\theta} \mathcal{L} \\
d &= \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta} \mathcal{L} \\
cbdc &= \gamma_{cbdc} \left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta} \mathcal{L} \\
Y &= AK^{\alpha} N^{1-\alpha} \\
\frac{W}{P} &= (1-\alpha)\frac{\varphi - 1}{\varphi}\frac{Y}{N} \\
z &= \alpha\beta\frac{\varphi-1}{\varphi}\frac{Y}{K} \\
z(1+i) &= \left[\psi\left(i^l + \delta\right)^{1-\theta^k} + (1-\psi)\left(i + \varrho + \delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}} \\
K^P &= \psi\left(\frac{i^l+\delta}{z(1+i)}\right)^{-\theta^k} K \\
K^{NP} &= (1-\psi)\left(\frac{i+\varrho+\delta}{z(1+i)}\right)^{-\theta^k} K \\
\omega^d_{\mathcal{L}} &= \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega^d_{\mathcal{L}})\theta + \omega^d_{\mathcal{L}}\frac{\partial \ln \mathcal{L}}{\partial \ln(1+i^{\mathcal{L}})}\right] \\
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i-\mu^d) \\
\omega_K^{K_{NP}} &= \psi\left(\frac{i^l+\delta}{z(1+i)}\right)^{1-\theta^k} \\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[\theta^k(1-\omega_K^{K_{NP}}) + \frac{1}{1-\alpha}\omega_K^{K_{NP}}\right]\right\}\frac{1+i^l}{i^l+\delta} \\
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left[1+i+\mu^l+\kappa\nu\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\right] \\
\frac{X}{P} &= i\frac{F}{P} + (i^l - \mu^l - i)\frac{L}{P} + (i - \mu^d - i^d)\frac{D}{P} - \Psi\left(\frac{L}{F}\right)\frac{F}{P} \\
\varsigma\frac{F}{P} &= \omega\frac{X}{P} \\
\frac{L}{P} + \frac{H}{P} &= \frac{F}{P} + \frac{D}{P} \\
Y &= C + \delta(K^P + K^{NP}) + gY + \mu^l\frac{L}{P} + \mu^d\frac{D}{P} + \varsigma\frac{F}{P} + \Psi\left(\frac{L}{F}\right)\frac{F}{P} + \varrho K^{NP}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \Phi(\mathcal{L}) - \frac{M+D+CBDC}{P} \\
\frac{L}{P} &= K^P
\end{aligned}
$$

We can further simplify these. The one for the bond rate disappears (just defines the steady state bond rate). The definitions of W/P, X/P, and H/P can be eliminated, as well as $L/P = K^P$. With these changes our equilibrium system becomes:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\frac{Y}{N} \\
\beta(1+i^{\mathcal{L}}) &= ab\mathcal{L}^{b-1} \\
1+i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{M}{P} &= \gamma_m\left(\frac{1}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
\frac{D}{P} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
\frac{CBDC}{P} &= \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}\mathcal{L} \\
Y &= K^\alpha N^{1-\alpha} \\
z &= \alpha\beta\frac{\varphi-1}{\varphi}\frac{Y}{K} \\
\frac{z}{\beta} &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k} + (1-\psi)\left(1/\beta-1+\varrho+\delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}} \\
K^P &= \psi\left(\frac{i^l+\delta}{z/\beta}\right)^{-\theta^k} K \\
K^{NP} &= (1-\psi)\left(\frac{1/\beta-1+\varrho+\delta}{z/\beta}\right)^{-\theta^k} K \\
\omega^d_{\mathcal{L}} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{1}{n}\left[(1-\omega^d_{\mathcal{L}})\theta + \omega^d_{\mathcal{L}}\frac{\partial\ln\mathcal{L}}{\partial\ln(1+i^{\mathcal{L}})}\right] \\
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1/\beta-\mu^d) \\
\omega^{K_{NP}}_K &= \psi\left(\frac{i^l+\delta}{z/\beta}\right)^{1-\theta^k} \\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{1}{n}\left[\theta^k(1-\omega^{K_{NP}}_K) + \frac{\omega^{K_{NP}}_K}{1-\alpha}\right]\right\}\frac{1+i^l}{i^l+\delta} \\
1+i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left[1/\beta + \mu^l + \kappa\nu\left(\ln\left(\frac{K^P}{F/P}\right) - \ln(\nu)\right)\right] \\
\frac{\varsigma}{\omega}\frac{F}{P} &= \left(\frac{1}{\beta}-1\right)\left(\frac{F}{P}+\frac{D}{P}-K^P\right) + (i^l-\mu^l)K^P - (\mu^d+i^d)\frac{D}{P} - \Psi\left(\frac{K^P}{F/P}\right)\frac{F}{P}
\end{aligned}
$$

$$
\begin{aligned}
Y &= C + \delta K^P + \delta K^{NP} + gY + \mu^l K^P + \mu^d \frac{D}{P} + \varsigma \frac{F}{P} + \Psi\left(\frac{L}{F}\right)\frac{F}{P} + \varrho K^{NP} \\
&+ a\mathcal{L}^b - q - \frac{M + D + CBDC}{P}
\end{aligned}
$$

This is a system of 19 equations in 19 unknowns. The unknowns are $N, C, Y, i^{\mathcal{L}}, \mathcal{L}, i^d, M/P, D/P, CBDC/P$, K, K^P, K^{NP}, z, i^l, $\omega^d_{\mathcal{L}}$, ϵ^d, $\omega^{K_{NP}}_K$, ϵ^l, F/P. Getting rid of the equations for z, Y, $\omega^d_{\mathcal{L}}$, $\omega^{K_{NP}}_K$, $\mathcal{L}$, M/P, and $CBDC/P$, we can write:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-\alpha)\frac{\varphi-1}{\varphi}\left(\frac{K}{N}\right)^{\alpha} \\
1 + i^{\mathcal{L}} &= \left(\gamma_m + \gamma_d(1+i^d)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \\
\frac{D}{P} &= \gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} \\
\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha} &= \left[\psi\left(i^l+\delta\right)^{1-\theta^k} + (1-\psi)\left(1/\beta - 1 + \varrho + \delta\right)^{1-\theta^k}\right]^{\frac{1}{1-\theta^k}} \\
K^P &= \psi\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k} K \\
K^{NP} &= (1-\psi)\left(\frac{1/\beta - 1 + \varrho + \delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{-\theta^k} K \\
\epsilon^d &= \frac{n-1}{n}\varepsilon^d + \frac{\theta}{n} - \frac{\gamma_d}{n}\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}\left[\theta - \frac{1}{b-1}\right] \\
1 + i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1/\beta - \mu^d) \\
\epsilon^l &= \left\{\frac{n-1}{n}\varepsilon^l + \frac{\theta^k}{n} - \frac{\psi}{n}\left(\frac{i^l+\delta}{\alpha\frac{\varphi-1}{\varphi}\left(\frac{N}{K}\right)^{1-\alpha}}\right)^{1-\theta^k}\left[\theta^k - \frac{1}{1-\alpha}\right]\right\}\frac{1+i^l}{i^l+\delta} \\
1 + i^l &= \frac{\epsilon^l}{\epsilon^l-1}\left[1/\beta + \mu^l + \kappa\nu\left(\ln\left(\frac{K^P}{F/P}\right) - \ln(\nu)\right)\right] \\
\frac{\varsigma}{\omega} + \Psi\left(\frac{K^P}{F/P}\right) &= \left(\frac{1}{\beta}-1\right)\left(1 + \frac{D}{F} - \frac{K^P}{F/P}\right) + (i^l - \mu^l)\frac{K^P}{F/P} - (\mu^d + i^d)\frac{D}{F} \\
(1-g)K^{\alpha}N^{1-\alpha} &= C + \delta K^P + \delta K^{NP} + \mu^l K^P + \mu^d\frac{D}{P} + \varsigma\frac{F}{P} + \Psi\left(\frac{K^P}{F/P}\right)\frac{F}{P} \\
&+ \varrho K^{NP} + a\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{b}{b-1}} - \frac{D}{P} - \gamma_m\left(\frac{1+i^m}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} \\
&- \gamma_{cbdc}\left(\frac{1+i^{cbdc}}{1+i^{\mathcal{L}}}\right)^{\theta}\left(\frac{\beta(1+i^{\mathcal{L}})}{ab}\right)^{\frac{1}{b-1}} - q
\end{aligned}
$$

This is a system of 12 equations in 12 unknowns: N, C, $i^{\mathcal{L}}$, i^d, D/P, K, K^P, K^{NP}, i^l, ϵ^d, ϵ^l, F/P. Recall that i^{cbdc} would be given by an assumption (like $i^{cbdc} = 0$ in the case of our baseline calibration), and the number of banks (n) would also given.

Appendix B.10 Welfare Change Measure

We define the (multiplicative) consumption equivalent variation required to keep the representative household indifferent between an initial scenario (for example the pre-CBDC deterministic steady state) and a new scenario (for example the post-CBDC deterministic steady state) to be the scalar ζ that satisfies the following equation:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t^{POST}) - v(N_t^{POST}) \right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(\zeta C_t^{PRE}) - v(N_t^{PRE}) \right].$$

For example, if the scalar, ζ, that satisfies the previous equation comes out to be 1.0030, this indicates that the representative household needs to be given 0.3% of its initial-scenario consumption path to be indifferent between the initial and final scenarios. In the case where $u(\cdot) = \ln(\cdot)$, and when we are comparing two steady states, the previous equation becomes:

$$\begin{aligned} \ln(\overline{C}^{POST}) - v(\overline{N}^{POST}) &= \ln(\zeta \overline{C}^{PRE}) - v(\overline{N}^{PRE}) \\ \zeta &= \exp\{[\ln(\overline{C}^{POST}) - v(\overline{N}^{POST})] - [\ln(\overline{C}^{PRE}) - v(\overline{N}^{PRE})]\} \end{aligned}$$

In our exposition, when comparing the pre-CBDC and the post-CBDC steady states, we refer to $(\zeta - 1) \cdot 100$ as the "welfare change from CBDC introduction".

Appendix C Additional Results and Robustness

Appendix C.1 Welfare Across Kappa and Theta

Figure C.1 plots the welfare change between the pre-CBDC scenario and the post-CBDC scenario for the baseline specification (where CBDC pays an interest rate of zero percent once it is introduced), but for different levels of κ (the importance of bank equity for lending) and different levels of θ^k (the elasticity of substitution between pledgeable and non-pledgeable capital). As the importance of bank equity for lending increases, the welfare gain from introducing CBDC goes down. This makes sense because "dis-intermediating" banks, by lowering their profitability through the introduction of CBDC, decreases lending more when κ is high. Recall our baseline value is $\kappa = 12$ basis points.

Across the different lines, we see that when θ^k is higher (the blue line), the welfare gains from introducing CBDC are higher (except for $\kappa = 0$). This is also to be expected, because when the substitutability between bank and non-bank intermediation is higher, then firms can more easily switch between bank and non-bank borrowing when banks are dis-intermediated, and the detrimental aspects of CBDC introduction are muted (leading to higher overall welfare gains).

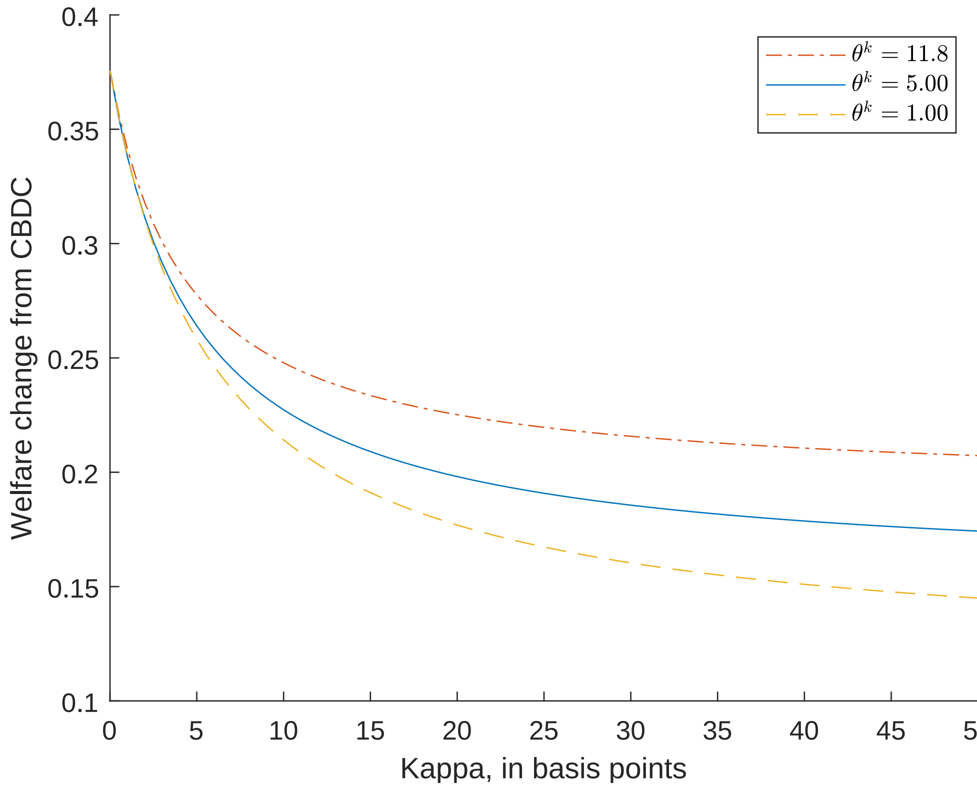

Figure C.1: This figure shows the welfare change (gain if positive, loss if negative) from CBDC introduction for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital.

Appendix C.2 Transition Between Steady States

Figures C.2 and C.3 depict the transition between the pre-CBDC and the post-CBDC steady state for several variables of interest. The transitions use our baseline calibration and a CBDC that pays an interest rate of zero percent. In orange, we have the initial steady state, in the dashed yellow line we have the final (post-CBDC) steady state, and in blue with have the transition between the two.

We can see that labor falls between the initial and final steady state, and it actually falls by more than that in the transition. Similarly, consumption increases between the initial and final steady state, and increases even more in the transition. This is possible because aggregate capital actually contracts in the new steady state (so the transition has disinvestment). Final output is lower in the new steady state, both due to the lower labor and lower capital. Nevertheless, consumption can end up higher because government spending, investment, and waste all fall in the new steady state, and allow consumption to be higher despite the lower final output.

Deposits, and the share of deposits in liquidity all fall in the new steady state due to the introduction of CBDC, but not by much. The fraction of deposits in liquidity ($\omega^d_{\mathcal{L}}$) falls from 80% to 74%. The loan rate increases by roughly 0.1% in the new steady state, due to commercial banks having less equity, as can be seen in the bottom right panel of Figure C.3. Both the deposit rate and the rate on liquidity increase substantially in the new steady state as can be seen from the bottom row of Figure C.2.

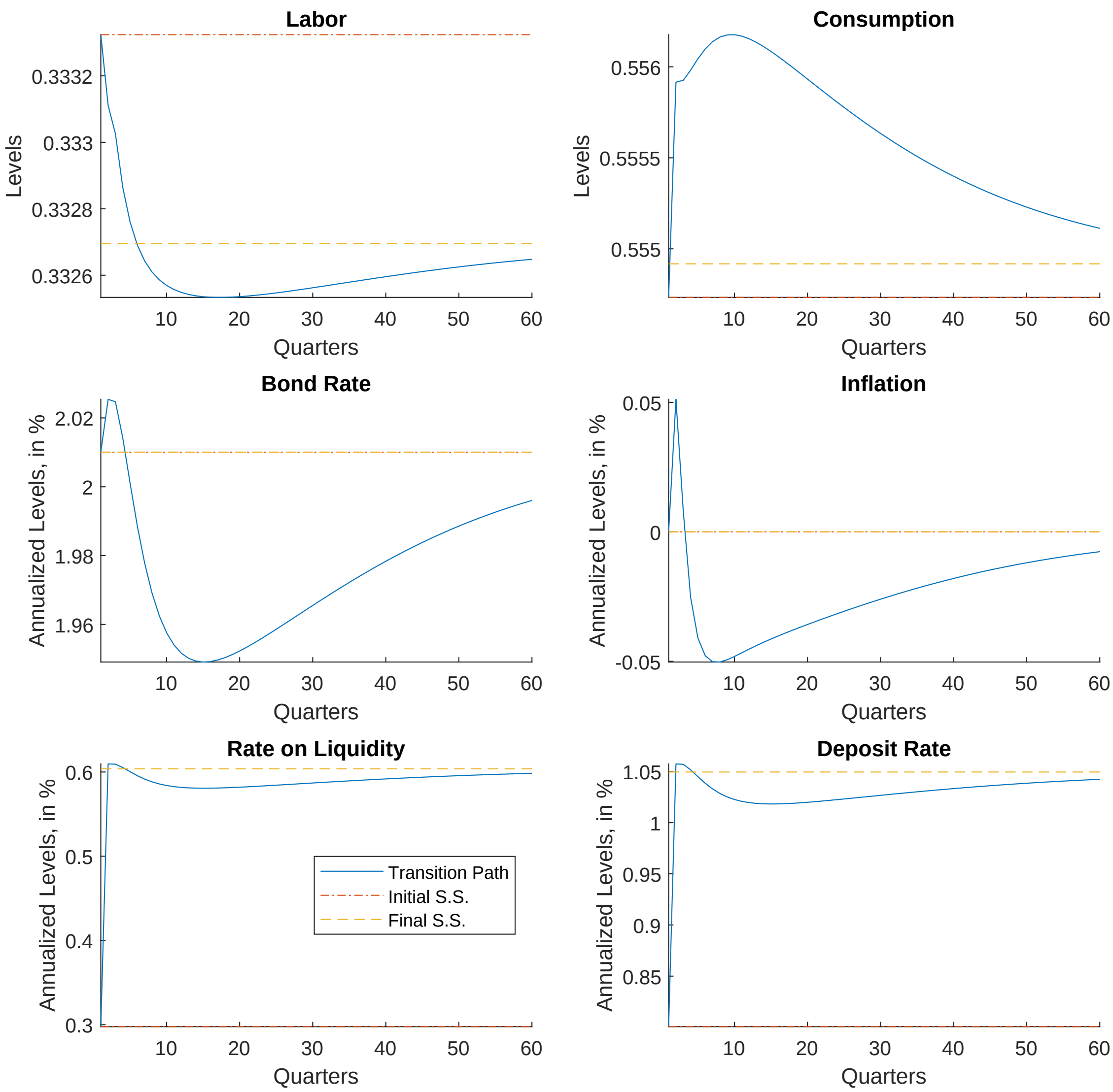

Figure C.2: This figure depicts the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for several variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

Importantly, even though banks pay a higher deposit rate (due to the greater competition with CBDC) and they have less equity in the new steady state, they also charge a higher loan rate and pay less operating costs in the new steady state (due to having less equity, recall that their operating costs are given as a fraction of their equity). Overall, their return on equity is essentially unchanged between the initial and final steady state. This alleviates concerns that our model is missing an entry margin in response to changes in bank profitability that could potentially change the results. Overall, labor falls by 0.18%, and consumption increases by around 0.04%. Overall, welfare is approximately 22 basis points higher in the post-CBDC steady state than in the initial (pre-CBDC steady state).

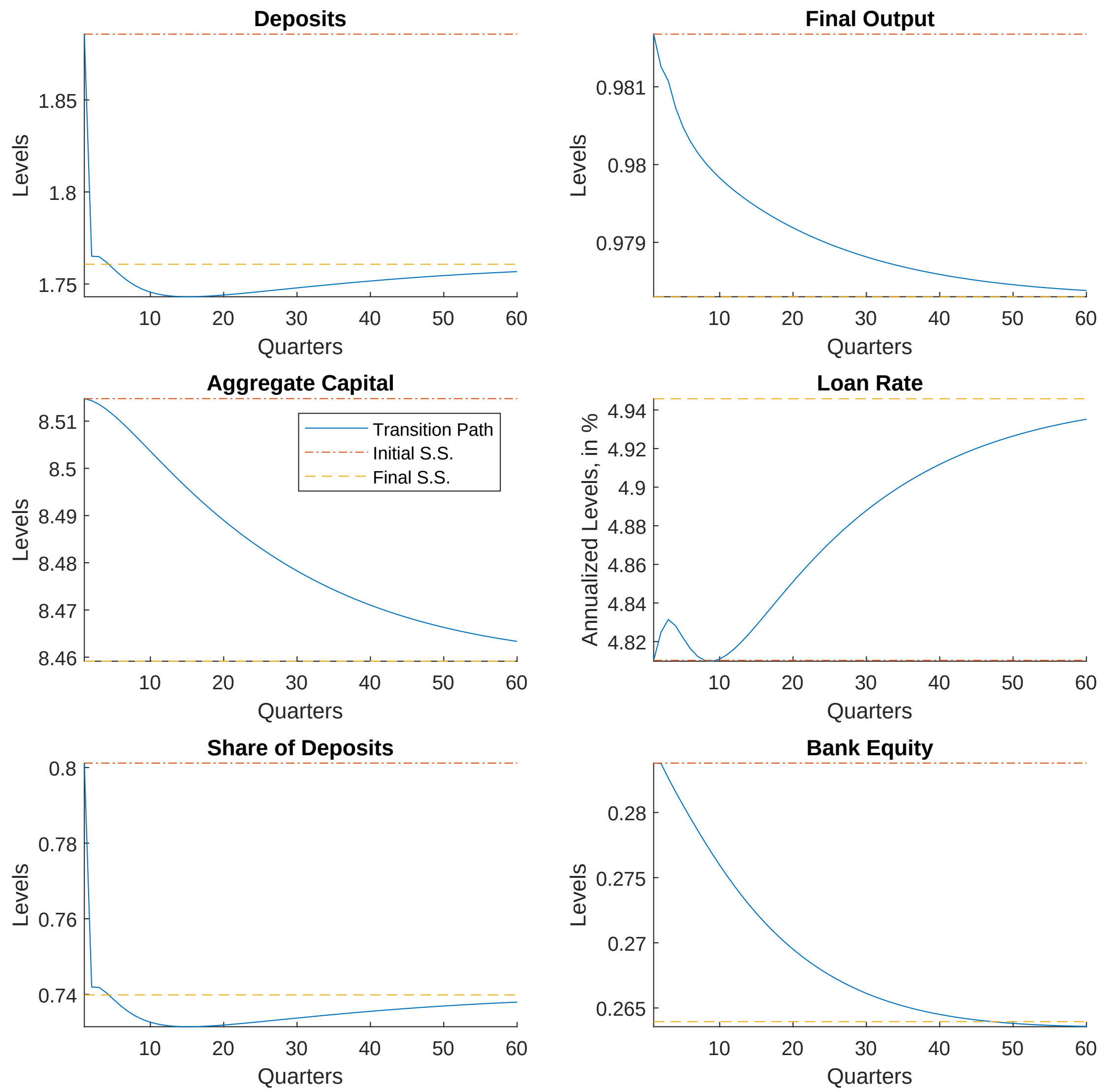

Figure C.3: This figure continues Figure C.2, depicting the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for additional variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

Appendix C.3 IRFs to a Technology Shock

Figure C.4 presents the IRFs of the economy to a 25-basis-points positive productivity shock, ϵ_t^a, with a persistence of 0.95 (see equation (3.26) for the law of motion of the technology shock). While the response of the economy to a technology shock is obviously different than the one to a monetary policy shock depicted in Figure 5.9, our main conclusion that the response to the shock is very similar across different remuneration schemes for CBDC is preserved.

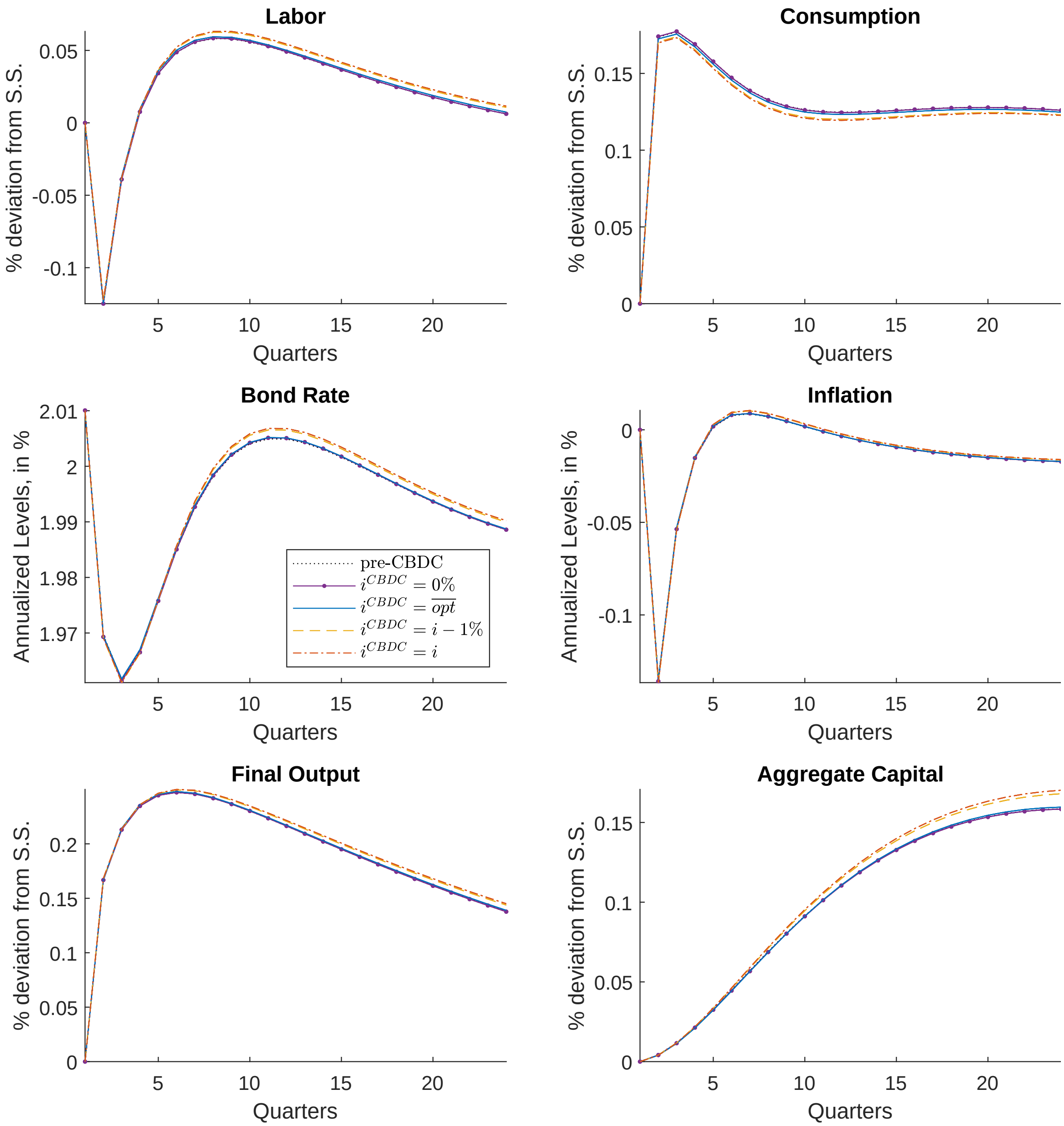

Figure C.4: This figure depicts the IRFs to a 25 basis points positive productivity shock, with a persistence of 0.95, for different CBDC remuneration schemes.

Appendix C.4 Robustness to Recalibrating Additional Parameters

Sections 5.2 and 5.3 analyzed several CBDC-related outcomes for different levels of the policy rate. In those sections, only the discount factor, β, was changing to generate the different levels of the policy rate, and no other underlying parameters where changing along with it. In this section, along with the discount factor, we vary additional parameters to continue to match some targets which we matched in our baseline calibration. Namely, we recalibrate the values of the disutility of labor parameter χ, the q parameter in the liquidity-cost function Φ, the exogenous elasticity of substitution between different banks in loans ε^l, the

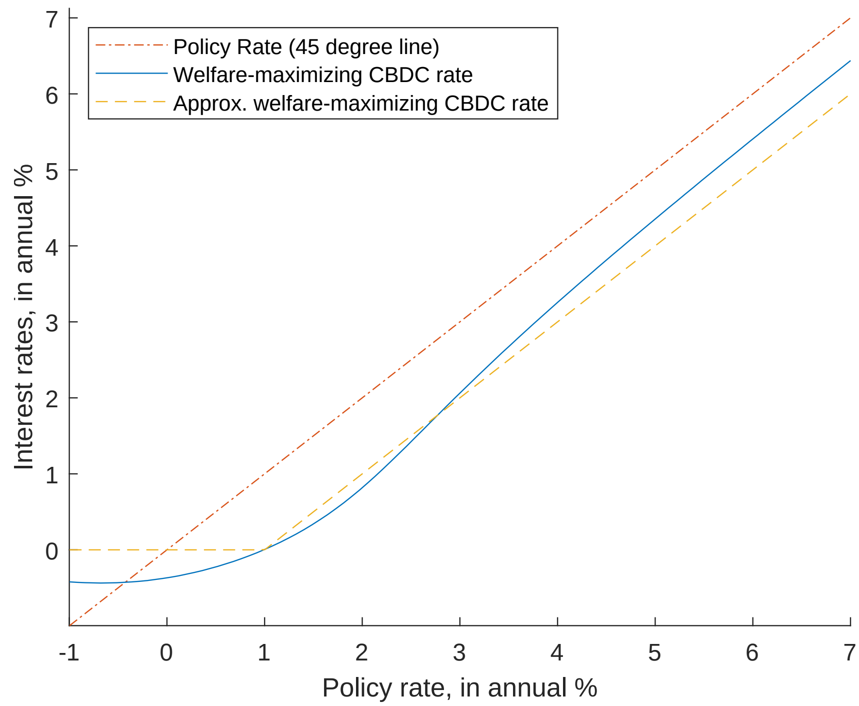

Figure C.5: This figure displays the policy rate, in orange (in both axes, so it is the 45 degree line), the welfare-maximizing level of the CBDC rate, in blue, and an approximate welfare-maximizing rule of thumb rate which is the maximum between 0 and the policy rate minus 1%, in yellow.

managerial cost of operating the bank ς, and the fraction of bank profits that stay in the bank ω. We do so to continue to match the following targets in different steady states associated with different policy-rate levels: 1) labor is equal to one third, 2) $\Phi(\cdot) = m + d + cbdc$, 3) the endogenous share of loans in firm borrowing is equal to the exogenous share, $\omega_K^B = \psi$, 4) Banks are at their loan-to-equity ratio target, $\mathcal{L}/F = \nu$, and 5) bank return on equity is 2.25% quarterly.

The results that we obtained in Sections 5.2 and 5.3 are qualitatively robust to this recalibration. Quantitatively, the results do change but to a fairly small degree. As an illustration, we reproduce Figure 5.5, but now with the recalibration of the aforementioned parameters. Figure C.5 provides the results. The orange dash-dot line for the policy rate and the yellow dashed line for the rule-of-thumb CBDC rate are still the same as those in Figure 5.5. The blue line is now different, and increases a lit bit more steeply with the policy rate, but the differences are fairly small. Further results with this recalibration procedure are available upon request.